Effect of Feshbach resonances on quasi-one-dimensional collisions

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Outline

Renormalization of scattering parameters in a confined geometry.

T matrix for resonant scattering under cylindrical harmonic confinement.

Two-channel one-dimensional problem.

Scattering: confinement vs. 1D and 3D.

Bound states: confinement vs. 1D and 3D.

$$\hbar = 1$$

NON-RESONANT SCATTERING UNDER CYLINDRICAL CONFINEMENT

Olshanii, 1998

$$E\psi(\mathbf{r}) = \left[-\frac{1}{m}\frac{\partial^2}{\partial z^2} + \hat{H}_\perp + \hat{V}_{Fermi}(r)\right]\psi(\mathbf{r}),$$

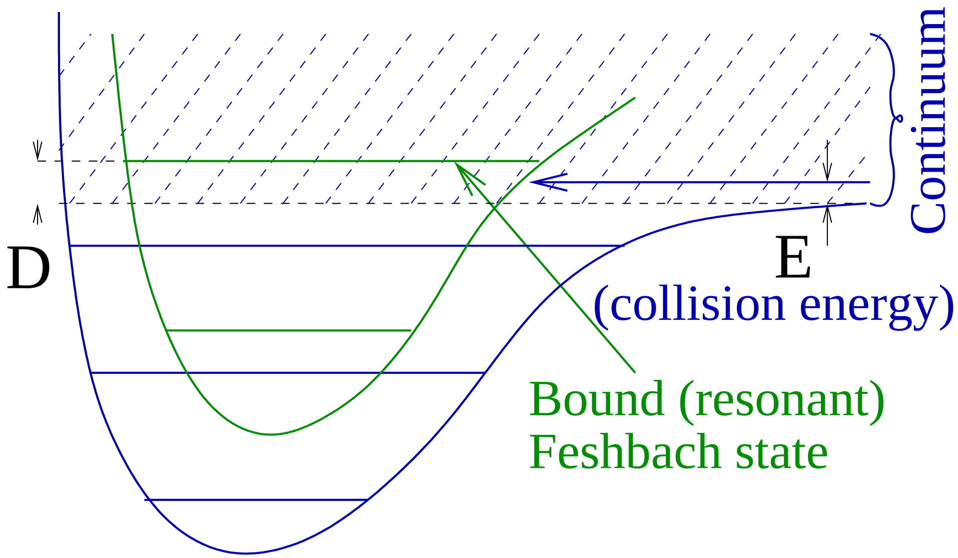
$$\hat{H}_\perp = -\frac{1}{m}\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2}\right) + \frac{m}{4}\omega_\perp^2\rho^2$$

$$\hat{V}_{Fermi}(r)\psi = \frac{4\pi}{m}a_{3D}\delta(r)\frac{\partial}{\partial r}(r\psi)$$ — Fermi pseudopotential

One-dimensional potential $$\underbrace{\frac{4a_{3D}}{ma_\perp^2}}\underbrace{\left(1 - C\frac{a_{3D}}{a_\perp}\right)^{-1}}\delta(z)$$

Transverse ground state average Confinement Induced Resonance

3D TWO-CHANNEL SCATTERING



$$\hbar = 1$$

$$E\psi_a = \left[-\frac{1}{m}\nabla^2 + V_a(\mathbf{r})\right]\psi_a + V_{am}^*(\mathbf{r})\,\psi_m$$

$$E\psi_m = \left[-\frac{1}{m}\nabla^2 + V_m(\mathbf{r})\right]\psi_m + V_{am}(\mathbf{r})\,\psi_a$$

$$V_{am}(r)\,\psi \sim \delta(r)\frac{\partial}{\partial r}(r\psi) \quad V_{am}^* \quad \text{??? Hermicity ???}$$

$$V(\mathbf{r}) \sim \delta(\mathbf{r})$$

One-channel Schroedinger equation with an energy-dependent potential:

$$E\psi_a = \left[-\frac{1}{m}\nabla^2 + V_{\text{eff}}(E)\,\delta(\mathbf{r})\right]\psi_a, \quad V_{\text{eff}}(E) = V_a + \frac{|V_{am}|^2}{E-D}$$

RENORMALIZATION IN A FREE SPACE

$$E\psi_a = \left[-\frac{1}{m}\nabla^2 + V_{\text{eff}}(E)\,\delta(\mathbf{r})\right]\psi_a, \quad V_{\text{eff}}(E) = V_a + \frac{|V_{am}|^2}{E-D}$$

Ultraviolet divergence !!!

Renormalization procedure Kokkelmans, Milstein, Chiofalo, Walser, and Holland (2002)

$p_c \to \infty$ — momentum cutoff

Non-renormalized parameters are related to a_{3D}, B_0, Δ, and μ

$$V_a = \tfrac{4\pi}{m} a_{3D}\left(1 - \tfrac{2}{\pi} a_{3D} p_c\right)^{-1}$$

$$|V_{am}|^2 = \tfrac{4\pi}{m} a_{3D}\mu\Delta\left(1 - \tfrac{2}{\pi} a_{3D} p_c\right)^{-2}$$

$$D_{3D} = \mu\left[B - B_0 - \Delta + \Delta\left(1 - \tfrac{2}{\pi} a_{3D} p_c\right)^{-1}\right]$$

$$a_{\text{res}} = a_{3D}\left(1 - \tfrac{\Delta}{B - B_0}\right)$$

$$T_{\mathrm{free}}\left(p_0\right) = -i\frac{4\pi}{m}a_{3D}\frac{p_0^2 - mD_{3D}}{a_{3D}p_0^3 - ip_0^2 - ma_{3D}D_{3D}p_0 + im\left(D_{3D} + \mu\Delta\right)}.$$

RENORMALIZATION UNDER CONFINEMENT

(cond-mat/0308465)

One-channel Schroedinger equation

$$E\psi_a(\mathbf{r}) = \left[-\frac{1}{m}\frac{\partial^2}{\partial z^2} + \hat{H}_\perp + V_{\text{eff}}(E)\,\delta(\mathbf{r})\right]\psi_a(\mathbf{r}),$$

Transverse Hamiltonian

$$\hat{H}_\perp = -\frac{1}{m}\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2}\right) + \frac{m}{4}\omega_\perp^2\rho^2$$

The eigenstates of $\hat{H}_\perp$ — two-dimensional harmonic oscillator

$$\hat{H}_\perp|Nm_z\rangle = (N + |m_z| + 1)\,\omega_\perp|Nm_z\rangle,$$

The matrix elements of the interatomic interaction

$$\langle 2n'm_z'|V_{\text{eff}}\delta(\mathbf{r})|2n, m_z\rangle = \frac{1}{\pi a_\perp^2}V_{\text{eff}}\delta_{0m_z}\delta_{0m_z'}\delta(z)$$

are independent of n and n'. Other matrix elements are zero.

Expansion in momentum (axial) and oscillator (transverse) modes

$$\psi_a(\mathbf{r}) = (2\pi)^{-1/2} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} dq \tilde{\psi}_n(q)\, e^{iqz} |2n0\rangle .$$

Collision of two atoms in a transverse state n with a relative axial momentum $p_n = \sqrt{m\left[E - (2n+1)\,\omega_\perp\right]}$

$$\tilde{\psi}_{n'}(q) = \delta(q - p_n)\,\delta_{n'n} + \frac{m}{p_{n'}^2 - q^2 + i0} \frac{1}{2\pi} T_{n'n}(p_0) .$$

The T matrix $T_{n'n}(p_0)$ satisfies the Lippmann-Schwinger equations

$$T_{n'n}(p_0) = \frac{1}{\pi a_\perp^2} V_{\text{eff}}(E) \left[1 - \frac{i}{2} m \sum_{n''=0}^{n_c} \frac{1}{p_{n''}} T_{n''n}(p_0)\right] ,$$

n_c — the level cutoff

$T_{n'n}(p_0)$ is independent of n and n', $T_{n'n}(p_0) = T_{\rm conf}(p_0)$

$$T_{\rm conf}(p_0) = \frac{4}{ma_\perp \left\{ \frac{4\pi a_\perp}{mT_{\rm free}(p_0)} + \sum_{n=0}^{n_c} \left[n - (p_0 a_\perp/2)^2 \right]^{-1/2} - \frac{2}{\pi} p_c a_\perp \right\}}$$

Finite limit at

$n_c \to \infty, p_c \to \infty$, $a_\perp p_c = \pi \sqrt{n_c - (p_0 a_\perp/2)^2}$

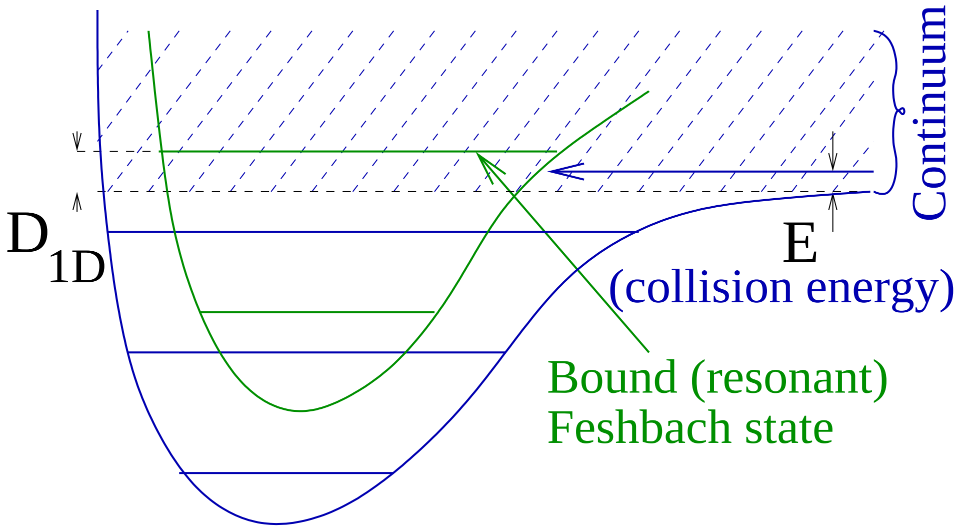
$$T_{\rm conf}(p_0) = \frac{4}{ma_\perp \left[\frac{4\pi a_\perp}{mT_{\rm free}(p_0)} + \zeta\left(\frac{1}{2}, -\left(\frac{a_\perp p_0}{2}\right)^2\right)\right]}$$

ζ — Hurwitz zeta function.

Non-resonant scattering — Bergeman, Moore, and Olshanii (2003)

$$a_{3D} \Longleftarrow \frac{m}{4\pi} T_{\rm free}(p_0)$$

ONE-DIMENSIONAL TWO-CHANNEL PROBLEM



$$E\varphi_a(z) = \left[-\frac{1}{m}\frac{d^2}{dz^2} + U_{1D}\delta(z)\right]\varphi_a(z) + g_{1D}^*\delta(z)\varphi_m$$

$$E\varphi_m = D_{1D}\varphi_m + g_{1D}\varphi_a(0)$$

$$E\varphi_a(z) = \left[-\frac{1}{m}\frac{d^2}{dz^2} + U_{\text{eff}}(E)\delta(z)\right]\varphi_a(z), \quad U_{\text{eff}}(E) = U_{1D} + \frac{|g_{1D}|^2}{E - D_{1D}}$$

Wavefunction in the momentum representation

$$\tilde{\varphi}_0(q) = (2\pi)^{-1/2}\int\limits_{-\infty}^{\infty} dy\,\psi_0(y)\exp(-iqy) = \delta(p_0 - q) + \frac{m}{p_0^2 - q^2 + i0}\frac{1}{2\pi}T_{1D}(p_0)$$

One-dimensional T matrix $\qquad T_{1D}(p_0) = U_{\text{eff}}(E)\left[1 + \frac{im}{2p_0}U_{\text{eff}}(E)\right]^{-1}$

EXPLICIT EXPRESSIONS FOR 1D SCATTERING PARAMETERS

$$p_0 \to 0 : \quad T_{conf}(p_0) \sim T_{1D}(p_0)$$

$$\Downarrow$$

$$U_{\text{eff}}(E) = \frac{1}{\pi a_\perp^2} T_{\text{free}}(E) \left[1 - \frac{Cm}{4\pi a_\perp} T_{\text{free}}(E) \right]^{-1}$$

$$U_{1D} = \frac{4 a_{3D}}{m a_\perp^2} \left(1 - C \frac{a_{3D}}{a_\perp} \right)^{-1} \text{ — Olshanii formula}$$

$$g_{1D}^2 = \frac{2 a_{3D} \mu \Delta}{m a_\perp^2} \left(1 - C \frac{a_{3D}}{a_\perp} \right)^{-2}$$

$$D_{1D} = \mu \left[B - B_0 + \underbrace{C \frac{a_{3D} \Delta}{a_\perp} \left(1 - C \frac{a_{3D}}{a_\perp} \right)^{-2}} \right] - \omega_\perp$$

Shift of Feshbach resonance due to the confinement induced resonance

SCATTERING UNDER CONFINEMENT

DIMENSIONLESS PARAMETERS

$$k = \frac{p_0 a_\perp}{2} = \sqrt{\frac{E}{2\omega_\perp} - \frac{1}{2}} \quad \text{—scattering momentum}$$

$$a = \frac{a_{3D}}{a_\perp} \quad \text{— scattering length}$$

$$b = \mu \frac{B - B_0}{2\omega_\perp} - \frac{1}{2} \quad \text{— detuning}$$

$$d = \frac{a_{3D}\mu\Delta}{2a_\perp\omega_\perp} \quad \text{— resonance strength}$$

Confined T matrix

$$T_{\text{conf}} = \frac{4}{m\omega_\perp} \frac{ak^2 - ab + d}{\left[ak^2 - ab + d\right]\zeta\left(1/2, -k^2\right) + k^2 - b}.$$

Small k — one-dimensional T matrix

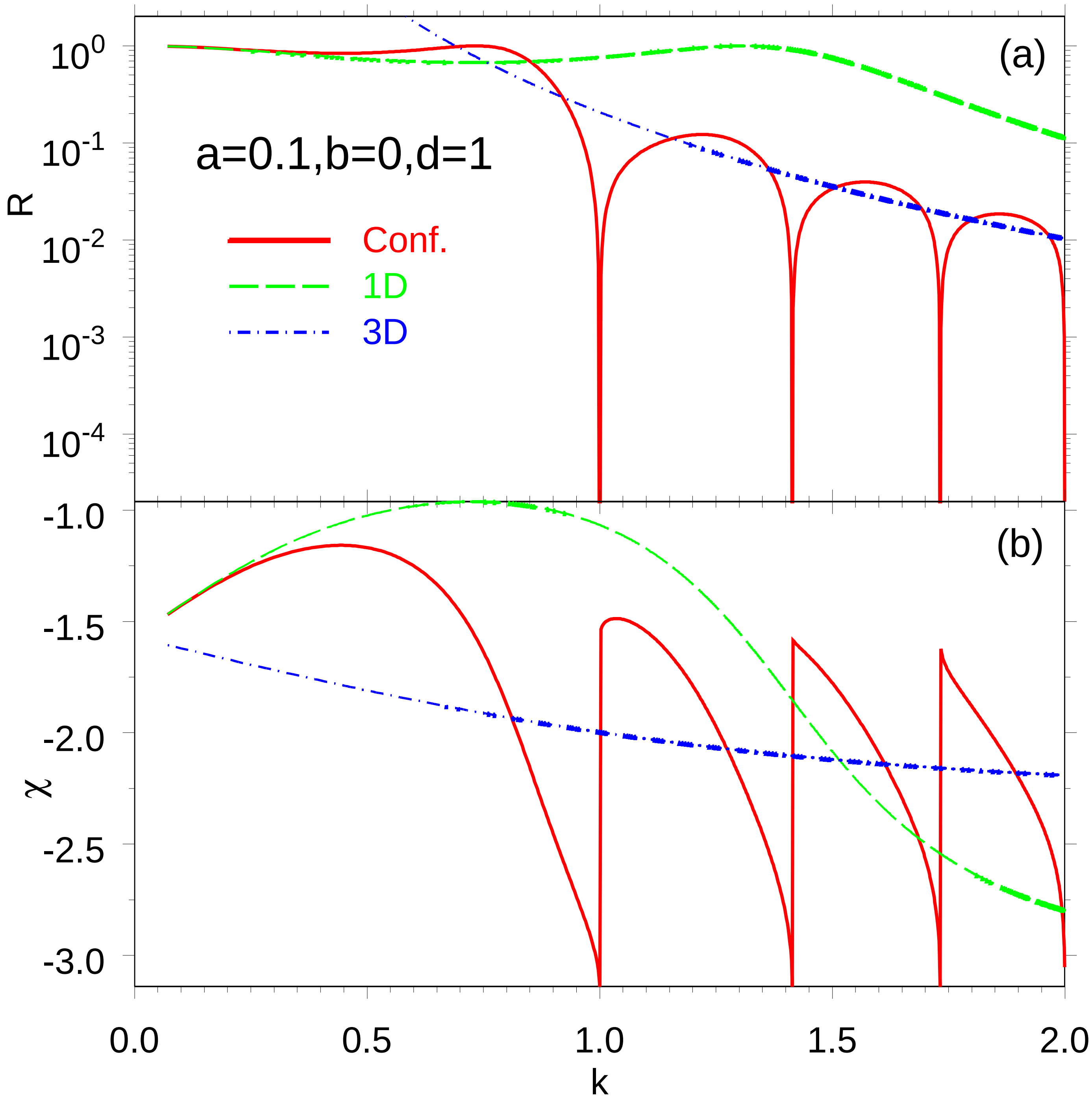
$$T_{1D} = \frac{4}{m\omega_\perp} \times \frac{ak^3 - (ab + d)\,k}{(1 - Ca)\,k^3 + iak^2 - [b\,(1 - Ca) + Cd]\,k - i\,(ab - d)}.$$

High k — three-dimensional T matrix

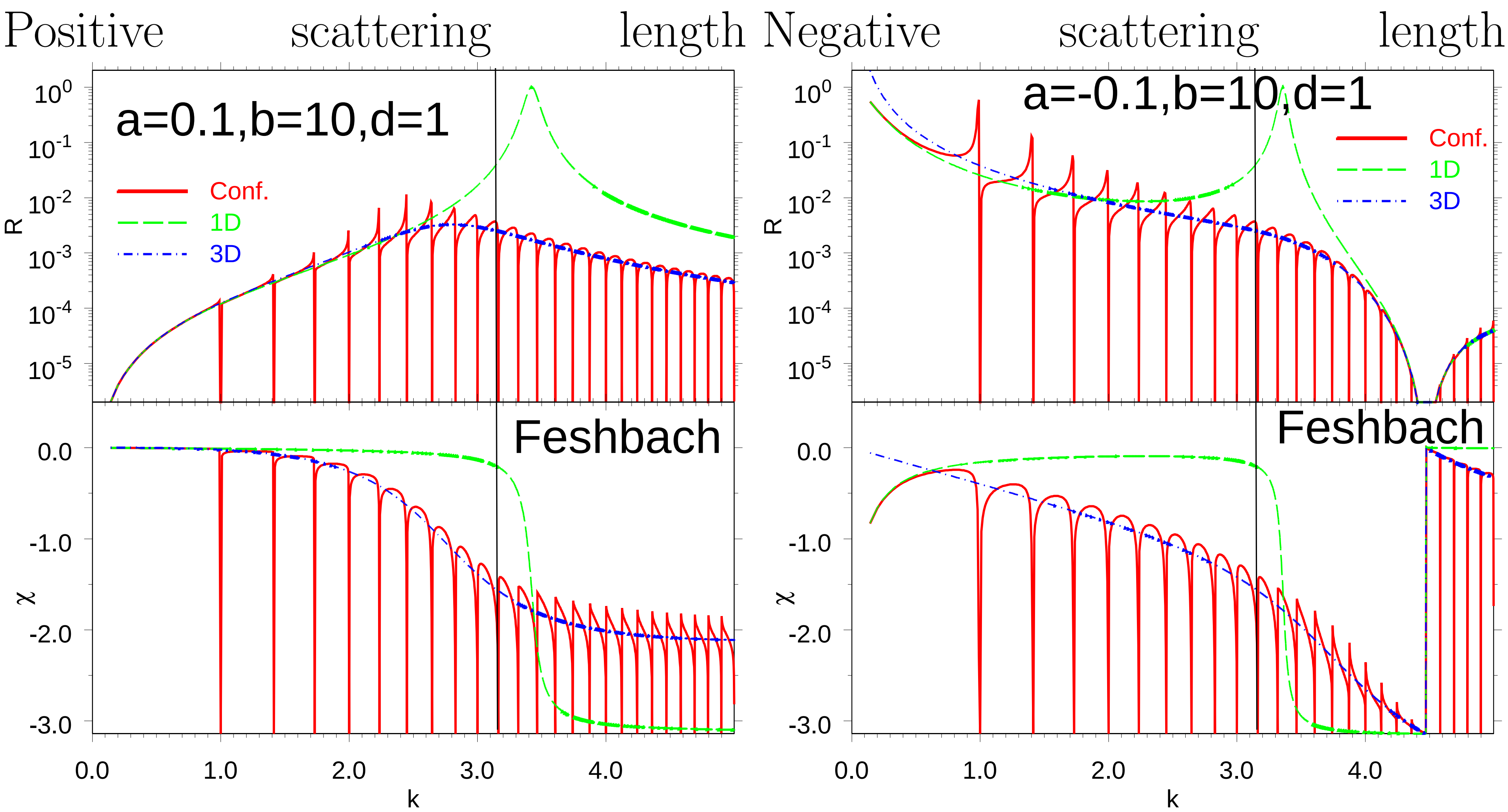
$$T_{3D} = -i\frac{4}{m\omega_\perp} \frac{ak^2 - ab + d}{2ak^3 - ik^2 - 2\,(ab - d)\,k + ib} \sim T_{\text{free}}$$

$$R = |f_{\text{even}}|^2 = |\frac{m}{2p_0} T_{\text{conf}}|^2 \quad \text{— reflection probability}$$

$$\chi = \arg T_{\text{conf}}\,(k) - \frac{\pi}{2} \quad \text{— phase of the scattering amplitude}$$



(a)
a=0.1,b=0,d=1
Conf.
1D
3D
R
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
(b)
χ
-1.0
-1.5
-2.0
-2.5
-3.0
0.0
0.5
1.0
1.5
2.0
k



Positive scattering length
Negative scattering length
a=0.1,b=10,d=1
a=-0.1,b=10,d=1
Conf.
1D
3D
Feshbach
R
χ
k

BOUND STATES

Poles of $T_{\text{conf}}(p_0)$ on the positive imaginary axis of p_0
$x = -ik$, binding energy $\epsilon_b = x^2$

$$\frac{x^2+b}{ax^2+ab-d} = -\zeta\left(\frac{1}{2}, x^2\right), x > 0,$$

$b < d/a$ — two bound states; $b > d/a$ — one bound state

$b \to -\infty \quad \Rightarrow$ bound state in the closed channel

$|b| \to \infty \quad \Rightarrow$ bound state in nonresonant system [Bergeman, Moore, and Olshanii (2003)]

Shallow bound states ($\epsilon_b \ll 1$),

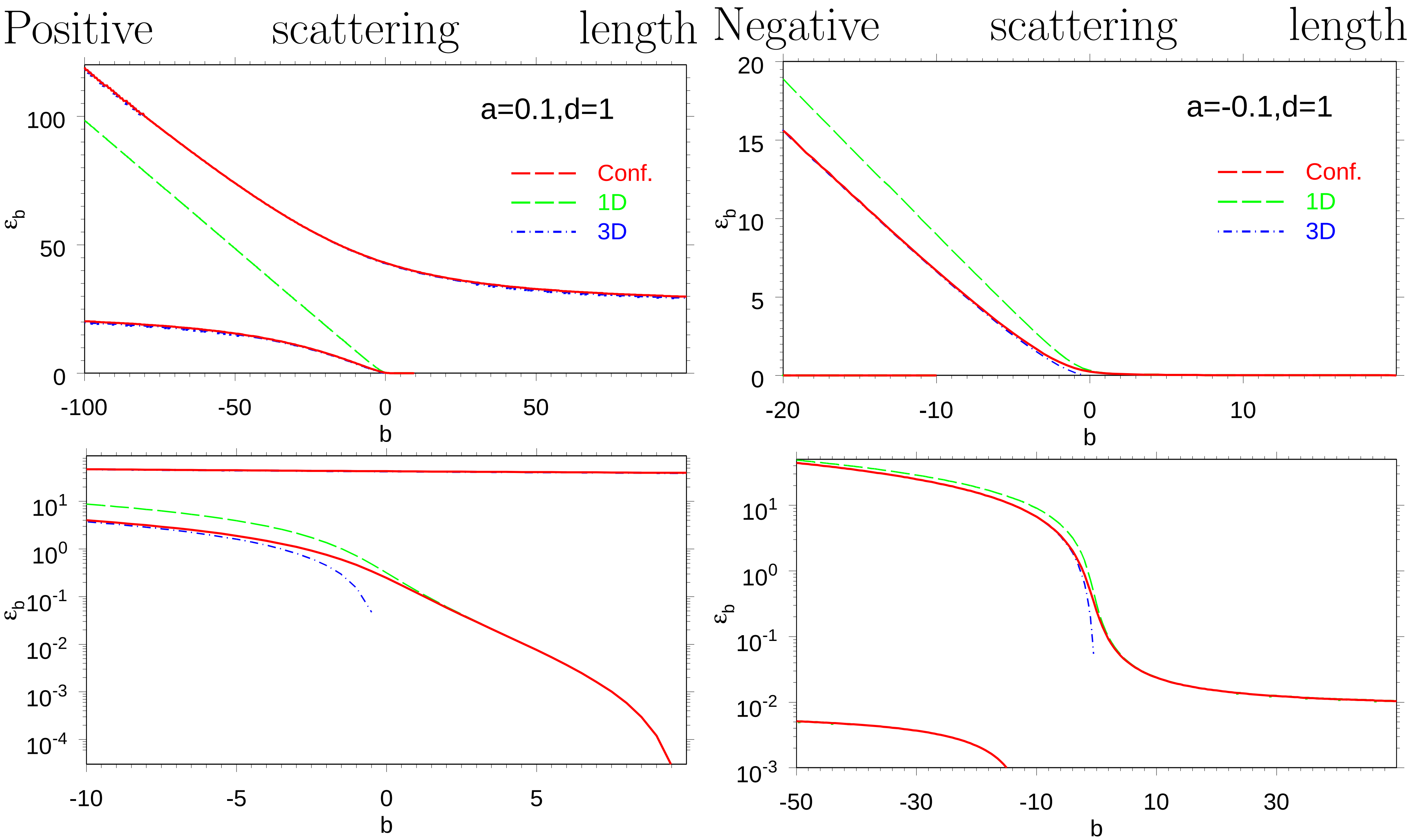
$$(1 - Ca)\,x^3 + ax^2 + [b\,(1 - Ca) + Cd]\,x + ab - d = 0$$

[As in one dimension, Kheruntsyan and Drummond (1998)]

Deep bound states ($\epsilon_b \gg 1$)

$$2ax^3 - x^2 + 2\,(ab - d)\,x - b = 0,$$

[As for free three-dimensional atoms, Green (2004), Julienne (2004), Drummond and Kheruntsyan (2004)]



Positive scattering length
Negative scattering length
a=0.1,d=1
a=-0.1,d=1
Conf.
1D
3D
Conf.
1D
3D
ε_b
b
ε_b
b
ε_b
b
ε_b
b

Conclusions

A problem of two-channel scattering of atoms under cylindrical harmonic confinement can be solved using a renormalization procedure.

One-dimensional scattering parameters can be related to three- dimensional ones.

Scattering amplitudes and bound states of the confined system incorporate both proprieties of one-dimensional and three-dimensional systems, as well as specific peculiarities.

Feshbach resonance is not just a change of an elastic scattering length !!!