# Inhomogeneous phases driven by competing orders


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# Message

**Inhomogeneous states *and glassiness* may occur spontaneously in uniform systems as a result of competing interactions or competing orders.**

*S. Brazovskii, 1975; J. Schmalian and P. Wolynes, 2000-2004;*
*ZN, IV, AVB 2004.*

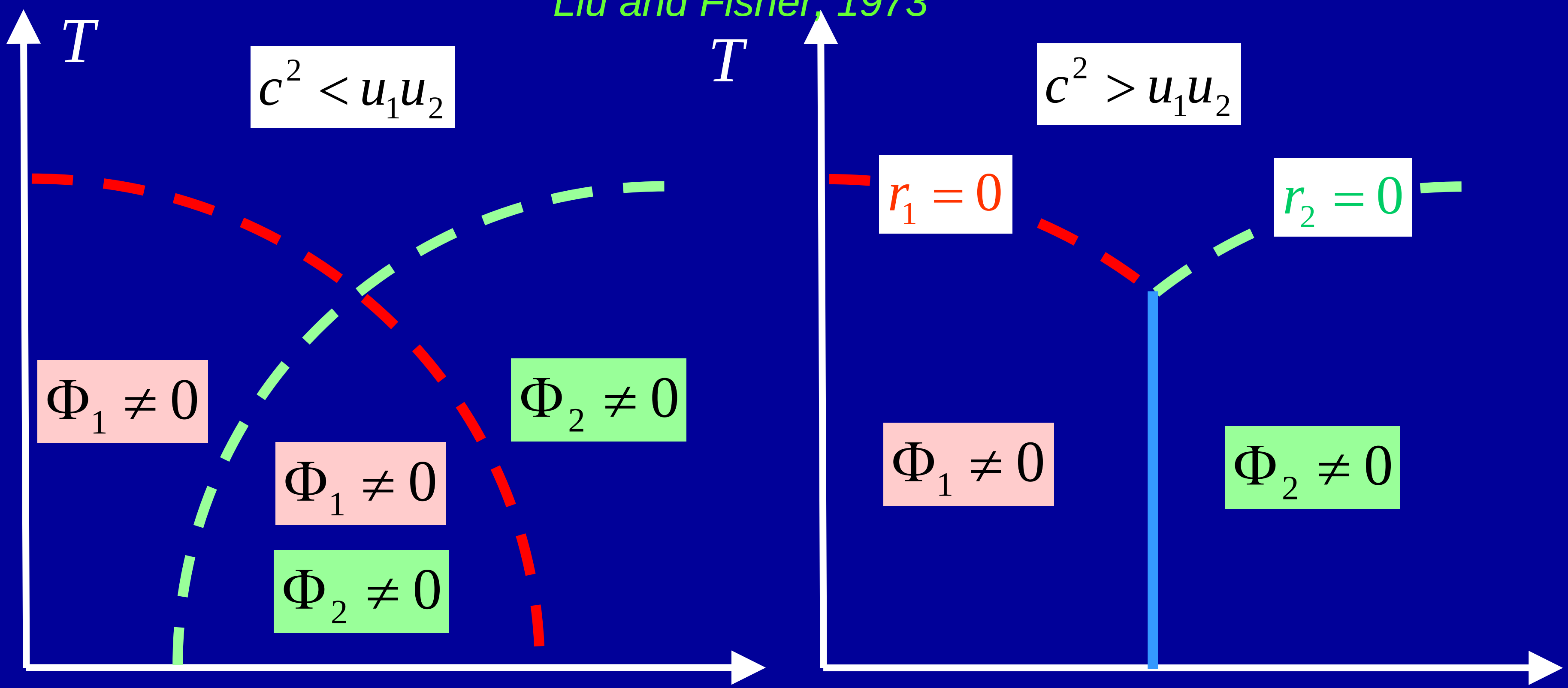
# Inhomogeneous ordered states

- Coexistence of different orders:
  *manganites, heavy fermions, cuprates,...*
- Intrinsic inhomogeneities on the macroscopic scale
- Microscopic origin: competing interactions
- Description at the level of effective theories (Ginzburg-Landau)?

# Competing orders: a reminder

$$F_0 = \frac{r_1}{2}|\Phi_1|^2 + \frac{u_1}{4}|\Phi_1|^4 + \frac{r_2}{2}|\Phi_2|^2 + \frac{u_2}{4}|\Phi_2|^4 + \frac{c}{2}|\Phi_1|^2|\Phi_2|^2 + \frac{1}{2}(\nabla\Phi_1)^2 + \frac{1}{2}(\nabla\Phi_2)^2$$

*E. M. Lifshitz (1944), K. Wilson and M. Fisher (1972), Liu and Fisher, 1973*




**Gradient coupling: inhomogeneous states in a uniform system**

# Gradient couplings

- System has a preferred wave vector:

$$F_1 = a|\Phi_1|^2|\Phi_2|^2(\nabla\varphi_2 - q_0), \text{ where } \Phi_2 = |\Phi_2|\exp(i\varphi_2)$$

**manganites**, *G. Milward, M. Calderon, P. Littlewood, 2004*

- System selects the wave vector from interactions
  - Structural transitions: *not for general symmetry OP*

$$F_1 = a(\Phi_1\nabla\Phi_2 - \Phi_2\nabla\Phi_1)$$

*V. Heine & J.McConnell, 1984*

  - General gradient coupling

$$F_1 = a|\Phi_1|^2|\nabla\Phi_2|^2, \ a < 0$$

***ZN, IV, and AVB, 2004-05***

$$F_1 = a|\nabla\Phi_1|^2|\nabla\Phi_2|^2, \ a < 0$$

**e.g. stripes: charges like to sit at magnetic domain walls**

# Inhomogeneous states

$$F_0 = \frac{r_1}{2}|\Phi_1|^2 + \frac{u_1}{4}|\Phi_1|^4 + \frac{r_2}{2}|\Phi_2|^2 + \frac{u_2}{4}|\Phi_2|^4 + \frac{c}{2}|\Phi_1|^2|\Phi_2|^2$$

$$F_1 = \boxed{\frac{a}{2}|\Phi_2|^2|\nabla\Phi_1|^2} + \frac{p_2}{4}|\nabla\Phi_1|^4 + \frac{1}{2}|\nabla\Phi_1|^2 + \frac{w}{2}|\nabla\Phi_2|^2 + higher$$

- If $\Phi_2 = 0$: uniform state

  Inhomogeneity only in the coexistence region

- If $\Phi_2 \neq 0$:

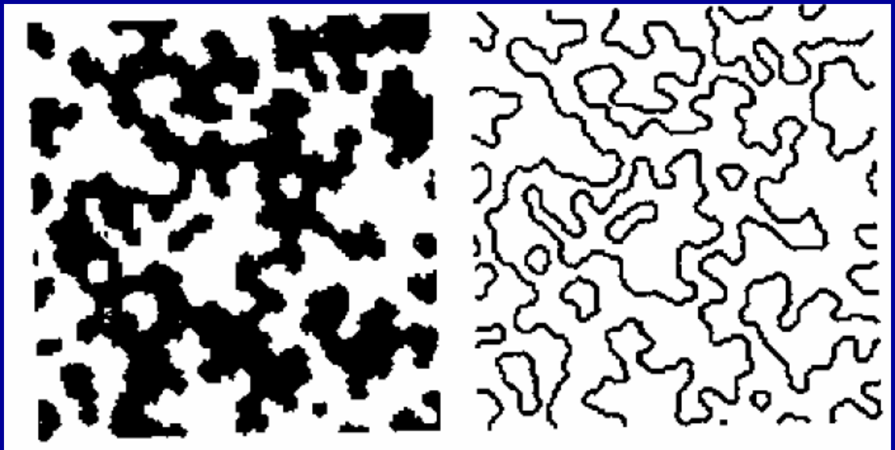
  Inhomogeneous state: disfavored in $F_0$, favored in $F_1$

- Different possibilities for the inhomogeneous state depending on the mean field transition temperatures of each field.

- Simplest case: mean field $\Phi_2 \approx \Phi_0$ sufficiently large effective negative gradient term for $\Phi_1$

$$F_{11} = -\frac{|a|}{4} |\Phi_2|^2 q^2 |\Phi_1|^2$$

- Modulated phase with weakly T-dependent $q_0 \propto \Phi_0$ (or any other extrinsically given wave vector)
- Effective model similar to surfactants
- Complicated inhomogeneous states



*M. Laradji et al. 1992*

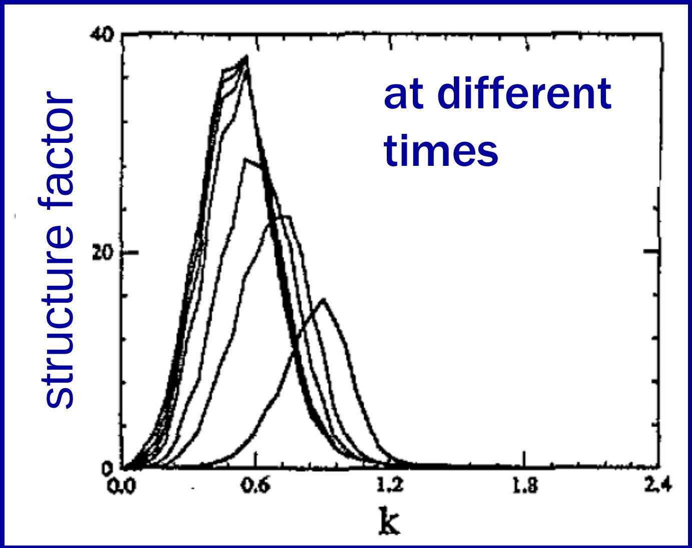
# Classical surfactant models

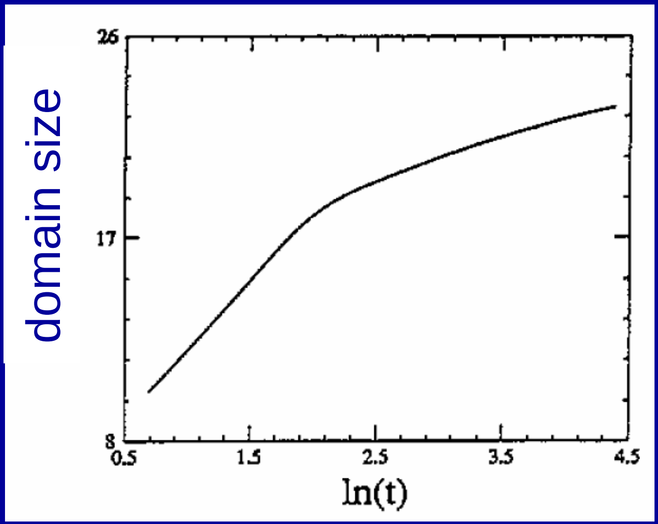
*M. Laradji et al. 1992*

- Inhomogeneous states with no long range order
- Structure factor peaked at finite k (*depends on conservation laws*)

$$S(k,t) \propto \left\langle \left| \sum_x \Phi(x,t) e^{ikx} \right|^2 \right\rangle$$

- Slow dynamics
- Glassy?



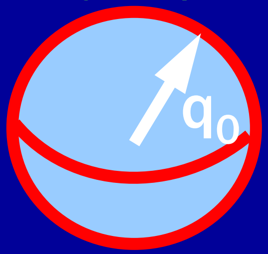
# Effective theory

*S. Brazovskii, 1975*

$$H = \frac{1}{2}\sum_{\mathrm{k}} v(\mathrm{k})\Phi_1(\mathrm{k})\Phi_1(-\mathrm{k}) + \frac{u}{4}\sum_{\mathrm{k}_1+\mathrm{k}_2+\mathrm{k}_3+\mathrm{k}_4=0}\Phi_1(\mathrm{k}_1)\Phi_1(\mathrm{k}_2)\Phi_1(\mathrm{k}_3)\Phi_1(\mathrm{k}_4)$$

$$v(\mathrm{k}) \approx r_0 + (\mathrm{k}^2 - q_0^2)^2, \qquad r_0 \equiv \xi_0^{-2} = a(T - T_{c1})$$

- **From competing interactions** *(J. Schmalian and P. Wolynes)* **or competing orders** (*ZN, IV, AVB*).
- **Isotropic model - shell of modes $|\mathrm{k}|=q_0$.**




- **Large phase space for fluctuations:**

**classical dynamics**

$$\langle \Phi^2 \rangle = bTq_0^2/\sqrt{r_0}$$

**or quantum dynamics**

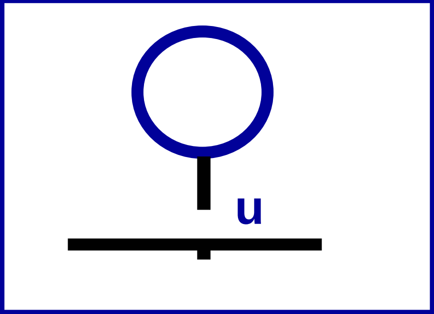
$$\langle \Phi^2 \rangle = bq_0^2 \log \frac{E_0}{r_0}$$

# $1^{st}$ order transition

*Brazovskii transition: S. Brazovskii, 1975*

- Drives the system away from transition: self-consistency

$$r = r_0 + u\left\langle \Phi^2 \right\rangle = r_0 + ubTq_0^2/\sqrt{r}$$




**$2^{nd}$ order transition impossible (large N) ; real transition may occur at $0 < T_c << T_{c1}$)**

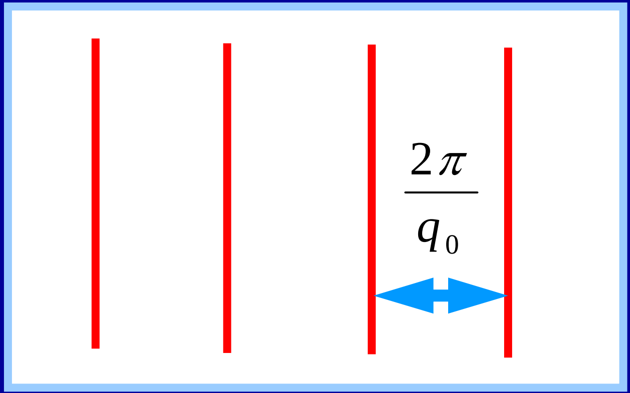
- Fluctuation driven (entropy) $1^{st}$ order transition

$$r^{3/2} \approx ubTq_0^2$$

- At the mean field level:

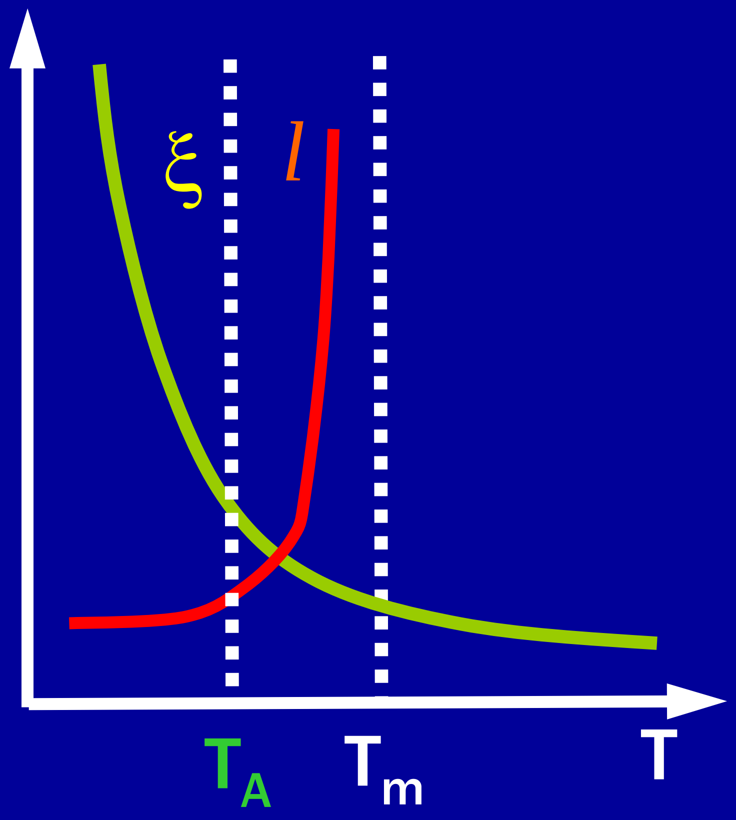
  transition into a lamellar phase

  $\Phi_1 = A_1 \cos(q_0 r)$



# Two length scales and glassiness

- Competition between
  - correlation length, $\xi^{-2}=r(T)+q_0^2$
  - modulation length $l^{-2}=q_0^2-r(T)$
  - at $q_0^2=r$ *short range correlations emerge ("liquid")*
  - at $r+q_0^2=0$ **long range order appears**




- ***Glass emerges when*** $\xi/l>2$

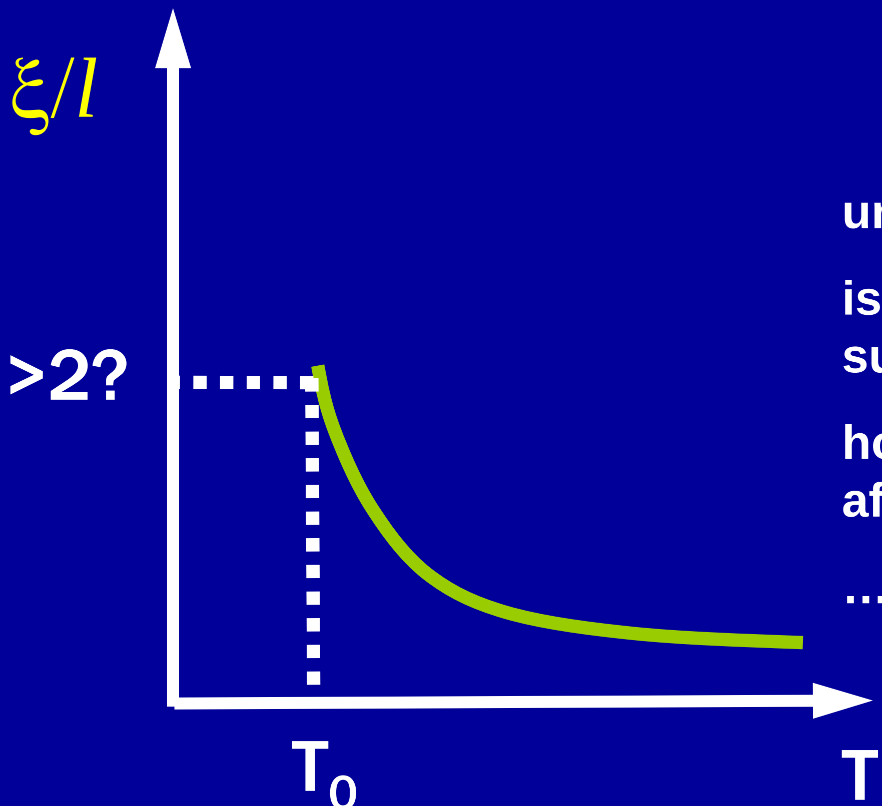
  *J. Schmalian and P. Wolynes,2000*
  - $N\propto\exp(q_0^3V)$ *metastable states below* $T_A(q_0)$
  - *Low cost of creating regions of order parameter* ($\xi^{-2}$) *correlated over short distance of order* $l$

**The transition to modulated phase is kinematically impossible and system is likely to become glassy instead**

- *J. Schmalian and P. Wolynes, 2000*

- As bare transition temperatures become closer $T_{c2} \approx T_{c1}$, 1st order transition becomes more likely, but details depend on parameters.

- $T_{c2} > T_{c1}$: possible to have 1st order into modulated phase
- glassiness depends on details: $q_0$ depends on T




under investigation:

is single modulation sufficient?

how do parameters affect the transition?

………

# Summary

- Competing orders which give modulated coexistence are likely to produce very inhomogeneous and glassy states in a nominally uniform system.

- A particular case of the more general situation: transition to a finite-q state can be glassy.

- Details matter (unfortunately?)

Open question:

- Hamiltonian from which such GL follows?