

# Competing Phases in Cuprate Superconductors

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W. A. Atkinson, Phys. Rev. B **71** 024516 (2005)

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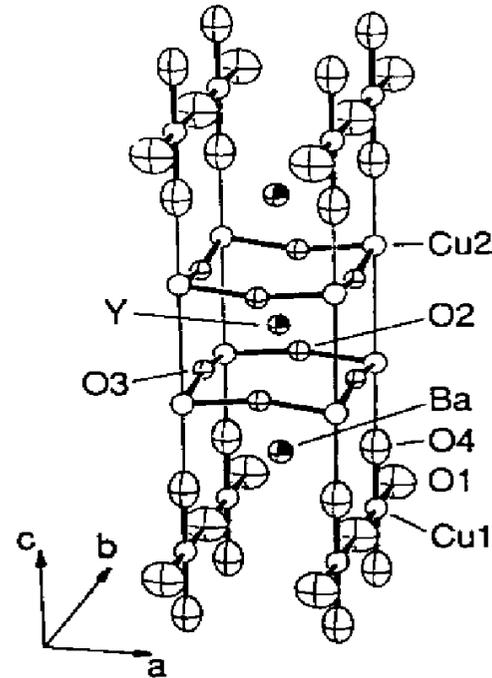
# Overview



- Introduction to High  $T_c$  Superconductors; Evidence for Coexisting Order.
- The calculations; self-consistent solutions.
- Excitation spectrum.

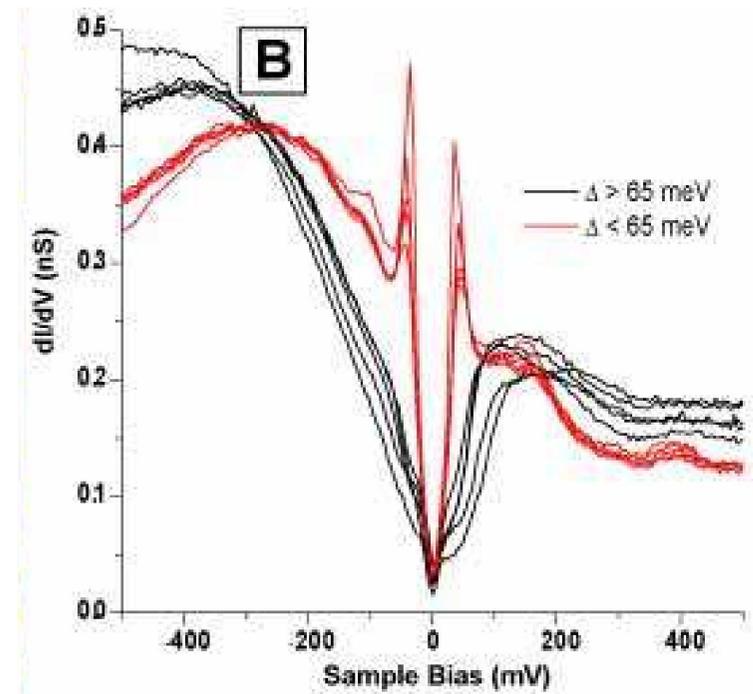
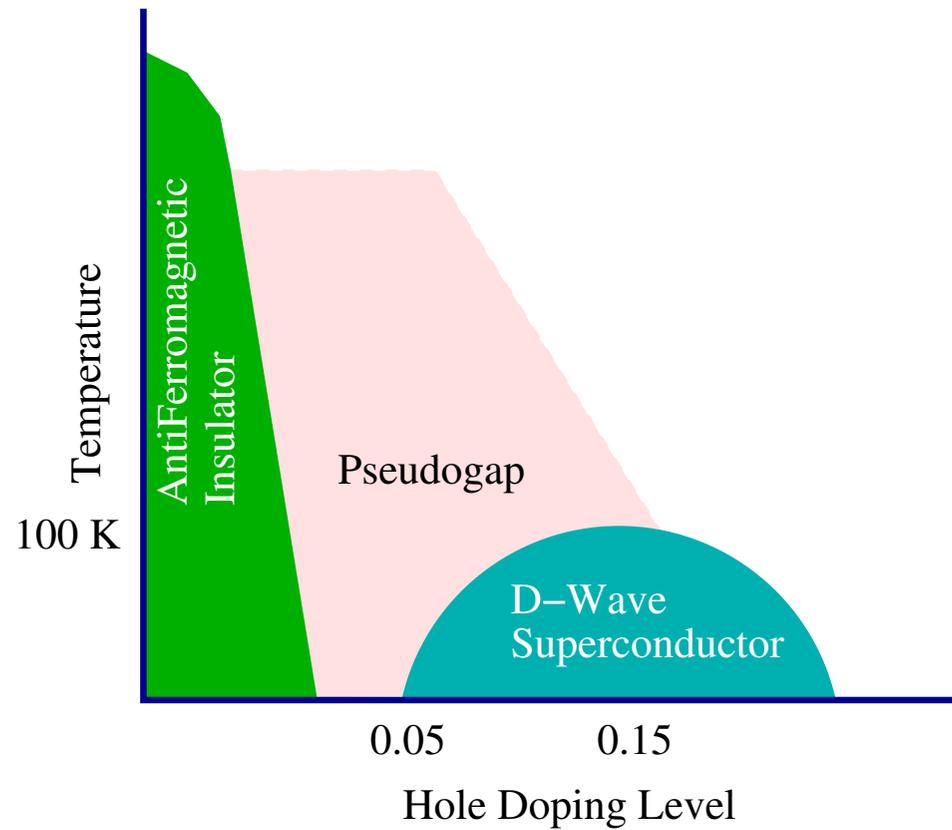
# Crystal Structure

- Electrons move within 2D Cu-O planes.
- Weak  $c$ -axis conductivity.
- Tune properties by chemical doping (O1 sites or Ca/Y substitution).



Narrow Bands & Low Dimensions  $\Rightarrow$  Strong Correlations

# Generic Phase Diagram



McElroy et al., cond-mat/0406491

# Evidence for Phases Coexisting with Superconductivity



**Stripes** (spin-charge separation). particularly in La-based cuprates, but also  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ .

**Glassy magnetic phases** (particularly in  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ )

**Weak charge modulations** in  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$

Tranquada et al., Nature **375**, 561 (1995);  
Mook et al., Nature **395**, 580 (1998).

Buyers et al. in  $\text{YBCO}_{6.35}$ ; BSCCO:  
Panagopoulos et al., Solid State Commun. **126**, 47 (2003).

McElroy et al., cond-mat/0406491; Howald et al., PRB **67** 014533 (2003).

Approach: take a simple model of competing AF and SC phases and study excitation spectrum.

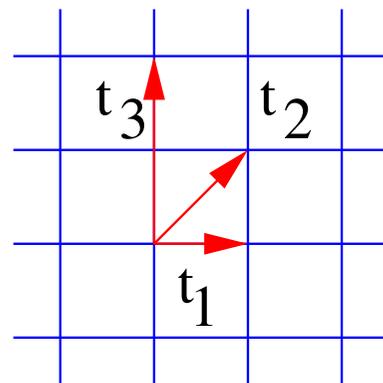
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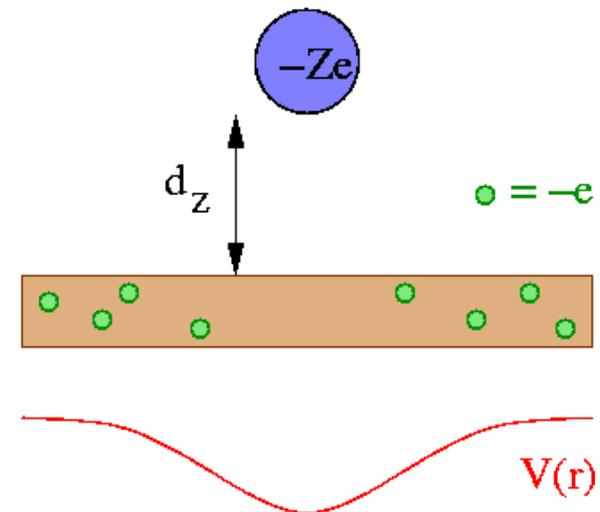
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# Mean-field Model

$$H = H_0 + H_{\text{int}}$$



$$t_1 = -1 \quad t_2 = 0.25 \quad t_3 = 0.1$$



$$H_0 = \sum_{i,j,\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} - Z \underbrace{\sum_{i,\mathbf{R}} V(\mathbf{r}_i - \mathbf{R}) \hat{n}_i}_{\text{Coulomb potential due to donors}}$$

Coulomb potential due to donors

$$H_{\text{int}} = \underbrace{\sum_i U \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}}_{\text{Hubbard}} + \underbrace{\frac{1}{2} \sum_{i \neq j} V(\mathbf{r}_i - \mathbf{r}_j) \hat{n}_i \hat{n}_j}_{\text{Long range Coulomb}} + \underbrace{\frac{J}{2} \sum_{\langle ij \rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j}_{\text{N.N. Exchange}}$$

$$n_{i\sigma} = \langle c_{i\sigma}^\dagger c_{i\sigma} \rangle \quad \Delta_{ij} = -\frac{3J}{4} \langle c_{j\downarrow} c_{i\uparrow} + c_{j\uparrow} c_{i\downarrow} \rangle$$

$$m_z(\mathbf{r}_i) = \frac{1}{2} e^{i\mathbf{Q} \cdot \mathbf{r}_i} (n_{i\uparrow} - n_{i\downarrow})$$

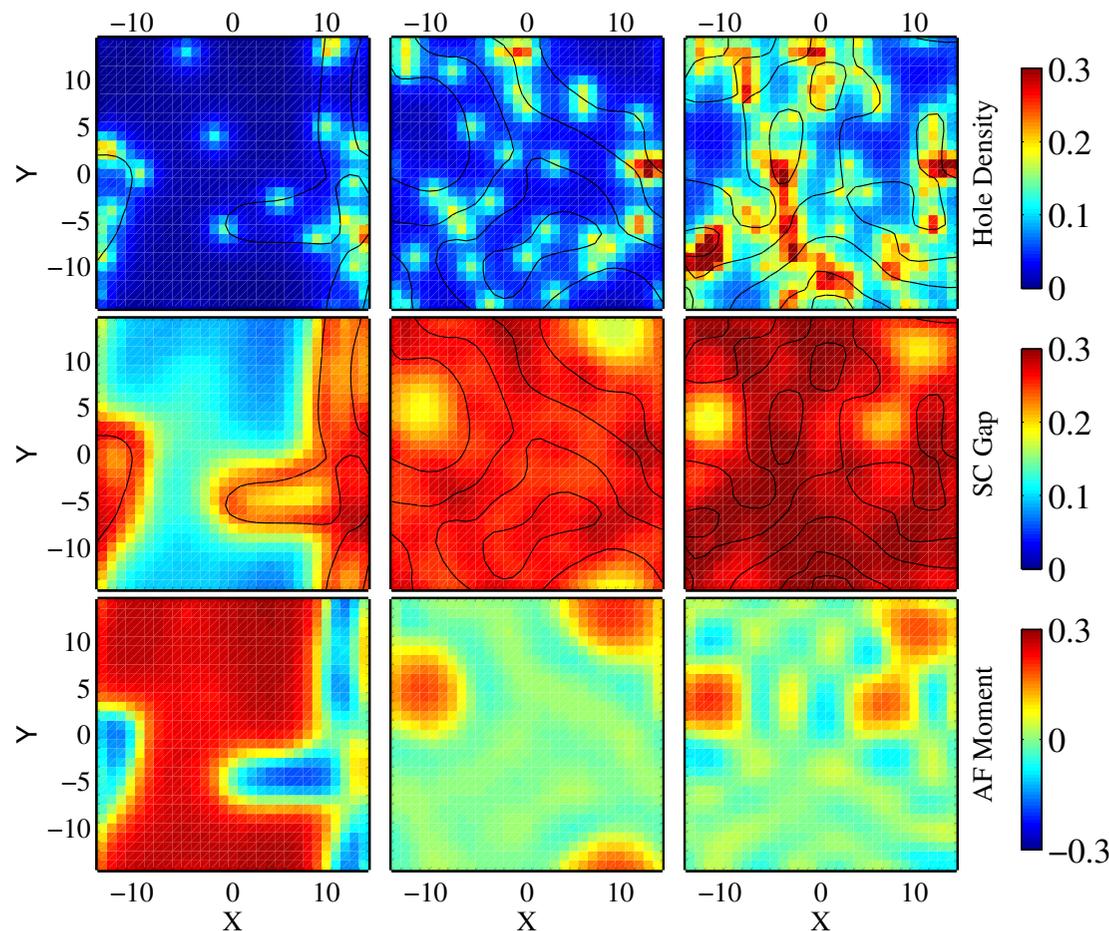
$$h(\mathbf{r}_i) = 1 - (n_{i\uparrow} + n_{i\downarrow})$$

## Comments about mean-field calculations



- In the neighbourhood of a negatively charged impurity, the local hole concentration is high. AF moments form preferentially in underdoped regions. **Local electronic spectrum is controlled by local potential.**  $\Rightarrow$  Other mechanisms possible. (eg. variations of effective mass)
- **RVB**  $\Rightarrow J \sim (1 - n/2)^{-2}$ . Calculations ignore variations of  $J$  (and therefore  $\Delta$ ) with doping.
- Missing dynamics (spin-waves, other collective modes)  $\Rightarrow$  tends to overestimate  $m_z$ .
- Calculations find **local** minimum of energy. Looking for glassy solutions.
- **Useful for studying large/disordered systems.**

# Self-Consistent Solutions



- SC order is suppressed by AF. Phases compete.
- AF phase is metallic (SDW  $\equiv$  AF here).
- Correlation length  $\xi_{AF} \sim t/U m_z$ .
- Substantial moment forms if distance  $d$  between domain walls is  $d > \xi_{AF}$ . However, moment is finite everywhere.
- AF moment is suppressed with doping by a proliferation of domain walls.

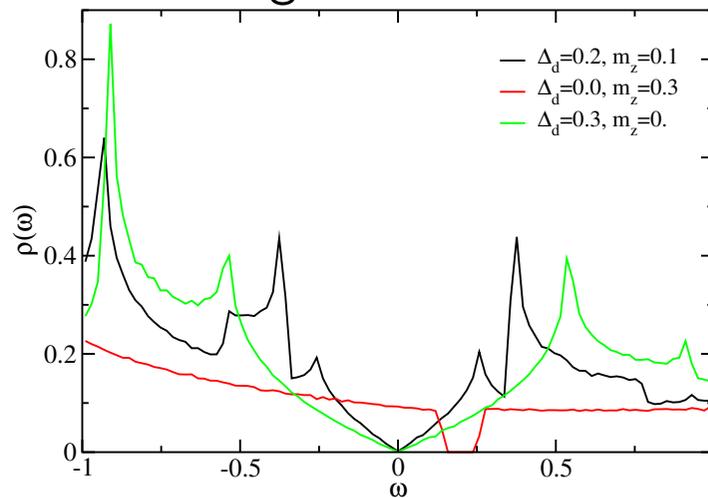
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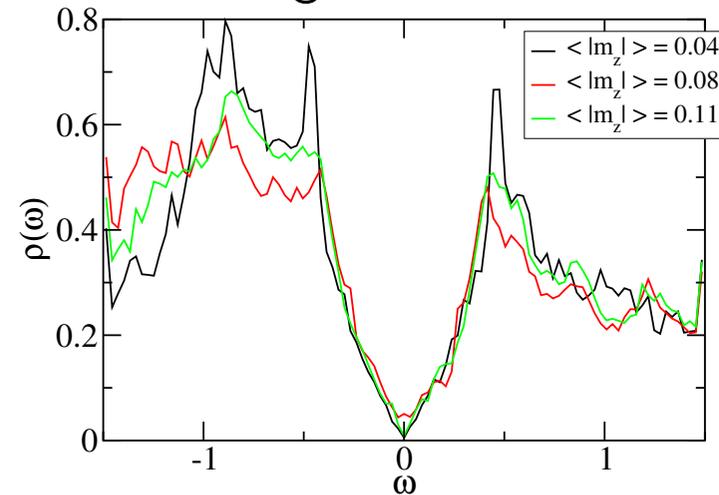
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# Density of States

## Homogeneous Order



## Inhomogeneous Order



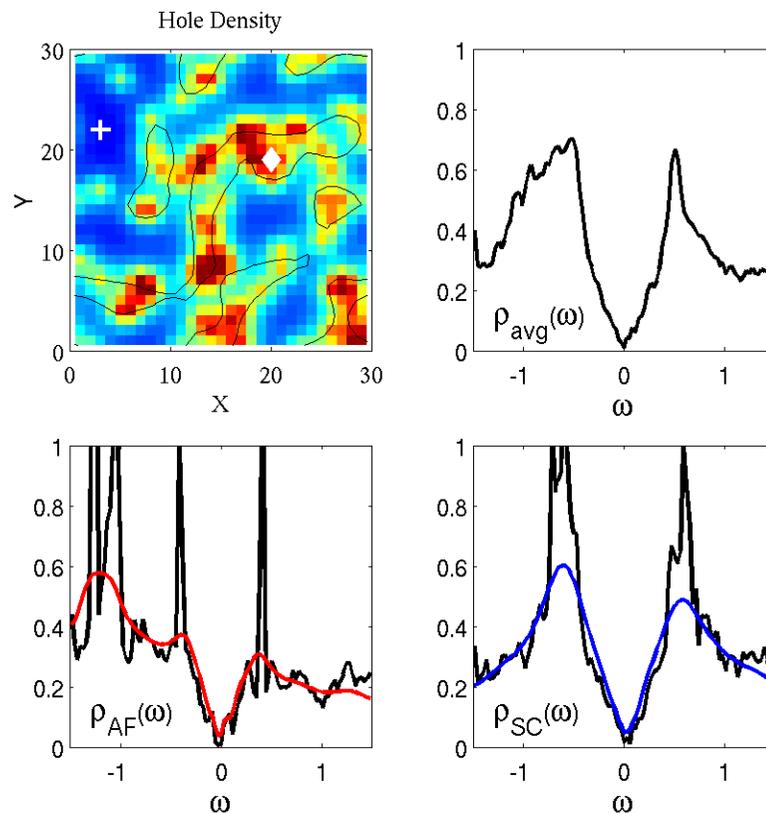
$\mathbf{H}_k =$

$$\begin{bmatrix} \epsilon_k & \Delta_k & -M & 0 \\ \Delta_k & -\epsilon_k & 0 & -M \\ -M & 0 & \epsilon_{k+Q} & \Delta_{k+Q} \\ 0 & -M & \Delta_{k+Q} & -\epsilon_{k+Q} \end{bmatrix}$$

$$M = U m_z$$

- Low energy DOS largely unaffected.
- Coherence peaks suppressed. Spectral weight shifted on energy scale  $M$ .

# Local Density of States



- Characteristic LDOS for different regions.
- Coherence peak weight is reduced in AF regions (wiped out by inelastic scattering).

$$\rho(\mathbf{r}_i, \omega) = \sum_{n, \sigma} \frac{|\Psi_{n, \sigma}(\mathbf{r}_i)|^2}{\omega - E_n - \Sigma(\omega)}$$

$$\Sigma(\omega) = \Gamma \omega^3 / (\omega^3 + \Delta^3)$$

## Conclusion

- Low  $T$  tunneling spectrum consistent with coexisting order provided  $m_z$  is not too big.
- Doping dependence of  $\lambda^{-2}(T)$  consistent with increasing  $m_z$  as one underdopes.