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Topological Excitations of Correlated Electronic States

One dimensional systems in the higher D world.

Which of their wonders are preserved?

Which new ones will appear?

What knowledge will they bring?

Solitons in 2000's, WHY?

New conducting polymers,

New events in organic conductors,

New accesses to Incommensurate CDWs,

New approach to strongly correlated systems as semiconductors.

Elementary excitations in electronic systems with spontaneous symmetry breaking: Superconductors, Antiferromagnetic semiconductors or Mott Insulator, Spin/ Charge Density Waves (commensurate or not)

Secure starting level: one- dimensional systems.

Solitons in the ground state and as elementary excitations.
Conversion of electrons to various solitons;
Separated or even anomalous charge, spin, currents.

Quasi one- dimensional route:

Confinement of solitons and dimensional crossover.
Spin-charge recombination due to 3D confinement.
Solitons acquire tails.

Arbitrary systems:

Solitons acquire feathers.
Combined symmetry of spin-charge transformations,
Topological constraints and coupling, Spin- or Charge- roton like excitations with charge- or spin- kinks localized in the core.

Combined symmetry breakings in polymeric and organic conductors -- *new routs to solitons.*

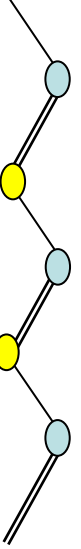
Charge Ordering together with Bonds Dimerization:

Peierls and Mott-Hubbard effects.

Build-in extrinsic and spontaneous intrinsic effects.

Nonequivalence of bonds and of sites.

In any case (organic crystals or polymers) this combination **lifts the inversion symmetry**, hence the **Ferroelctricity**.



FERROELECTRIC MOTT-HUBBARD PHASE and CHARGE ORDERING in QUASI 1D ORGANIC CONDUCTORS.

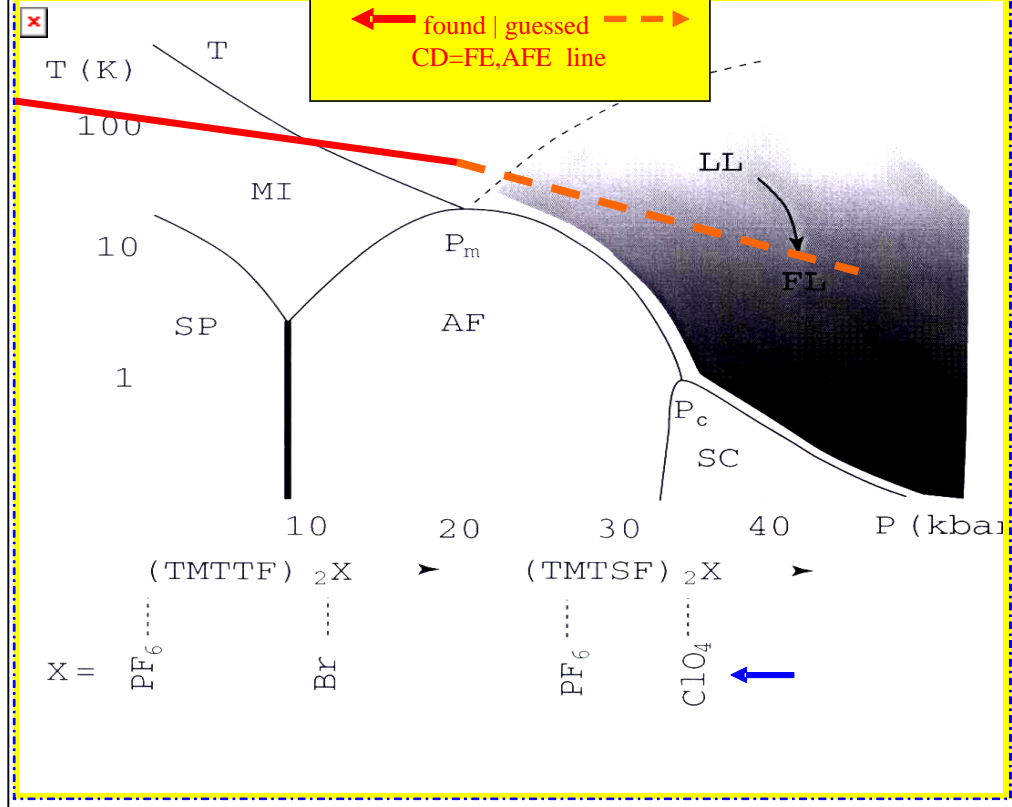
Collaboration: F. Nad Moscow and Grenoble
P. Monceau Grenoble and Saclay
J.M. Fabre & C. Carcel Montpellier.

NMR route: S. Broun et al, UCLA

Event: resolution for a longest (since 1985) story of misinterpretations: lost high T phase transitions.

A challenge to the High-Tc world:

Thought-to-be crossover lines may be true phase boundaries.



(TMTCF)₂X, 1980-2002

In black and white:

SC- superconductivity

AF- AFM = SDW

SP- Spin-Peierls

LL- Luttinger liquid

MI- Mott insulator

Red line - 2000 revolution:

= Ferroelectricity =

Charge disproportionation

Resolving the mystery of structureless transitions: *Coulon et al, 1985*

Gigantic anomaly in permittivity of $\epsilon'(T)$ (*Nad et al, Grenoble-Moscow*)

Charge Orderin seen by NMR (*Brown - UCLA, Fujiyama -IMS*).

Views and interpretations:

FerroElectric version of Mott-Hubbard state, mixed site/bond $4K_F$

CDW, nonsymmetrically pinned

Wigner crystal

Facility to see Solitons:

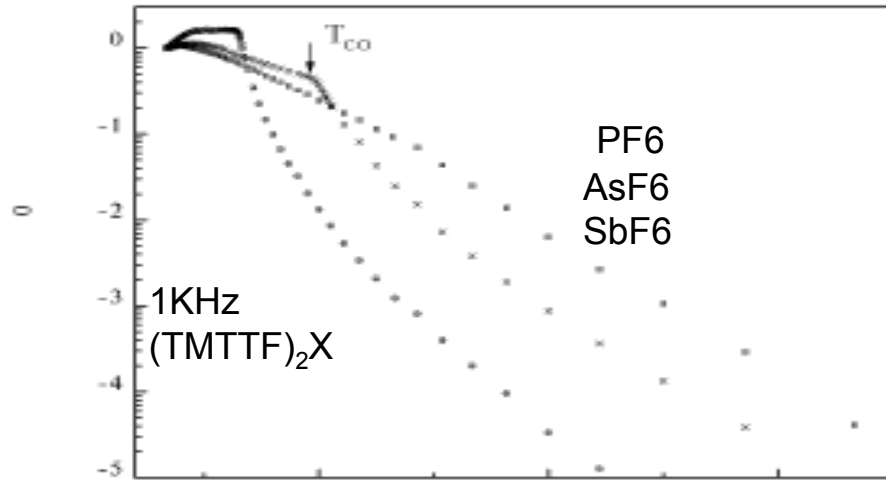
Purely 1D regime for

electrons - $T_{FE} \approx 150$ is

10 times above 3D

electronic transitions.

*Why solitons, and
as many as 4 types?*



$$G \sim \exp(-\Delta/T)$$

Solitons as spinless charge carriers:

Conductance G - normalized to RT , Arrhenius plot $\text{Log}G(1/T)$.

Gaps for thermal activation Δ range within 500-2000K.

Clearlest example for conduction by
charged spinless solitons - holons.

Conrarily to normal semiconductors - no gap in spin susceptibility:

Pauli-like $\chi = \text{cnst}$ as in the metallic high T phase.

COMBINED MOTT - HUBBARD STATE

$$H \sim (\hbar/4\pi\gamma) [v_\rho (\partial_x \varphi)^2 + (\partial_t \varphi)^2 / v_\rho] + H_U$$

Chiral phase $\varphi = \varphi(x, t)$ for electrons near $\pm K_F$

U: amplitude of the Umklapp scattering

2 types of dimerization \Rightarrow

2 interfering sources for two-fold commensurability

\Rightarrow 2 contributions to the Umklapp interaction.

Site dimerization: $H_U^s = -U_s \cos 2\varphi$ (spontaneous)

Bond dimerization: $H_U^b = -U_b \sin 2\varphi$ (build-in)

At presence of both site and bond types

$$H_U = -U_s \cos 2\varphi - U_b \sin 2\varphi = -U \cos (2\varphi - 2\alpha)$$

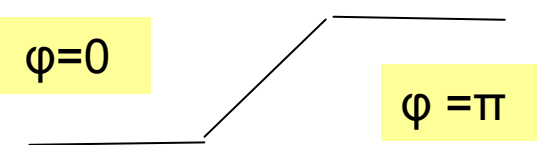
$$U = (U_b^2 + U_s^2)^{1/2}, \quad \tan 2\alpha = U_s / U_b$$

*$U_s \neq 0 \rightarrow \alpha \neq 0 \rightarrow$ phase $\varphi =$ “mean displacement of all electrons”
shifts from $\varphi = 0$ to $\varphi = \alpha$, hence the gigantic FE polarization.*

From a single stack to a crystal: Macroscopic FerroElectric ground state if the **same** α is chosen for all stacks,
Anti-FE state if the sign of α **alternates** as for $(\text{TMTTF})_2\text{SCN}$.

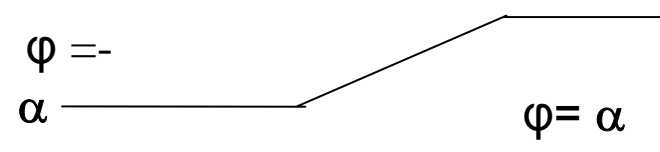
For a given U_s the ground state is still doubly degenerate between $\varphi = \alpha$ and $\varphi = \alpha + \pi$. $H_U = -U \cos(2\varphi - 2\alpha)$

It allows for **phase π solitons**, i.e. **holons** with the charge e .



Purely on-chain solitons, exist as conducting quasiparticles both above and below the T_{FE} .

Spontaneous U_s itself can change sign between different FE domains. Then the electronic system must also adjust its ground state from α to $-\alpha$. Hence the domain boundary $U_s \leftrightarrow -U_s$ requires for the phase soliton of the increment $\delta = -2\alpha$ which will concentrate the *non integer* charge $q = -2\alpha/\pi$ per chain.



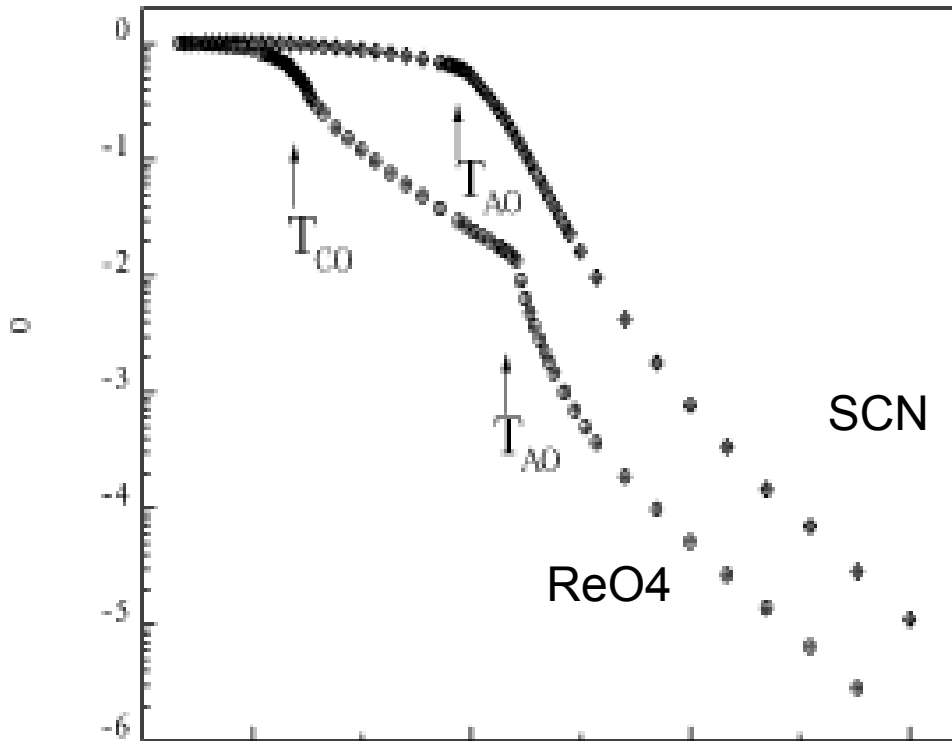
Alpha- solitons are the walls between domains of opposite FE polarizations.

They are on-chain conducting particles only above T_{FE} . Below T_{FE} they aggregate into macroscopic walls. They do not conduct any more, but determine the FE depolarization dynamics.

Effects of subsequent transitions.

Combined solitons. Spin-Charge reconfinement.

Another present from the Nature:
tetramerization in $(\text{TMTTF})_2\text{ReO}_4$ at $T_{\text{AO}} < T_0$



Spin-charge reconfinement below T_{AO} of tetramerisation. Enhanced gap Δ comes from topologically coupled π - solitons in both sectors of the charge and the spin. The last is weakly localized. What does it mean ?

$$\Psi_{\pm\sigma} \sim \exp[i(\varphi + \sigma\theta/2)]$$

Charge φ and spin θ phases as modulation of Fermi momenta.

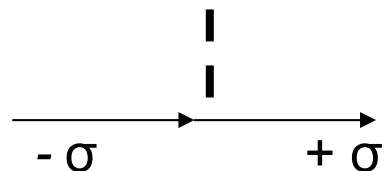
θ -- chiral phase of the spin spin $\sigma = +/-1$,
 $\theta'/\pi =$ smooth spin density

Former translations $\mathbf{x} \rightarrow \mathbf{x} + 2$ are weakly prohibited.

Allowed ones are $\mathbf{x} \rightarrow \mathbf{x} + 4$ hence only $\varphi \rightarrow \varphi + 2\pi$ **at first sight**.

Invariant Hamiltonian: $H_U = -U \cos(2\varphi - 2\alpha) - V \cos(\varphi - \beta) \cos \theta$
 $V \sim \Delta_\sigma^2 \neq 0$ at $T < T_{a0}$ Δ_σ is the spin gap

Origin of V : singlet pair creation by external periodic potential.



$2K_F$ scattering due to tetramerization

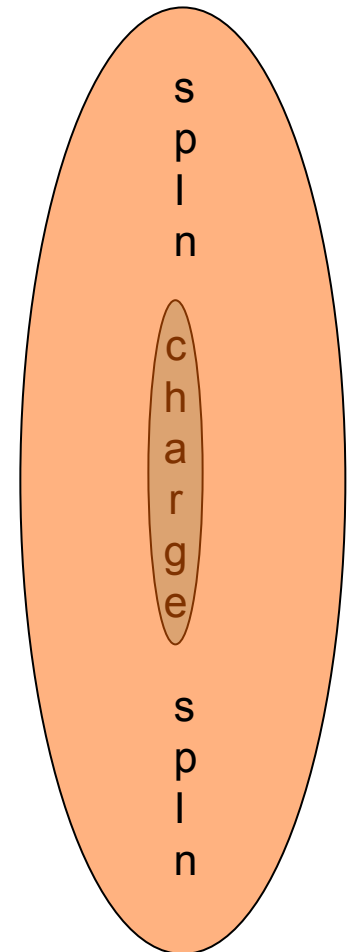
Further symmetry lifting after the tetramerization mixes charge and spin channels: additional energy $V \cos(\varphi - \beta) \cos \theta$

φ and θ -- chiral phases counting the charge and the spin
 φ / π and $\theta / \pi =$ smooth charge and spin densities

Major effects of V-term :

- Opens spin gap $2\Delta_\sigma$:
triplet pair of $\delta\theta = \pi$ solitons at $\varphi = \text{const}$
- Prohibits $\delta\varphi = \pi$ solitons –
now bound in pairs by spin strings
- Allows for combined spin-charge topologically bound solitons:
 $\{\delta\varphi = \pi + \delta\theta = \pi\}$ – leaves the V term invariant

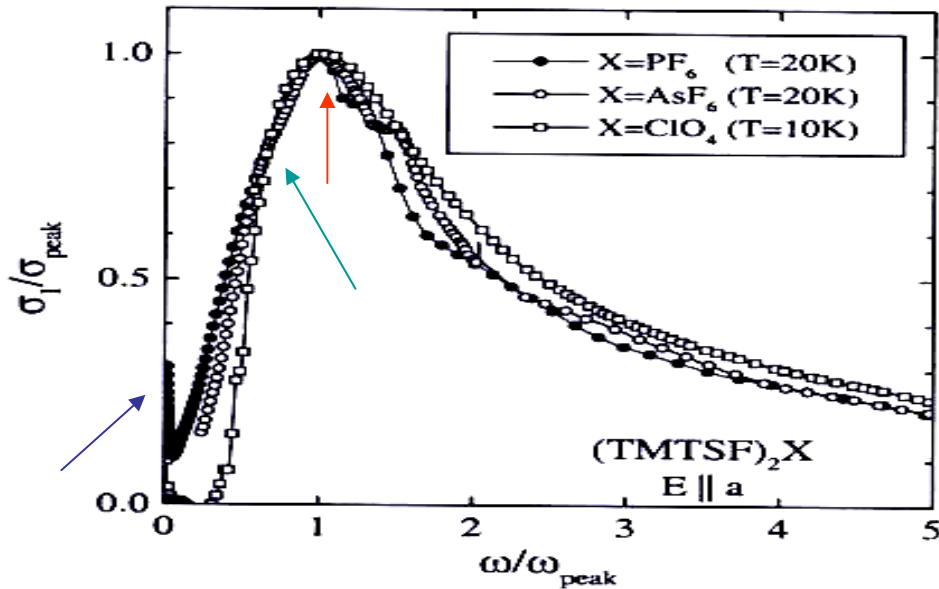
Quantum numbers of the compound particle --
charge e , spin $1/2$ but differently localized:
charge e , $\delta\varphi = \pi$ sharply within $\hbar v / \Delta_\rho$
spin $1/2$, $\delta\theta = \pi$ loosely within $\hbar v / \Delta_\sigma$



Solitons in Optics

ETH group

$\sigma(\omega)$: Optical Conductivity



PSEODGAP REGION IN TMTSF SUBFAMILY.

Plausible interpretations in terms of solitons=kinks:

Low ω rise:

optically active mode of the FE polarization or Drude peak.

Kink at $\omega < \omega_{\text{peak}}$:

Collective phason mode = exciton = bound pair of kinks

$\omega_{\text{peak}} = 2\Delta$ - 2-particle gap: pair of kink

-- call for photoconductivity experiments.

Solitons in tunneling: Incommensurate CDWs.

Yurii Latyshev, P. Monceau, S.B., et al; *Moscow, Grenoble, Orsay, Sendai.*

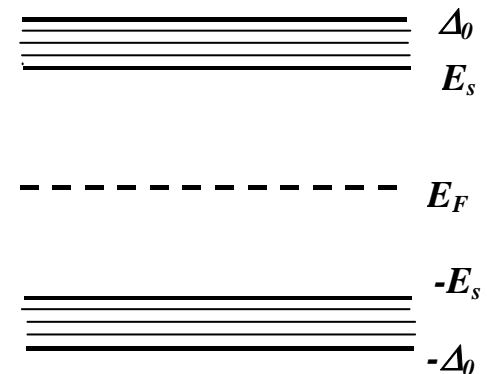
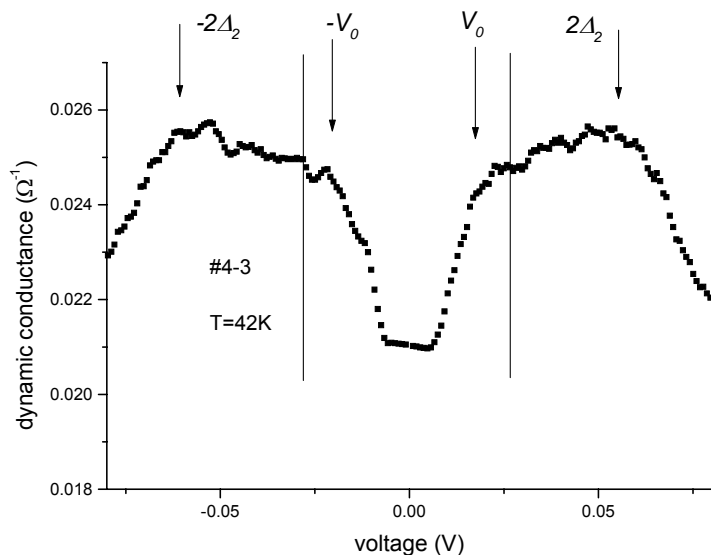
Direct observation of solitons in tunneling on **NbSe₃** and **TaS₃**

Expected thresholds at 2Δ for two electrons, followed through a pseudogap down to the edge for electron \rightarrow soliton transition at $2\Delta/\pi$.

Preliminary results:

Yu.Latyshev, P.Monceau, A.Sinchenko, L.Bulaevskii, S.Brazovskii, T.Kawae, T.Yamashita,
“Interlayer tunneling spectroscopy of the charge density wave state in NbSe₃”, in
Proceedings of the International Workshop on Strongly Correlated Electrons in New Materials (SCENM02),
Journal of Physics A: **36**, (2003) 9323.

Details to be published.



CONTINUOUS SYMMETRIES

Solitons are stable energetically but not topologically.

Special significance: allowance for a direct transformation of **one electron into one soliton**.

(Only $2 \rightarrow 2$ were allowed for kinks in discrete symmetries)

Incommensurate CDWs = chiral Gross-Neveu model

Order Parameter $\sim \mathbf{A}(x)\mathbf{COS}[Qx+\varphi]$

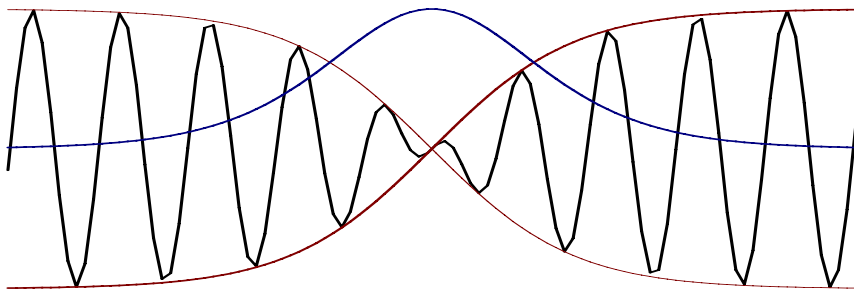
$\Delta = \mathbf{A} \exp[i\varphi]$; \mathbf{A} - amplitude , φ - phase

Ground State with an odd number of particles:

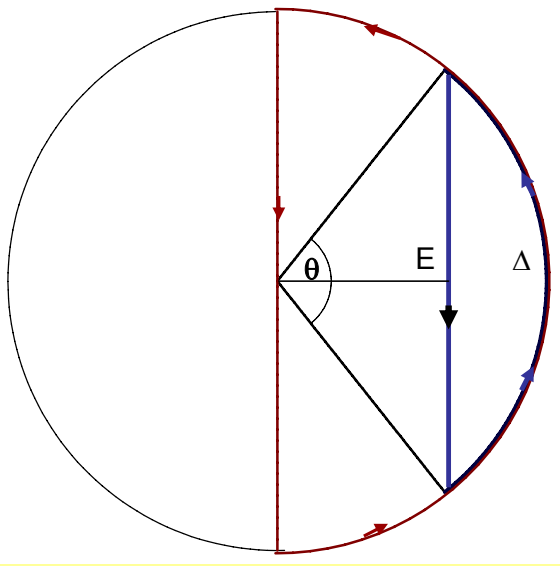
In 1D - *Amplitude Soliton AS* $\Delta(x=-\infty) \Leftrightarrow -\Delta(x=\infty)$

via $\mathbf{A} \Leftrightarrow -\mathbf{A}$ at arbitrary $\varphi = \text{cnst.}$

Spin $\frac{1}{2}$ and Charge $\mathbf{0}$



Oscillating electronic density,
Overlap soliton $A(x)$,
Midgap state = spin shape

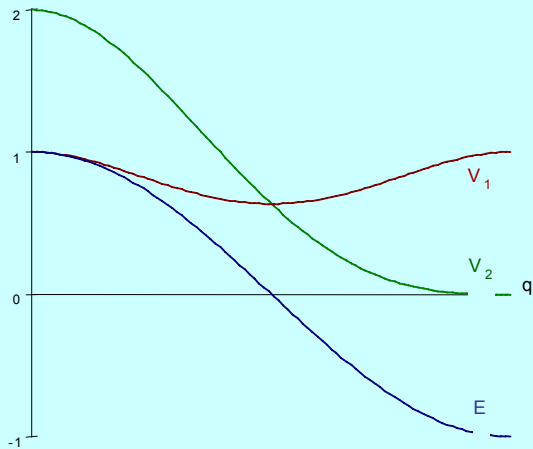


Soliton trajectories in the complex plane of the order parameter.

Red line: stable amplitude soliton.

Blue line: intermediate chordus soliton within chiral angle θ (black radial lines).

The value $\theta=100^\circ$ is chosen which corresponds to the optimal configuration for the interchain tunnelling.



Selftrapping branches $V_n(\theta)$ for chordus solitons for fillings $n=1$ (red line) and $n=2$ (green line),

the energy $E_0(\theta)$ (blue line) of localized split-off state

as functions of the chiral angle θ .

Phase tails of the amplitude soliton AS:

Prohibited in $D > 1$ (including $1+1$) environment (confinement !)

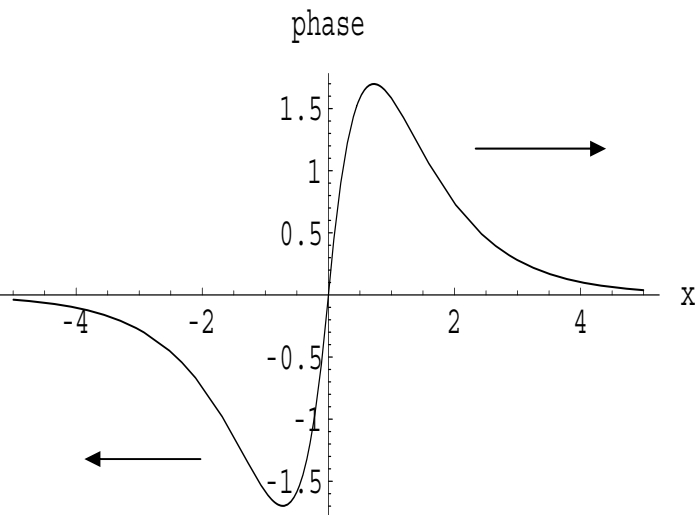
Allowed in $D > 1$ if acquires phase tails with the total increment

$\delta\varphi = \pi$ $\{A \rightarrow -A, \varphi \rightarrow \varphi + \pi\}$:

selfmapping of the order parameter $\Delta = A \exp[i\varphi]$

Result: allowed particle with the AS core carrying the spin $1/2$

plus the phase twisting wings carrying the charges $e/2$



Injecting a bare electron at a time $t=0$, AS core develops fast but the phase tails propagate (arrows) with the phase sound velocity u .

Tails contribute to the action $\sim u^{-1} \ln[t]$.

Hence the probability $I(E)$ to create the AS near its threshold energy $E_{AS} = 2/\pi\Delta$ is $I \sim (E - E_{AS})^{-1/u}$

This is the pseudogap between Δ and E_{AS}

S. Matveenko and S.B.

Can the solitons cross the boarder to the higher D world ?
Are they allowed to bring their anomalies like spin-charge separation or midgap states?

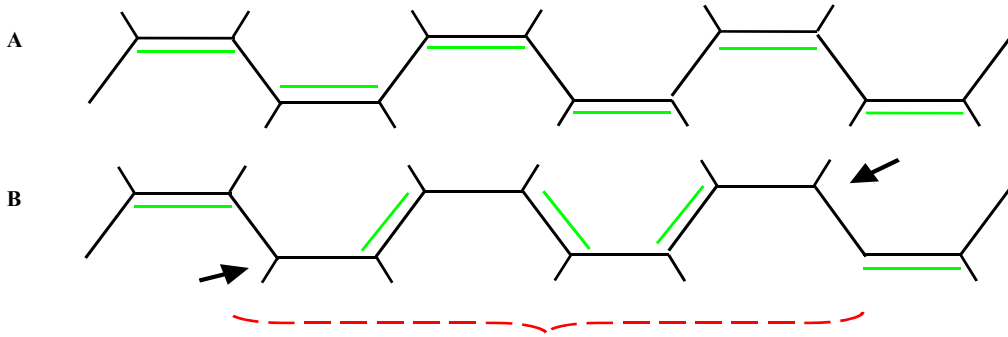
Password : confinement.

As topological objects connecting degenerate vacuums, solitons acquire an infinite energy unless they reduce or compensate their topological charges.

Various scenarios :

- Compensation by the gapless mode *S.B. 1980, 2000*
- Aggregation into domain walls versus their melting by thermal deconfinement or long range Coulomb forces
S.B. & T.Bohr 1983, S. Teber 2001
- Coupling to structural defects in polymers.
- Binding to kink-antikink pairs, origin of bipolarons.
S.B. & N.Kirova, 1981- 90's
- *Today's speciality :*
Topological binding to gapless degrees of freedom

Fatal effect upon kinks: lifting of degeneracy, hence confinement.
Trivial but spectacular example: global lifting of symmetry.



Nature present:
cis-isomer of $(\text{CH})_x$
Build-in slight inequivalence
of bonds.

Confinement of kinks pairs into
 $2e$ charged (bipolaron) or neutral (exciton) formation.

Symmetry determined picture of optical differences for
trans- and cis $(\text{CH})_x$.

Photoconductivity versus photoluminescence,
new optical features due to hybridization of midgap states.

S. B. and N. Kirova, 1981: Interpretation of experiments and
«accidental» exact solution for the model with confinement.

FINITE TEMPERATURE, ENSEMBLES OF SOLITONS,
 PHASE TRANSITIONS OF CONFINEMENT AND AGGREGATION.
 DISCRETE SYMMETRY only.

Fatal effect upon kinks: lifting of degeneracy, hence confinement.

Nontrivial but still spectacular:

local lifting in the state with long range order.

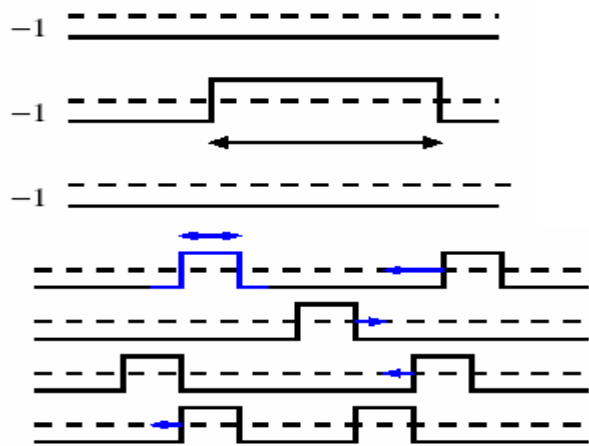
Interchain coupling of the order parameter.

$$H_I = - \sum_{\langle \alpha, \beta \rangle} \int dx V_{\perp} \Delta_{\alpha}(x) \Delta_{\beta}(x)$$

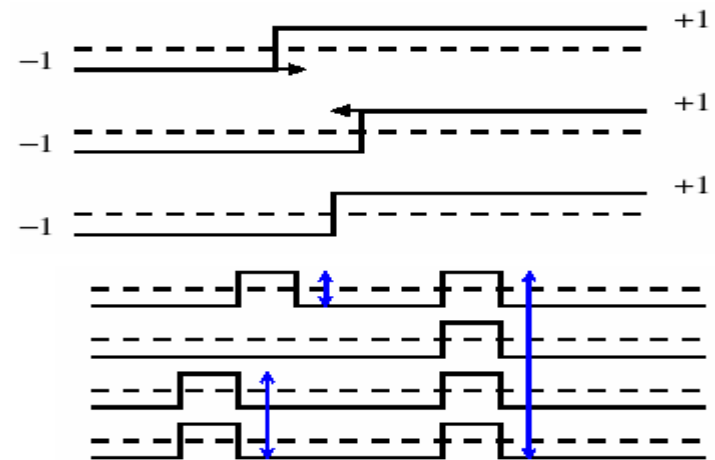
Two competing effects:

Binding of kinks into pairs at $T < T_c$;

Aggregation into macroscopic domain walls at $T < T_0 < T_c$.



$T > T_0$



$T < T_0$

Solution for a statistical model *T.Bohr and S.B. 1983, S.Teber et al 2000's*

Quantum models based on bosonisation.

MIXED DISCRETE AND CONTINUOUS SYMMETRIES.

SPIN-GAP cases.

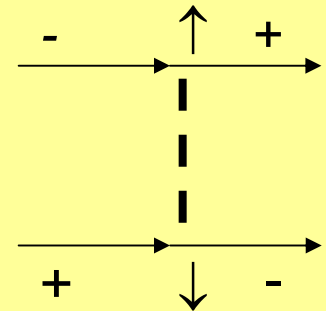
$$H_{1D} \sim (\partial\theta)^2 - V \cos(2\theta) + (\partial\varphi)^2$$

V - from the backward exchange scattering

In **1D** : Spinon as a soliton $\theta \rightarrow \theta + \pi$ hence $s=1/2$;

Gapless charge sound in φ .

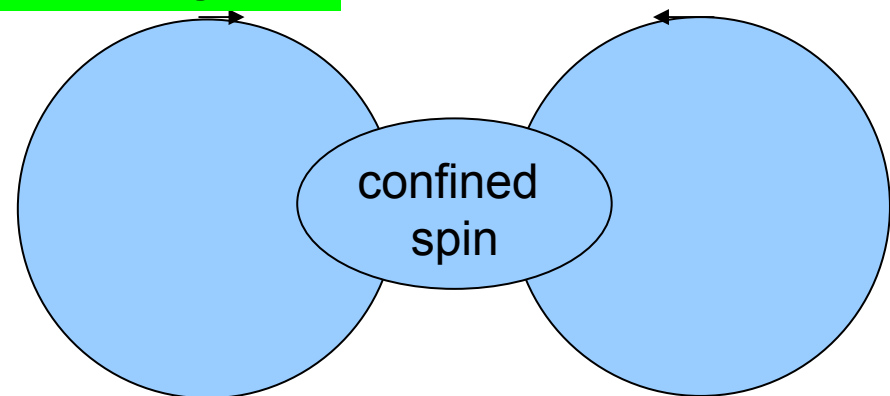
$$CDW \text{ order parameter} \sim \psi_{+\uparrow}^\dagger \psi_{-\uparrow} + \psi_{+\downarrow}^\dagger \psi_{-\downarrow} \sim \exp[i\varphi] \cos\theta$$



At higher D : allowed mixed configuration

$\theta \rightarrow \theta + \pi, s=1/2$	$\varphi \rightarrow \varphi + \pi, e=1$
↑ spin soliton ↑	↑ charged wings ↑

Spinon as a soliton + semi dislocation loop = π - vortex of $\varphi \equiv$ confined spin + semi dislocation loop



Singlet Superconductivity: $D=1 \rightarrow D>1$:

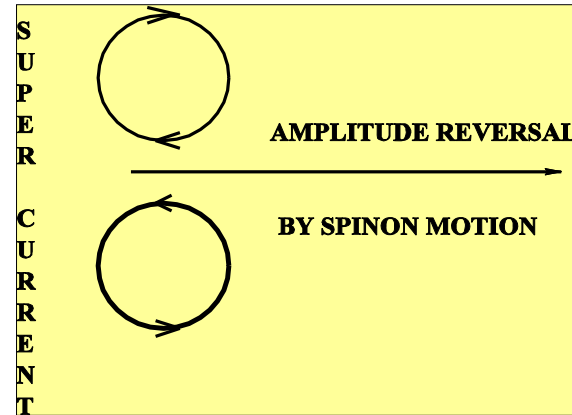
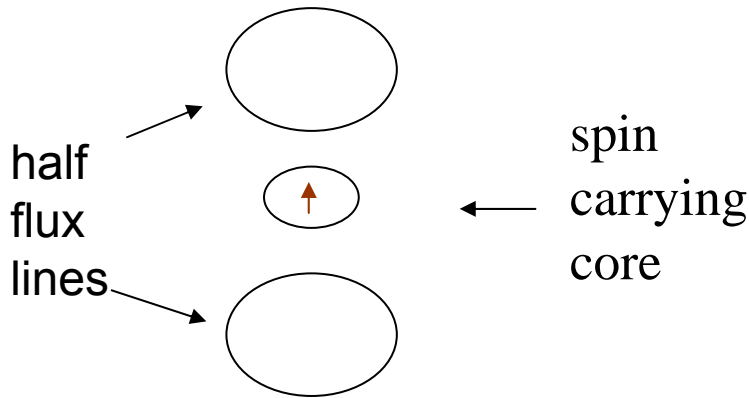
$$\eta_{SC} \sim \psi_{+\uparrow} \psi_{-\downarrow} + \psi_{+\downarrow} \psi_{-\uparrow} \sim \exp [i\varphi^*] \cos\theta$$

$$\theta \rightarrow \theta + \pi \quad s=1/2$$

$$\varphi^* \rightarrow \varphi^* + \pi$$

↑ spin soliton ↑
 supercurrents ↑

↑ wings of



Quasi 1d view : spinon as a π - Josephson junction in the superconducting wire (applications: Yakovenko et al).

2D view : pair of π - vortices shares the common core bearing unpaired spin.

3D view : half-flux vortex stabilized by the confined spin.

Best view: nucleus of melted FFLO phase in spin-polarized SC

Half filled band with repulsion.
SDW rout to the doped Mott-Hubbard insulator.

$$H_{1D} \sim (\partial\varphi)^2 - U \cos(2\varphi) + (\partial\theta)^2$$

U - Umklapp amplitude

(*Dzyaloshinskii & Larkin ; Luther & Emery*).

φ - chiral phase of charge displacements

θ - chiral phase of spin rotations.

Degeneracy of the ground state:

$\varphi \rightarrow \varphi + \pi =$ translation by one site

Excitations in 1D :

holon as a π soliton in φ , spin sound in θ .

Higher D : A hole in the AFM environment.

Staggered magnetization \equiv AFM=SDW order parameter:

In a stretched frame of particles : $S_{\pm} \sim \exp\{\pm i(Qx + \theta)\}$

In the absolute frame of sites : $O_{SDW} \sim \cos\varphi \exp\{\pm i(Qx + \theta)\}$

To survive in $D > 1$:

The π soliton in φ $\cos\varphi \rightarrow -\cos\varphi$

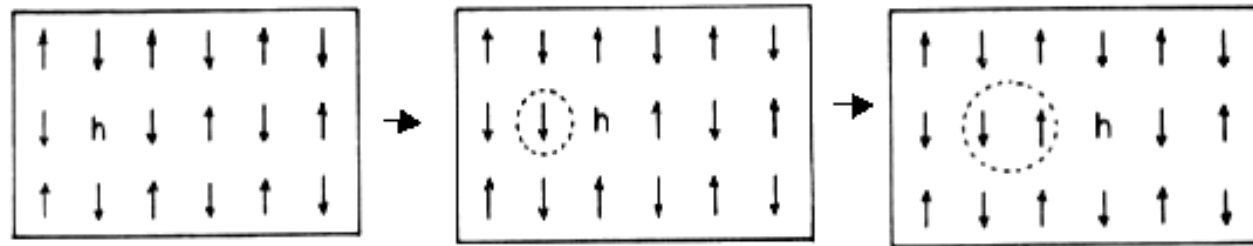
enforces a π rotation in θ to preserve O_{SDW} .

Propagating hole as an amplitude soliton.

Its motion permutes AFM sublattices \uparrow, \downarrow
creating a string of the reversed order parameter:
the staggered magnetization.

It blocks the direct propagation.

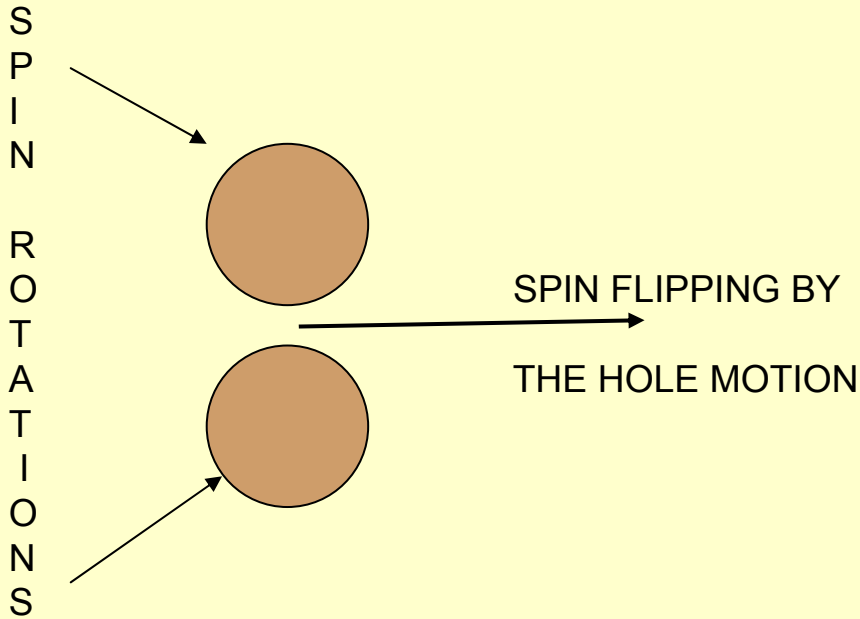
Brinkman and Rice, Nagaev et al



Adding the semi-vorticity to the string end heals the permutation
thus allowing for propagation of the combined particle.

Resulting Elementary Excitations:

half integer vortex ring of staggered magnetization = $\frac{1}{2}$ roton with the holon confined at its core.



Alternative view:

Nucleus of the stripe phase or
the minimal element of its melt.

- Physical variables :
- Electric charge -- Localized in the core;
- Delocalized AFM staggered and net ferro magnetizations.
- Well away from the core:

$$e^{i\theta} = \sqrt{\frac{x+i(y-1)}{\sqrt{x^2+(y-1)^2}} \frac{x-i(y+1)}{\sqrt{x^2+(y-1)^2}}}$$

$$S_{\text{afm}} \propto \frac{x}{x^2 + y^2} \quad S_{\text{fer}} \propto \frac{(x^2 - y^2)}{(x^2 + y^2)^2}$$

E.g. to be tested by the NMR

Paradox:

the central chain, $\mathbf{y}=\mathbf{0}$: $\delta\theta=\pi \rightarrow \mathbf{s}=\mathbf{1}/2$
like an added electron of spin $\mathbf{s}=\mathbf{1}/2$, charge $\mathbf{e}=\mathbf{1}$.

But integrally over cross section:

$$\int \delta\theta d^2r_{\perp} = 0 \quad \Rightarrow \quad \text{net spin} \quad \mathbf{s} = \mathbf{0}$$

3D quantum numbers are like for normal electron spin
 $\mathbf{s}=\mathbf{1}/2$ (while in wings) charge $\mathbf{e}=\mathbf{1}$ (while in the core).

But integrally over a perpendicular cross-section:

$$\text{net AFM spin} \quad \mathbf{S}_{\text{afm}} = \mathbf{0};$$

integrally over any cross section:

$$\text{net magnetization} \quad \mathbf{S}_{\text{fer}} = \mathbf{0}$$

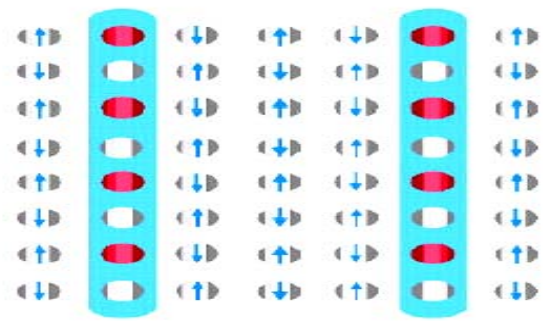
The LRO in $D>1$ reconfines the charge and the spin but only in a core.
But integrally one of the two is transferred to the collective mode.
Locally we restore single electronic quantum numbers
but with different scales of localization.
Integrally – still the spin without the charge.

$1D \rightarrow \text{quasi } 1D \rightarrow 2D, 3D$ route to dopping of AFM insulator.

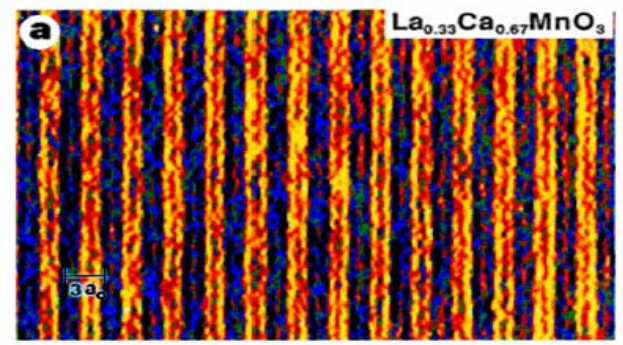
Aggregation of holes (extracted electrons) into stripes.

Left: scheme derived from neutron scattering experiments.

Right: *direct visualization via electron diffraction microscope.*



J.Orenstein et al Science 288, 468 (2000)



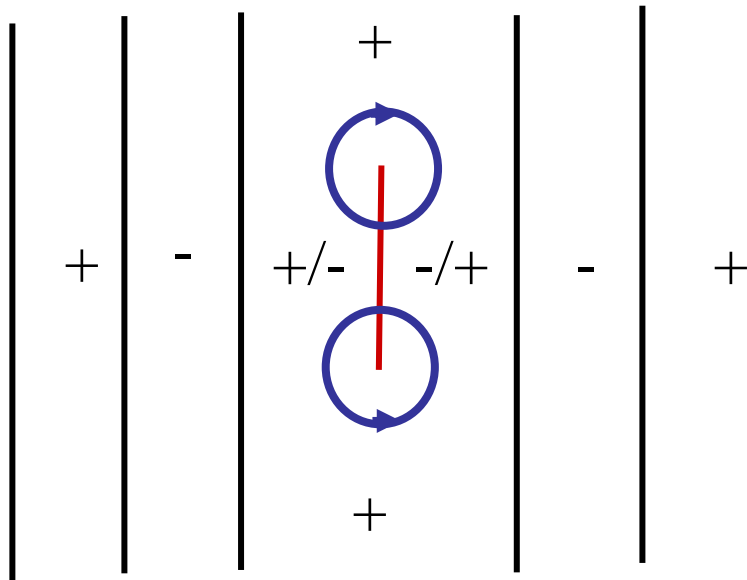
S.Mori et al Nature 392, 473 (1998)

Equivalence for spin-gap cases:

Fulde-Ferrell-Larkin-Ovchinnikov FFLO phase in superconductors

Solitonic lattices in CDWs above the magnetic breakdown

Kink-roton complexes as nucleuses of melted macro structures:
 FFLO phase for superconductors or strips for doped AFMs.



A defect embedded into the regular stripe structure (black lines).

+/- are the alternating signs of the order parameter amplitude.

Termination points of a finite segment (red color) of the zero line must be encircled by semi-vortices of the π rotation (blue circles) to resolve the signs conflict.

The minimal segment corresponds to the spin carrying kink.

SUMMARY

- Existence of solitons is proved experimentally in single- or bi-electronic processes of 1D regimes in quasi 1D materials.
- They feature selftrapping of electrons into midgap states and separation of spin and charge into spinons and holons, sometimes with their reconfinement at essentially different scales.
- Topologically unstable configurations are of particular importance allowing for direct transformation of electrons into solitons.
- Continuously broken symmetries allow for solitons to enter the $D > 1$ world of LRO states: SC, ICDW, SDW.
- They take forms of amplitude kinks topologically bound to semi-vortices of gapless modes.
- Charge and spin can be viewed as reconfined or not depending on the type of measurement.
- These combined particles substitute for electrons in all quasi-1D.
- The picture is extrapolated to strongly correlated isotropic cases.

Supplementary sets of viewgraphs

Page: <http://ipnweb.in2p3.fr/~lptms/membres/brazov/seminars.html>

<http://ipnweb.in2p3.fr/~lptms/membres/brazov/FE-vg-web.pdf>

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<http://ipnweb.in2p3.fr/~lptms/membres/brazov/junctions.pdf>

<http://ipnweb.in2p3.fr/~lptms/membres/brazov/solitons.pdf>

Related publications:

cond-mat/9911143

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