Transient properties of a quantum dot in the Kondo regime

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Work supported by NSF, and the Robert A. Welch Foundation
NEQDIS06, Dresden April 2006
The SET provides several means of tunability of a quantum dot in the Kondo regime.

Time dependent modulation of $V_d$, $V_B$, and $V_T$ allow the investigation of the nonadiabatic properties of the Kondo state.

Outline

- Introduction
- Nonequilibrium NCA
  - Dot level modulation
    - Response to AC modulation of the dot level for low bias
    - Response to a sudden shift of the dot level with finite constant bias
    - Response to a sudden shift of bias across the leads
    - Response to a sudden shift of the coupling of the dot to its leads
- Conclusions and References
Mechanism (changing $V_d$)

For a sufficiently small dot, the SET can be switched between conducting and nonconducting by the addition or removal of a single electron

Single electron transistor! Small size, low power consumption …………

How fast can it be switched?
Linear response \((V_B=0)\)

Wingreen, Meir 1992

\[
G(T) = \frac{e^2}{\hbar} \frac{\Gamma_{dot}}{2} \int d\varepsilon \rho_{dot}(\varepsilon) \left( -\frac{\partial f_{FD}}{\partial \varepsilon} \right)
\]

Formula valid in time-dependent case and when many-electron interactions are present. Conductance depend on spectral function around the \(\varepsilon_F\)

Spectral function of the dot level.

If many-electrons interactions (No Kondo) are neglected, the device function is trivial

Low Temperature
High Temperature
Effect of many-electron interactions

When dot contains an odd number of electrons, the interaction with the leads can result in the Kondo effect (spin fluctuations)

\[ T_K \propto D \exp\left[-\frac{\varepsilon_F - \varepsilon_{\text{dot}}}{\Gamma_{\text{dot}}}\right] \]

A Kondo resonance is induced just above the Fermi level for \( T \approx 10T_K \)

Empty impurity

Kondo regime
Spin fluctuations

Mixed –valence regime
Charge and spin fluctuations
Nature of the Kondo state

The Kondo state results when the occupation of a localized degenerate (spin) level is constrained (Coulomb repulsion).

The spin of the dot level can be flipped by the exchange of the dot electron with an electron from the Fermi level.

The slow spin flips are screened by electron-hole pair excitations.

The Kondo resonance a coherent superposition of electron-hole pairs of excitation energies around $k_B T_K$

The Kondo state is suppressed by thermal excitation of electron-hole pairs
The Kondo state need long time (many spin fluctuations) to form

Gunnarsson & Schonhammer, 1983
Transport through a Kondo state

The Kondo resonance increases the conductance of the SET.
For \( T < T_K \), the dot becomes transparent!

The size and shape of the Kondo resonance depend on \( T \)

\[
T = 0.005 \quad (T_K = 0.0022)
\]

\[
T = 0.02
\]

\[
T = 0.08
\]

The enhancement of the conductance depend on \( T/T_K \)
The Kondo temperature depends on \( \exp[-(\varepsilon_F - \varepsilon_d)/\Gamma] \)
Transport through SET in the Kondo regime

The Kondo effect enhances the conductance when odd number of electrons in the dot.

Kondo enhanced conductance as a function of dot level.

Equilibrium conductance as a function of dot level.

The conductance peaks for odd numbers of electrons.


Anderson model

\[ H(t) = \sum_{\sigma} \varepsilon_{\sigma}(t)n_{\sigma} + \frac{1}{2} \sum_{\sigma \neq \sigma'} U_{\sigma,\sigma'} n_{\sigma} n_{\sigma'} + \sum_{k,\sigma_{\text{lead}}} \varepsilon_{k,\sigma}(t)n_{k\sigma} + \sum_{k,\sigma_{\text{lead}}} [V_{k,\sigma}(t)c_{k,\sigma}^+ c_{\sigma} + H.c.] \]

All parameters are expressed in units of \( \Gamma \) (which is tunable)

\( T \) is temperature, \( t \) denotes time

\[ \Gamma_{\sigma,\text{lead}}(t) = 2\pi \sum_{k} |V_{k,\sigma_{\text{lead}}}(t)|^2 \delta(\varepsilon_{F} - \varepsilon_{k,\sigma_{\text{lead}}}) \]

The Coulomb parameter is assumed large, \( \varepsilon_{\sigma} + U >> \varepsilon_{F} + k_{B}T \)

We assume half-filled elliptic band, \( D=18 \)

We assume a symmetric dot i.e. \( \Gamma_{\text{left}} = \Gamma_{\text{right}} \)

Dot level is doubly degenerate (spin)

Total current is the sum of the left and right currents

Most units will be assumed to be one
Solution of time dependent NCA

Auxiliary boson
\[ c^+_\sigma c^+_{k\sigma} \rightarrow c^+_\sigma c^+_{k\sigma} b, \quad c^+_k c^-_\sigma \rightarrow c^+_k c^-_\sigma b^+, \quad Q = \sum_\sigma c^+_\sigma c^-_\sigma + b^+b = 1 \]

\[ iG(t,t') = \langle T_c c(t) c^+(t') \rangle, \]

\[ iB(t,t') = \langle T_c b(t) b^+(t') \rangle \]

G and B satisfies Dyson Eqs.
\[
\left[ i \frac{\partial}{\partial t} - \varepsilon_\sigma(t) \right] G_\sigma(t,t') = \delta(t-t') + \int d\tilde{t} \Sigma_\sigma(t,\tilde{t}) G_\sigma(\tilde{t},t')
\]

Projected upon the real axis
\[
\left[ i \frac{\partial}{\partial t} - \varepsilon_\sigma(t) \right] G^{\ uncertainties}_\sigma(t,t') = \int d\tilde{t} \Sigma^{R}_\sigma(t,\tilde{t}) G^{\ uncertainties}_\sigma(\tilde{t},t') + \Sigma^{\ uncertainties}_\sigma(t,\tilde{t}) G^{A}_\sigma(\tilde{t},t')
\]

The coupled Dyson’s equation are solved numerically on a discrete grid

Instantaneous spectral function and current are obtained from G and B

Method can be used for arbitrary \( \varepsilon_d(t), \Gamma(t) \) and \( V_B(t) \)
AC modulation of the dot level ($V_B=0$)

Equilibrium SPF and Conductance as a function of $\varepsilon_d$

$$\varepsilon_d(t) = \varepsilon_{dot} + \varepsilon_{ac} \cos \Omega t$$

Since the dot level oscillates in time, the Kondo state is not fully formed. The conductance depends on the average spectral function over a period. In the limit of $\Omega \rightarrow \infty$, the conductance is given by $G(T, \varepsilon_{dot})$

**Dot A**
- $\varepsilon_d(t) = -5 + 4 \cos \Omega t$
- $T_K = 10^{-6}$
- $G(\Omega \rightarrow \infty)$ small for finite $T$

**Dot B**
- $\varepsilon_d(t) = -2.5 + 2 \cos \Omega t$
- $T_K = 10^{-3}$
- $G(\Omega \rightarrow \infty)$ large for finite $T$
Conductance as a function of $\Omega$

Satellites develop at $\pm N\Omega$ around the Kondo peak causing a logarithmic fall-off of $G$

Satellites develop at $\pm N\Omega$ around $\varepsilon_{\text{dot}}$

When $\varepsilon_{\text{dot}} + N\Omega$ get close to $\varepsilon_F$, the Kondo effect increase the conductance causing the nonmonotonic $G(\Omega)$

The same calculation neglecting many-electron effects would have $G(\Omega) = \text{almost constant}$
Application of a sudden voltage to the dot

The sudden bias moves the dot level from the non-Kondo regime ($T_K << T$) to a state in the Kondo regime ($T_K \sim T$)

The instantaneous conductance of the dot reflects the timescales for formation of the Kondo state

How does the spectral function of the dot level evolve in time?
Sudden dot level change response ($V_B=0$)

Instantaneous spectral function
$T_K=0.0022$, $T=0.0025$

Fast time scale
$\tau_0 \sim 1/\Gamma$

Slow time scale
$\tau \sim 1/T_K(T)$

Thermal fluctuations reduce the Kondo effect

The Kondo resonance takes a long time to form, $\tau \sim 1/T_K$

For times $t$ smaller than $\tau$, the state can be viewed as a decohered Kondo state

Equivalency between finite time and equilibrium thermal decoherence
$T_{\text{eff}}(t) = T_{\text{coth}} \frac{\pi t}{2\hbar}$
Long time limit, Universality

System 1, S1: $\varepsilon_{\text{dot}}=-2.0$, $T_K=0.0022$
System 2, S2: $\varepsilon_{\text{dot}}=-2.225$, $T_K=0.0011$

Both the transient and final conductance satisfy universality, i.e. scaling with $T_K$

Deviations from universality because of admixture of mixed valent state

$$\frac{G}{G_0} = \frac{3\pi^2}{16\ln^2(T/T_K)}$$

Abrikosov, 1965
Timescale for $V_B=0$

The approach to equilibrium follows:

$$T_{\text{eff}}(t) = T \coth \frac{\pi T t}{2}$$

The rate constant for the approach to equilibrium is $1/\tau = \pi T$

Korringa rate: A spin flip requires the creation of a zero energy e-h pair. Phase space restriction=>

$$\int d\varepsilon f(\varepsilon)[1 - f(\varepsilon)] = T$$

Deviation from simple formula for $T<T_K$

$1/\tau \rightarrow \pi T_K$ and possibly at large $T$ ($\ln T/T_K$)

Reshaping of spectral functions continue beyond the time where the conductance has saturated for all $T$ studied suggesting the presence of a longer timescale
Experimentally accessible quantity is the integrated current

\[
G^{\text{int}}(\tau_{\text{on}}) = \int_{0}^{\tau_{\text{on}} + \tau_{\text{off}}} dt \ G(t; \tau_{\text{on}})
\]

The derivative of \(G^{\text{int}}(\tau_{\text{on}})\) with respect to \(\tau_{\text{on}}\) is a measure of the Time scale for formation of the Kondo state
Sudden change of $V_B$ ($\varepsilon_{\text{dot}}(t) = \text{const}$)

$T_K = 0.0025 \ (V_B=0)$

$V_B(t) = V_B[\Theta(t)-\Theta(t-\tau)]$

$V_B = 0.2$

A finite bias introduces a split Kondo resonance and extra broadening of the two Kondo resonances.

Instantaneous current $I(t; \tau)$

Solid line: $I(t, \tau)$.

Fast rise time of order $\tau_1 \sim 2\pi/V_B$ (for $V_B > T_K$)

Dotted line: $I(\tau, \tau) - I(t+\tau, \tau)$.

Slower decay of order $\tau_3$. The timescale of the order $1/T_K$ and $1/T_K(V_B)$

Dashed line conductance $G(t)$ for $V_B = 0$.

Rise time of order $1/T_K(V_B = 0)$
Time scales for sudden switching of $V_B$

Split Kondo Peak (SKP) oscillations!
Quantum beating between the two Kondo resonances.
Oscillation frequency $\omega = 2\pi/V_B$

The damping of SKP oscillations occur on a “new” longer time scale $\tau_2$

The integrated current

$$Q^{\text{int}}(\tau_{on}) = \int_0^{\tau_{on} + \tau_{off}} dt \ I(t; \tau_{on})$$

$$\frac{dQ^{\text{int}}(\tau_{on})}{d\tau_{on}}$$

Measurement of $dQ^{\text{int}}/d\tau$ provides estimates of $\tau_1$ and $\tau_2$
Sudden dot level, Finite constant $V_B$

Equilibrium spectral function as a function of $V_B$ and $T$

$T_K \ll T$

$V_B = 0.04$

$T = 0.0015$

$V_B = 0.2$

Instantaneous spectral function

The magnitude and shape of the split Kondo resonance depend on $V_B$ and $T$.

The timescale for formation will also depend on $V_B$ and $T$

$$\tau = \frac{1}{T_{K,\text{Eff}} (V_B, T)}$$
Instantaneous conductance \( (I(t)/V_B) \)

The rise time and saturation conductance depend strongly on \( V_B \) for \( V_B > 4T \).

SKP oscillations are small but remain for longer time \( \tau_2 \).

Nonuniversal contributions (ringing) of the conductance on timescale \( \Gamma \).
The rise time depend strongly on $T$ for $T > 0.25V_B$

The magnitude and lifetime of the SKP oscillations decrease with increasing $T$
Long time limit, $G_{Eq}(T,V_B)$

Rosch et Al, 2001

Finite $T$ consistent with $T=0$

Korringa rate for finite bias. The phase space is increased to:

$$\frac{1}{4} \sum_{i,i'} \int \! d\epsilon f_i(\epsilon)[1 - f_{i'}(\epsilon)] = TF\left(\frac{V}{T}\right)$$

$$\Rightarrow \frac{1}{\tau(T,V)} = \pi TF\left(\frac{V}{T}\right)$$

For large $V$

$$\frac{1}{\tau(T,V)} = \pi \frac{V}{4} \quad T \leftrightarrow \frac{V}{4}$$

Rise time defined as time $\tau$ when $G(\tau) = 0.99G_{Eq}(T,V_B)$

Excellent agreement with numerical simulations for $T > T_K$
SKP oscillations (S1)

Damping of SKP oscillations occur on a much slower time scale than the rise time.

Damping depend on $V_B$

Damping depend on $T$
Instantaneous spectral function

Conductance rises and saturates. Split Kondo peak not fully formed.

Conductance is crude measure of equilibration

SKP oscillations. Formation of split Kondo Resonances

Conductance only weakly Time dependent
SKP damping rate

Calculated SKP damping rates $\tau_2$ (divided by 2!?)

Our lowest temperature result agrees well with Rosch, Kroha and Wölfe, 2001

Lower temperature simulations may be needed to get rid of the factor of 2

Our TDNCA approach is presently being extended to low T regime

The integrated current

$$Q^{\text{int}}(\tau_{on}) = \int_0^{\tau_{on} + \tau_{off}} dt I(t; \tau_{on})$$

$$\frac{dQ^{\text{int}}(\tau_{on})}{d\tau_{on}}$$

Measurement of $dQ^{\text{int}}/d\tau$ provides estimates of rise time $\tau_1$ and SKP damping rate $\tau_2$
The sudden increase in $\Gamma$ moves the dot from a non-Kondo regime ($T_K << T$) to the Kondo regime ($T_K \approx T$)

The instantaneous conductance of the dot reflects the timescales for formation of the Kondo state.
Instantaneous conductance ($\Gamma$ change)

Red: Dot level change
Black: Gamma switching

Approach to equilibrium on the same timescale as for sudden shift of the dot level

$S1$: $T=0.02$
$T_K=0.0025$
Conclusions

The application of time dependent voltages on the gates of an SET in the Kondo regime provides novel insight into the Kondo problem

- **AC modulation of the dot level**
  - Nonmonotonic dependence of $G$ on $\Omega$
  - $G$ determined by interplay between pat and Kondo effect

- **Pulsed modulation of the dot level**
  - Kondo resonance is formed on a timescale $\tau=1/T_K$
  - For finite bias $\tau=1/T^*(T,V)$
  - SKP oscillations

- **Pulsed bias across the leads**
  - Very large amplitude SKP oscillations
  - Transient currents influenced by many different timescales

- **Sudden change of the tunneling barrier**
  - Timescale for formation of Kondo state same as for dot level shift

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