Relaxation of Hot Quasiparticles in a \textit{d}-Wave Superconductor

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- pump-probe experiments in high-temperature superconductors
- relaxation of high-energy quasi-particles
- momentum conservation and diffusion in momentum space
Phase Diagram of the Cuprates

best understood (??): physics within superconducting phase
Scenario 1: Quasiparticles in superconducting phase

Microwave conductivity $\sigma(\omega)$ in $\text{YBa}_2\text{Cu}_3\text{O}_{6-\delta}$ Hosseini et al. (1999) well fitted by thermal QPs Agreement with $\lambda^{-2}(T)$.

$\tau_{tr}^{-1}$ decreases fast below $T_c$ Umklapp scattering of QPs

Walker & Smith, PRB 61, 11285 (2000)
Scenario 2: Quantum Critical Scaling

\( \sigma(\omega) \) in Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_8\) requires more complicated model

\[ \rightarrow \hbar \tau^{-1}_{qp} \approx 0.8 k_B T \]

This agrees with ARPES:
Nodal QPs of energy \( \omega \) show ‘quantum critical scaling’:

\[ \hbar \tau^{-1}_{qp} \approx \max[\hbar \omega, k_B T] \]

above and below \( T_c \)
(marginal Fermi liquid ??)

\( \diamond = 48 \text{ K}, \triangle = 90 \text{ K}, \circ = 300 \text{ K} \)
Pump–Probe Experiments in Superconductors

**Pump** with laser pulse — **probe** optical response

Step 0: excitation of QP (1 eV)

Step 1 ( < 1ps):
cascade of pair–breaking down to gap

Step 2 (1−10ps):
This talk!

Step 3 (ns − μs):
phonons equilibrated with hot QP transport energy out of system
Rothwarf, Taylor (1967)
Pump–Probe Reflectivity on YBa$_2$Cu$_3$O$_{6.5}$

time-resolved reflectivity $\delta R \propto \int d\omega \omega^2 \delta \sigma(\omega)$ after pumping

Segre, Gedik, Orenstein, Bonn, Liang, Hardy et al., PRL 88, 137001 (2002)

QPs relax very fast down $\sim \Delta_0$, then slowly with $\tau^{-1} \propto T^3$

Mystery:
QP energy $\sim \Delta_0 \gg T$ so expect $\tau$ independent of $T$. 
More experiments...

in general: very different time-scales measured in pump-probe experiments in HTc depending on doping, sample quality, frequency, intensity, temperature

some recent experiments:

- **Diffusion of nonequilibrium quasi-particles in a cuprate superconductor**, (Gedik et al. Science 2003):
  writing a interference pattern on crystal to obtain diffusion

- **Dynamics of Cooper pair formation in Bi$_2$Sr$_2$CaCu$_2$O$_{8-\delta}$** (Kaindl et al., Science 2003, PRB, 2005):
  time-dependence of $\sigma(4\text{meV} < \omega < 11\text{meV})$

- **Nonequilibrium quasiparticle relaxation in the vortex state of La$_{2-x}$Sr$_x$CuO$_4$**, (Bianchi et al. PRL 2005)

- **Abrupt transition in quasiparticle dynamics at optimal doping in a cuprate superconductor system** (Gedik, et al. PRL 2005): different sign and different density dependence for under- and overdoped samples!
underdoping vs. overdoping (Gedik, et al. PRL 2005)

need other hot QP to free trapped excitations for underdoped system?
Fermi Liquid Interpretation

Photoinduced QPs create cascade of QPs, i.e. excite pairs out of condensate.

Some QPs end up near gap antinodes with energy $\Delta_0$.

**How do these ‘hot’ QPs relax?**

- $E$ and $k$ conservation $\Rightarrow$ **bottleneck** for pair-breaking processes.

Intermediate $T$:
- Umklapp **scattering from thermal QPs** with activation gap $\lesssim \Delta_0$.

Low $T$:
- Slow relaxation by **scattering from thermal QPs** at nodes.

NB: Clean crystals $\Rightarrow$ negligible impurity scattering
Bottleneck for Pair-Breaking

Scattering process $d^\dagger_{-p-q}d^\dagger_p d^\dagger_{k+q} d_k$

**Idea**: antinodal velocity $< \text{nodal velocity} \Rightarrow$
antinodal QP cannot lose enough energy to generate nodal QP-pairs

\[
E^\text{AN}_k = \Delta_0 \cos 2\theta + \frac{2k^2 E_F^2}{\Delta_0 \cos 2\theta}
\]

\[
E^\text{N}_p = 2\sqrt{E_F^2 p_\perp^2 + \Delta_0^2 p_\parallel^2}
\]

Energy lost by antinodal QP:

\[
E^\text{AN}_k - E^\text{AN}_{k+q} = 2q\Delta_0 \sin 2\theta
\]

Energy needed to create pair:

\[
E^\text{N}_p + E^\text{N}_{-p-q}
\]

\[
= 2\sqrt{E_F^2 q^2 \cos^2 \alpha + \Delta_0^2 q^2 \sin^2 \alpha} \approx 2qE_F \cos(\frac{\pi}{4} + \theta)
\]

⇒ Pair-breaking only possible if $\theta \approx \frac{\pi}{4}$

No scattering for $|k - k_F| \lesssim \frac{1}{\sqrt{2E_F^2}} \frac{\Delta_0^2}{\cos^2 2\theta}$
How else can Quasiparticles Relax?

Also bottleneck for phonons and spin waves. What about other QP scattering processes?

\[ \mathcal{H}_{\text{int}} = \sum_{1,2,3,4} \left[ V_{2-3}(u_1v_2 + v_1u_2)(u_3v_4 + v_3u_4)d_{4\uparrow}^\dagger d_{3\downarrow}^\dagger d_{2\uparrow}^\dagger d_{1\downarrow}^\dagger 
+ V_{2+1}(u_1u_2 - v_1v_2)(u_3v_4 + v_3u_4)d_{4\sigma}^\dagger d_{3\bar{\sigma}}^\dagger d_{2\bar{\sigma}}^\dagger d_{1\sigma}^\dagger 
+ V_{2+1}(u_1v_2 + v_1u_2)(u_3v_4 + v_3u_4)d_{4\sigma}^\dagger d_{3\bar{\sigma}}^\dagger d_{2\bar{\sigma}}^\dagger d_{1\sigma}^\dagger 
+ V_{2-3}(u_1u_4 - v_1v_4)(u_2u_3 - v_2v_3)d_{4\sigma}^\dagger d_{3\bar{\sigma}}^\dagger d_{2\bar{\sigma}}^\dagger d_{1\sigma}^\dagger 
+ \text{h.c.} \right] \]

Consider QP–QP scattering terms:
Low density of hot QPs, so **scatter off thermal QPs.**
Umklapp Scattering

QP–QP scattering: \(d_{k+q}^\dagger d_{p-q}^\dagger G d_p d_k\)

Large momentum transfer possible, \(q \sim k_F\), but thermal QP can’t be arbitrarily close to node

\[\Rightarrow \tau_{U}^{-1} \sim e^{-E_{\text{act}}/T}\]

with activation energy \(E_{\text{act}} \lesssim \Delta_0\)

exponentially suppressed at low \(T\)!
Scattering from nodal QPs: \( d_{k+q}^\dagger d_{p-q}^\dagger d_p d_k \)

Discrepancy between nodal and antinodal velocities
\[ \Rightarrow q = (q_\parallel, q_\perp) \sim \left( \frac{T}{\Delta_0}, \frac{T}{E_F} \right) \ll k_F \]

small transferred momentum \( \Rightarrow \) **diffusion in momentum space**

hot QP reaches **quasi-equilibrium** with nodal QPs: \( |k - k_F| \sim \sqrt{T \Delta_0}/E_F \)

- for \( T \ll \frac{\Delta_0^3}{E_F^2} \) \( \Rightarrow \) \( |k - k_F| \ll \frac{\Delta_0^2}{E_F^2} \) \( \Rightarrow \) hot QP inside bottleneck

- for \( T \gg \frac{\Delta_0^3}{E_F^2} \) \( \Rightarrow \) \( |k - k_F| \gg \frac{\Delta_0^2}{E_F^2} \) \( \Rightarrow \) hot QP outside bottleneck
Boltzmann Equation at $T \ll \Delta_0^3/E_F^2$ (1)

Calculate evolution of antinodal distribution $g_k(t)$.

$$\frac{\partial g_k}{\partial t} = -\int dq \left[ g_k G_{qq}(q, E_k - E_{k+q}) - g_{k-q} G_{qq}(q, E_{k-q} - E_k) \right],$$

$$G_{qq}(q, \varepsilon) = |V_{qq}|^2 \int dp f_p (1 - f_{p-q}) \delta(E_p - E_{p-q} + \varepsilon)$$

Hot QPs ‘thermalise’ in radial direction $\rightarrow$ transformation:

$$g_k(t) = \frac{C_k(t)}{h_\theta} e^{-\beta E_k}, \quad h_\theta = \int (k_F + k) dk \ e^{-\beta E_k}$$

Quasi-equilibrium in radial direction

$\Rightarrow C_k$ only weakly dependent on $|k|$.

$$C_k = C_\theta^0 + (k - \langle k \rangle) C_\theta^1 + \cdots$$
Boltzmann Equation at $T \ll \Delta_0^3/E_F^2$ (2)

Project out $C_\theta^0$ and do gradient expansion since $q \ll k_{th}$.

$$\frac{\partial}{\partial t} \left[ \frac{C_\theta^0(t)}{h_\theta} \right] = \left[ F \theta \frac{\partial}{\partial \theta} + D \frac{\partial^2}{\partial \theta^2} \right] \frac{C_\theta^0(t)}{h_\theta}$$

(1)

where $F = 4\beta \Delta_0 D = \frac{7\pi^5 |V_{qq}|^2 T^4}{80 \Delta_0^3 E_F^2}$

Recall $g_k(t) = \frac{C_k(t)}{h_\theta} e^{-\beta E_k}$ Equilibrium? $C_\theta^0(t) = h_\theta$. √

Particle conservation? $N = \int dk g_k = \int d\theta C_\theta^0$

Re-write (1) as

$$\frac{\partial}{\partial t} C_\theta^0 = -F \frac{\partial}{\partial \theta} (\theta C_\theta^0) + D \frac{\partial^2}{\partial \theta^2} C_\theta^0$$

Right-hand side has only gradient terms. √

⇒ Structure of equation + ratio $F/D$ fixed by symmetries!
Diffusion Equation

\[
\frac{\partial}{\partial t} C^0_\theta = -F \frac{\partial}{\partial \theta} \left( \theta C^0_\theta \right) + D \frac{\partial^2}{\partial \theta^2} C^0_\theta
\]

- force due to energy gradient from antinode
- diffusion

Dominant contribution from \( \phi \approx \frac{\pi}{4} \)

⇒ Zig-zig diffusion in momentum space

\[
C_\theta(t) = \frac{1}{\sigma(t) \sqrt{2\pi}} \exp \left[ -\frac{\theta^2}{2\sigma^2(t)} \right], \quad \sigma^2(t) = (D/F + a^2/2)e^{2Ft} - D/F
\]

Hot QP reaches node after time:

\[
\tau_\theta \sim \frac{1}{F} \ln \min \left[ \sqrt{\frac{F}{D}}, \frac{1}{a} \right] \sim \frac{\Delta^3_0 E^2_F}{|V_{qq}|^2 T^4}
\]
Check

What about next terms in $C_k = C_0^0 + (k - \langle k \rangle)C_1^1 + \cdots$? Project out equation for $C_1^1$:

$$C_1^1(t) = C_1^1(0) \exp \left[ -\frac{tE_F^2}{\tau_0 \Delta^2_0} \right]$$

→ assumption of radial equilibrium justified.
Intermediate Temperatures \( (T \gg \Delta_0^3/E_F^2) \)

\[ |k - k_F| \sim \frac{\sqrt{T\Delta_0}}{E_F} \gg \frac{\Delta_0^2}{E_F^2} \]

so outside pair-breaking bottleneck

But

- detailed balance satisfied (both pair-breaking and recombination)
- QPs are created near nodes \( \Rightarrow \) number of antinodal QPs conserved
- Small momentum transfer, \( q \ll k_F \)

\[ \rightarrow \text{same form of diffusion equation, with different } D \]

\[ \tau_\theta \sim \frac{E_F^5}{|V|^2 \sqrt{T^5 \Delta_0^3}} \]

with equal contributions from pair-breaking and QP–QP scattering.
Latest data (Gedik et al. 2004) consistent with activated behaviour with $E_{\text{act}} \approx 8 \text{ meV}$

Why does activated behaviour dominate at such low $T$?

Compare quantitatively:
(using $\frac{\Delta_0}{E_F} \approx \frac{1}{8}$ and $\Delta_0 \approx 20 \text{ meV}$ for Y123):

Umklapp scattering:

Quasielastic scattering:
(for $T \gg \Delta_0^3/E_F^2 \approx 3 \text{ K}$)

Only comparable at $T \approx 4 \text{ K}$  
⇒ need to measure at lower $T$ to see diffusive process
Conclusions

- Pump-probe experiments: New information about strongly correlated systems! How do QP interact? How does SC form dynamically? ...
- Within Fermi liquid picture: bottleneck for pair-breaking.
- Hot QPs relax slowly by scattering from thermal QPs.
- Analytic solution of Boltzmann equation: diffusion in momentum space and power-law behaviour at low temperature
- Many open questions and challenges from experiments Role of phonons? (main player in conv. SC) Main discrepancy: density dependence quantitatively inconsistent (2 orders of magnitude too weak)