Talking Through the Continuum: A Mesoscopic Multi-State Fano Resonance

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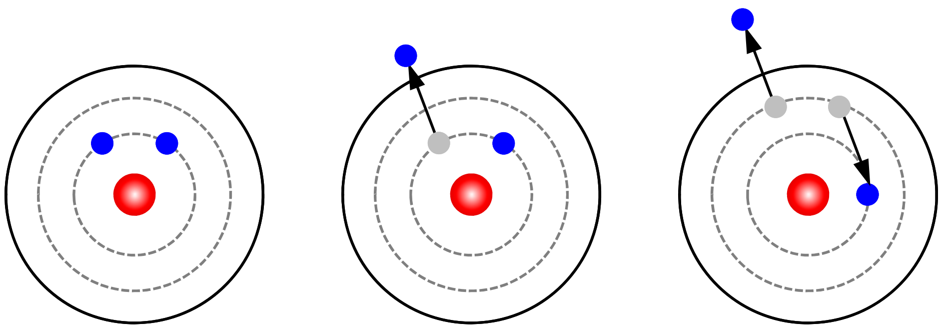
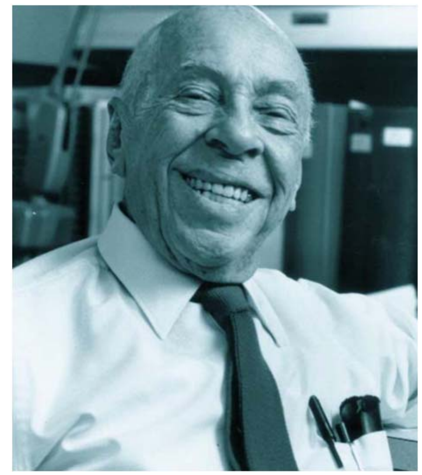
Introduction: The Fano Resonance … the Archetypal Open System

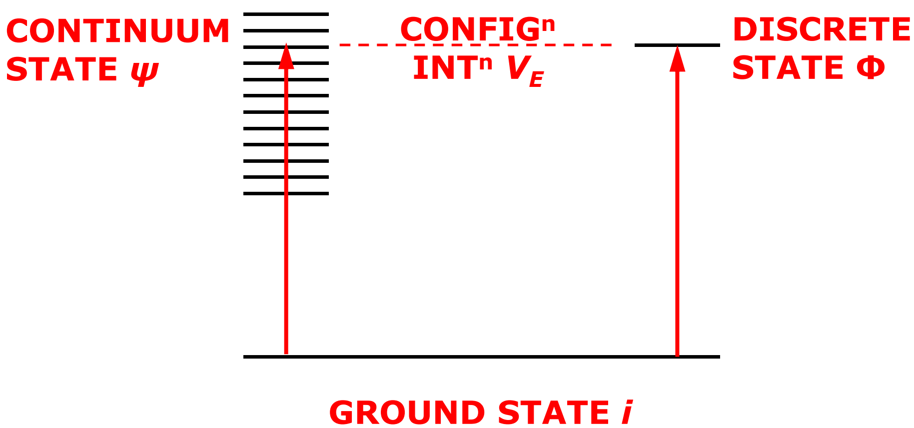
PHYSICAL REVIEW VOLUME 124, NUMBER 6 DECEMBER 15, 1961

Effects of Configuration Interaction on Intensities and Phase Shifts*

U. Fano
National Bureau of Standards, Washington, D. C.





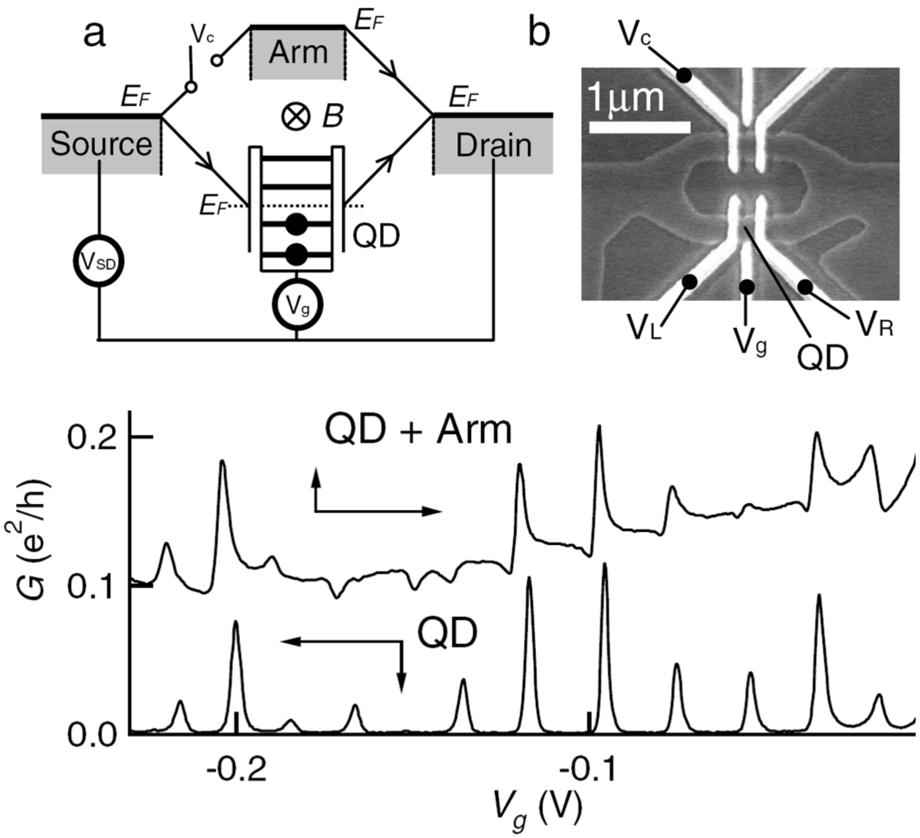


RESONANCE CROSS SECTION:

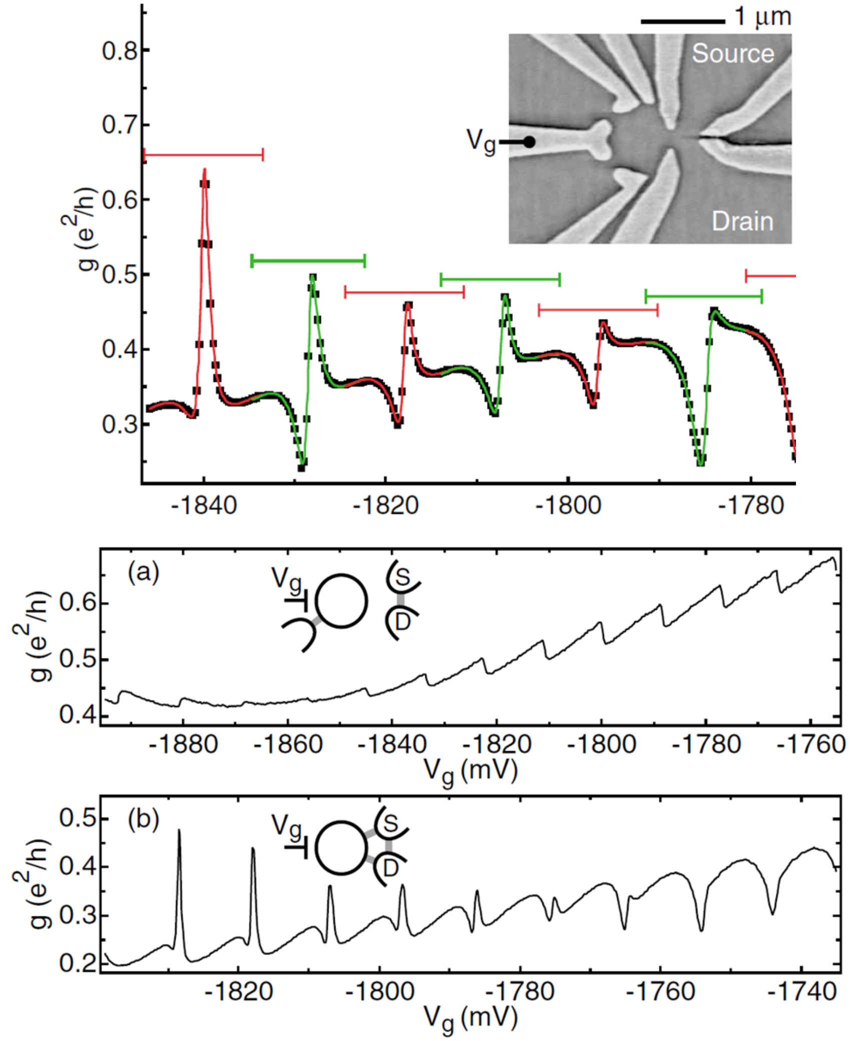
$$\sigma = \frac{(\varepsilon + q)^2}{\varepsilon^2 + 1}, \varepsilon = 2\frac{E - E_o}{\Gamma}$$

FANO SHAPE PARAMETER:

$$q = \frac{<\Phi \,|\, T \,|\, i>}{\pi V_E <\psi \,|\, T \,|\, i>}$$



K. Kobayashi et al.
Phys. Rev. Lett. 88, 256806 (2002)



A. T. Johnson et al.
Phys. Rev. Lett. 93, 106803 (2004)

MULTIPLE Demonstrations in Which a QUANTUM DOT is Used to Provide the DISCRETE State

PHYSICAL REVIEW VOLUME 124, NUMBER 6 DECEMBER 15, 1961

Effects of Configuration Interaction on Intensities and Phase Shifts*

U. Fano
National Bureau of Standards, Washington, D. C.
(Received July 14, 1961)

5. A NUMBER OF DISCRETE STATES AND ONE CONTINUUM

Consider now the situation where a set of discrete states $\varphi_1, \cdots, \varphi_n, \cdots$ experiences configuration interaction with a set of states $\psi_{E'}$ belonging to one continuous spectrum. The energy submatrix which we want to diagonalize is

$$(\varphi_m|H|\varphi_n)=E_n\delta_{mn}, \tag{45a}$$

$$(\psi_{E'}|H|\varphi_n)=V_{E'n}, \tag{45b}$$

$$(\psi_{E''}|H|\psi_{E'})=E'\delta(E''-E'). \tag{45c}$$

Equation (45a) implies that the smaller submatrix

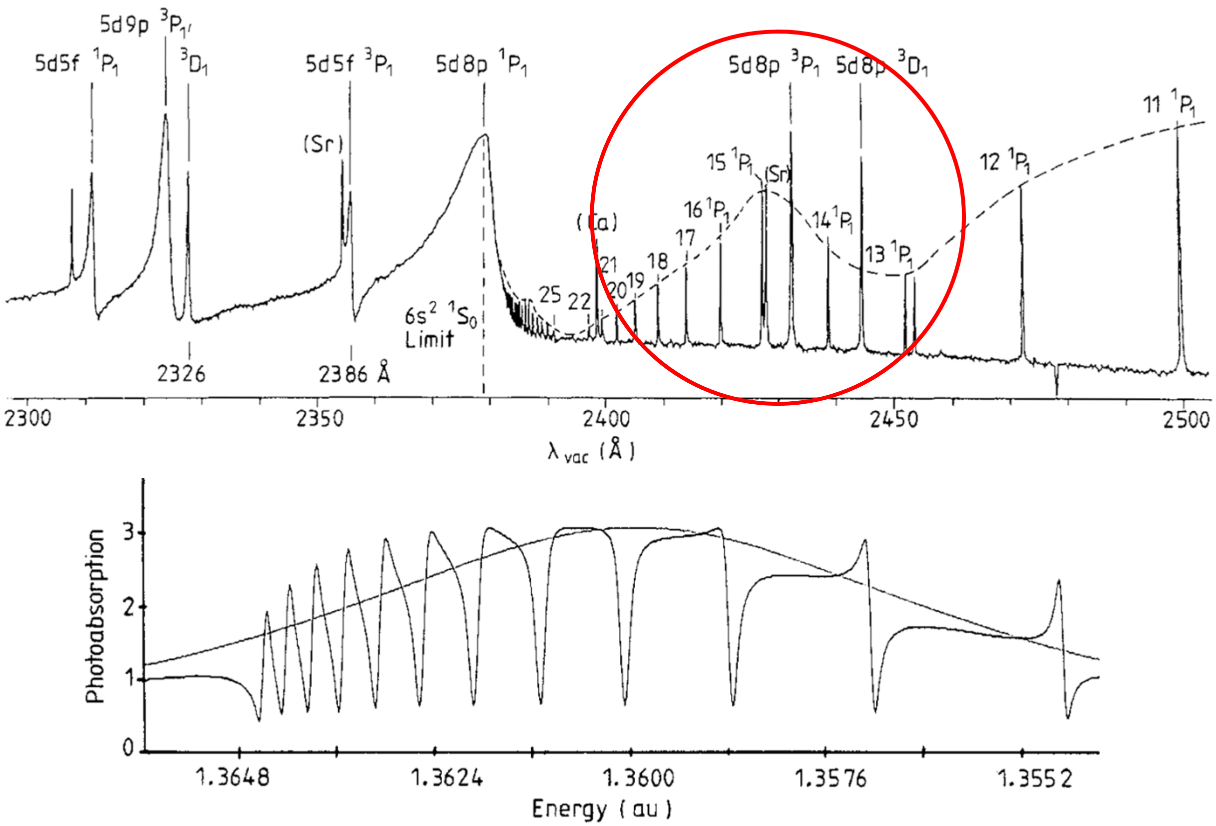
Fano ALSO Predicted MULTI-STATE Resonances Due to Interaction With COMMON Continuum

Rep. Prog. Phys. **51** (1988) 1439–1478. Printed in the UK

Interacting resonances in atomic spectroscopy

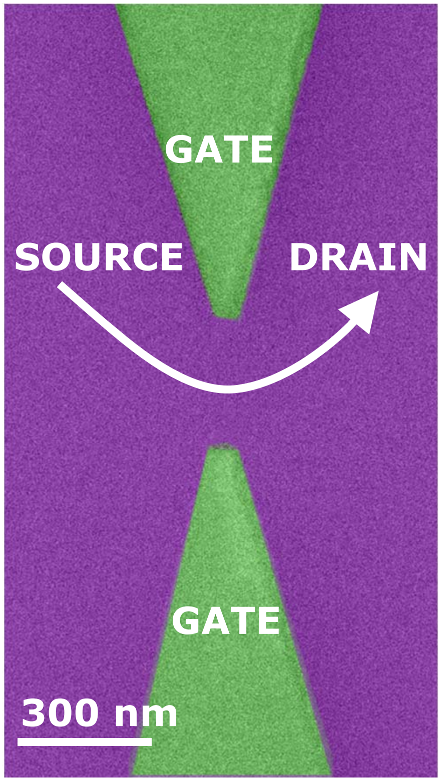
J-P Connerade† and A M Lane‡

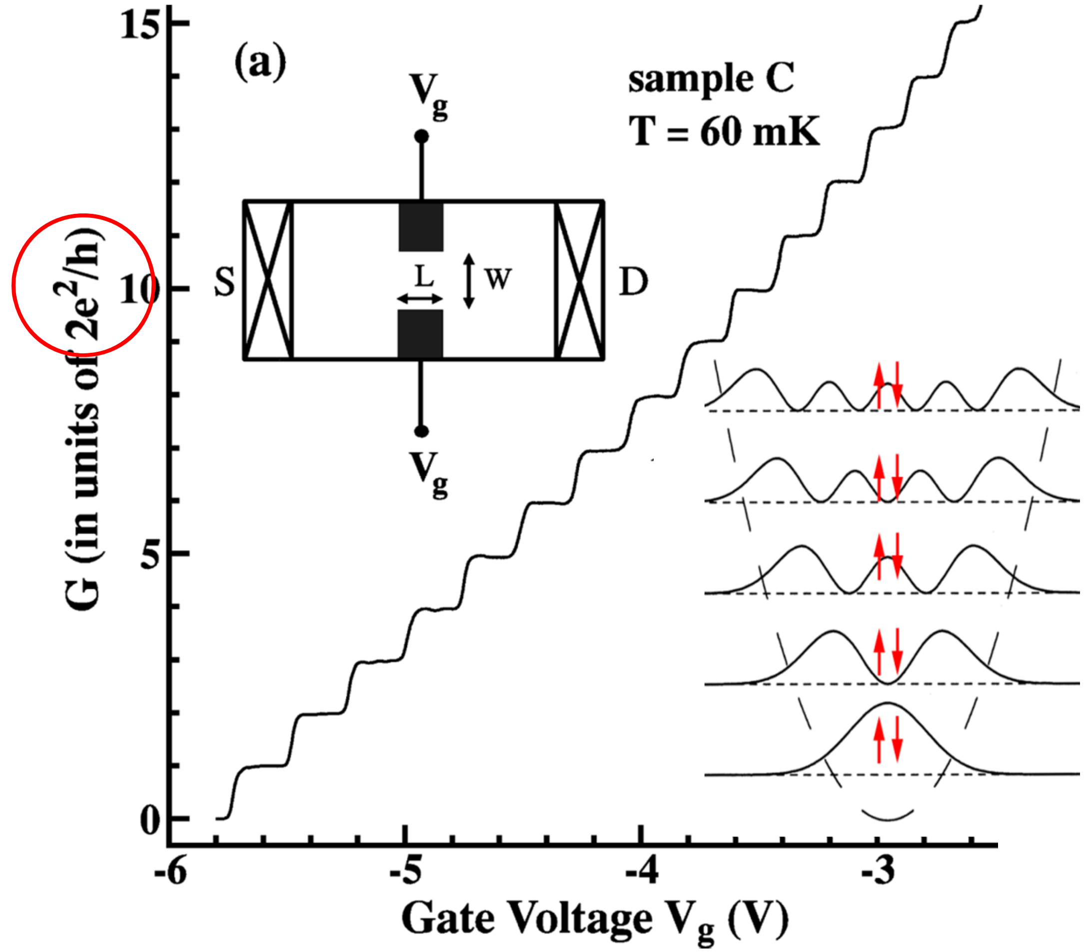
† Blackett Laboratory, Imperial College of Science and Technology, Prince Consort Road, London SW7 2BZ, UK and Physikalisches Institut der Universität Bonn, Nussallee 12, 53 Bonn, Federal Republic of Germany



FEW Examples: ***q*-REVERSAL** due to **LEVEL OVERLAP** in Rydberg Atoms Most Prominent

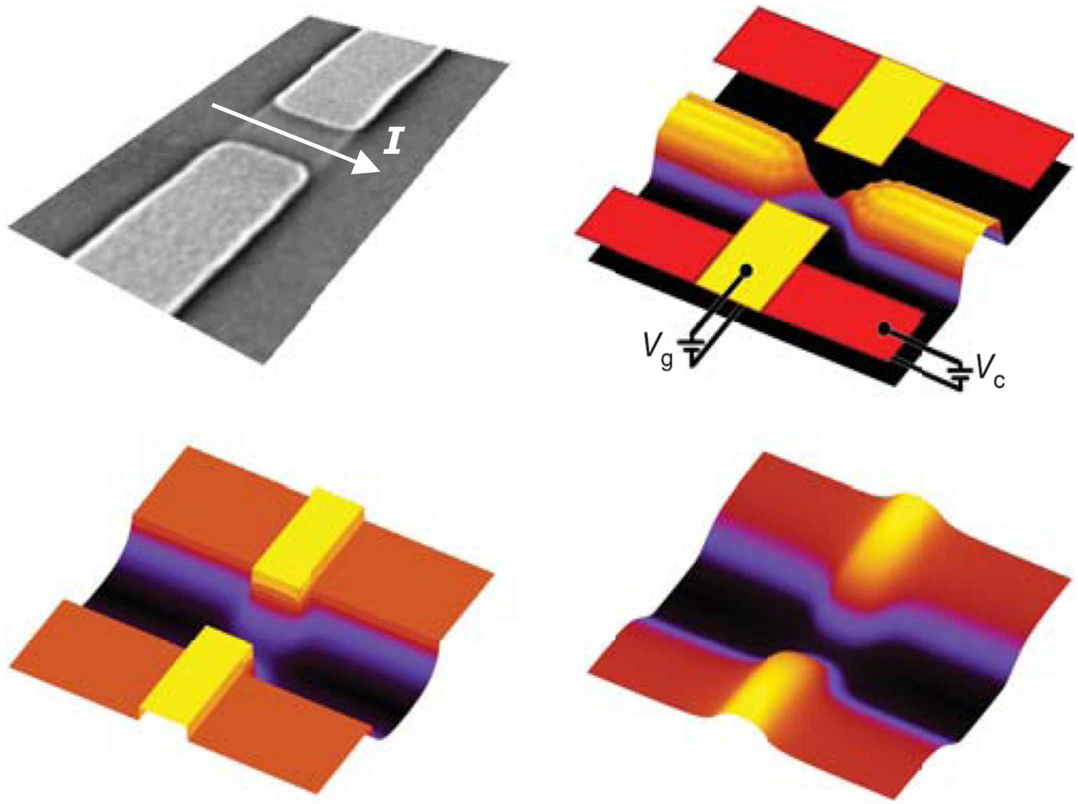
Here We Discuss a MESOSCOPIC Realization of a Multi-State Fano Resonance That Exploits the Unique Behavior of QUANTUM POINT CONTACTS





K. J. Thomas et al, Phys. Rev. B 58, 4846 (1998)

A Critical Aspect of These Devices For This Study is Their Ability to Act as a TUNABLE BARIER to Carriers ALONG their Direction of Flow



Barrier Allows Electron Density at QPC Center to Be LOWERED – Even PINCHED-OFF Completely

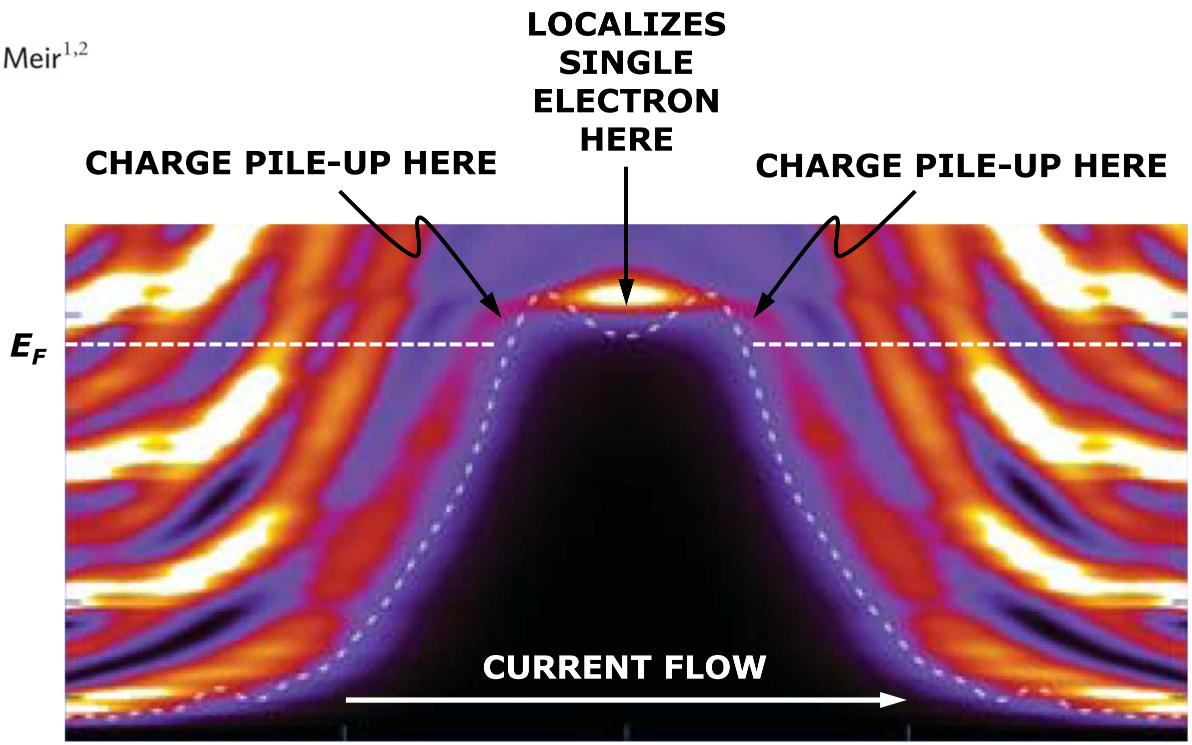
Quantum Point Contacts Near Pinch-Off

nature Vol 442|24 August 2006|doi:10.1038/nature05054

LETTERS

Magnetic impurity formation in quantum point contacts

Tomaž Rejec[1] & Yigal Meir[1,2]



BOUND STATE Thought to SPONTANEOUSLY Form in These Structures at PINCH-OFF

... Suggested by Numerous MANY-BODY Theories

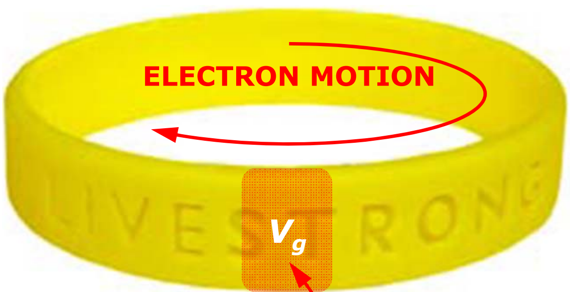
Reference	Approach	Features
T. Rejec, Y. Meir Nature **442**, 900 (2006)	Local Spin-Density Approximation	• Self-consistently formed **bound state** for single electron near pinch-off • Kondo effect from dynamically-fluctuating spin
A.D. Güçlü et al. PRB **80**, 201302 RC (2009)	Variational & Diffusion Quantum Monte Carlo	• Inhomogeneous wire with low-density region where interactions dominant • **Single-electron localization** at low density
E. Welander et al. PRB **82**, 073307 BR (2010)	Local Spin-Density Approximation	• Inhomogeneous wire with low-density region where interactions dominant • Electron **bound state** arises from Coulomb interactions & evolves sensitively with density
J.H. Hsiao, T.M. Hong PRB **82**, 115309 (2010)	Non-Equilibrium Green Functions with Spin-Orbit (Rashba) + Electron Interactions	• Spin-orbit interactions induce **local-moment** when QPC symmetry broken by source bias • Resulting spin polarization enhanced by role of Coulomb interaction
T. Song, K.H. Ahn PRL **106**, 057203 (2011)	Exact Diagonalization	• Scattering resonances due to 1D-2D transition • Resonant levels used to compute eigenstates of interacting electrons • Dependent on filling both **Kondo effect** & **ferromagnetic** character obtained!

PHYSICAL REVIEW B **82**, 073307 (2010)

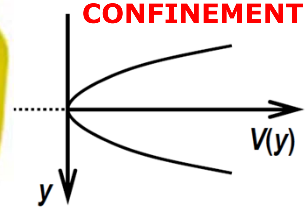
Localization of electrons and formation of two-dimensional Wigner spin lattices in a special cylindrical semiconductor stripe

E. Welander,* I. I. Yakimenko, and K.-F. Berggren

Department of Physics, Chemistry and Biology, Linköping University, S-58183 Linköping, Sweden



PARABOLIC LATERAL CONFINEMENT



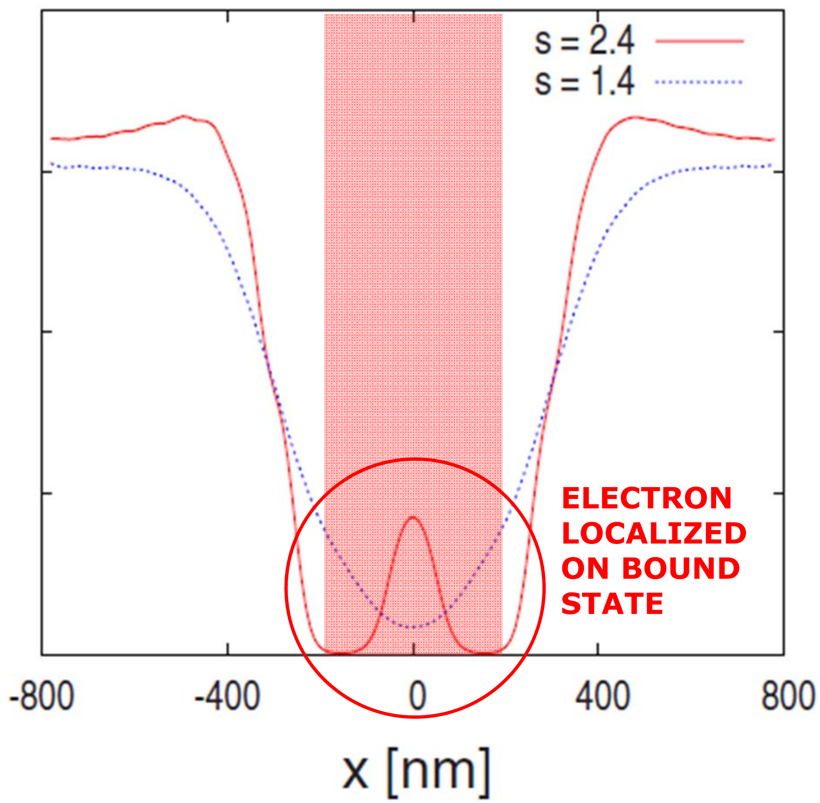
$$V_{\text{conf}} = \frac{1}{2} m^* \omega^2 y^2 + V_g \{\tanh[s(\theta + \theta_0)] - \tanh[s(\theta - \theta_0)]\}$$

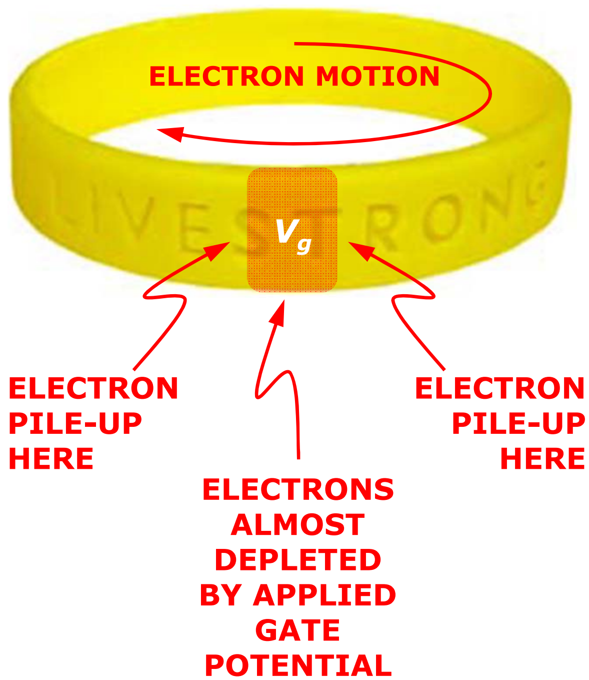
The barrier is effectively rectangular for s in the range of 4 or higher[8] but at lower values of s the edges become smoothed and eventually the barrier becomes more like a conventional QPC saddle

In the present case spatial and spin distributions are obtained from the self-consistent solution of the Kohn-Sham (LSDA) equations for the occupied electron orbitals $\Psi_k^\sigma(\sigma = \pm\frac{1}{2})$,

H^σ : V_{conf} + KINETIC ENERGY OPERATOR + COULOMB, EXCHANGE & CORRELATION TERMS

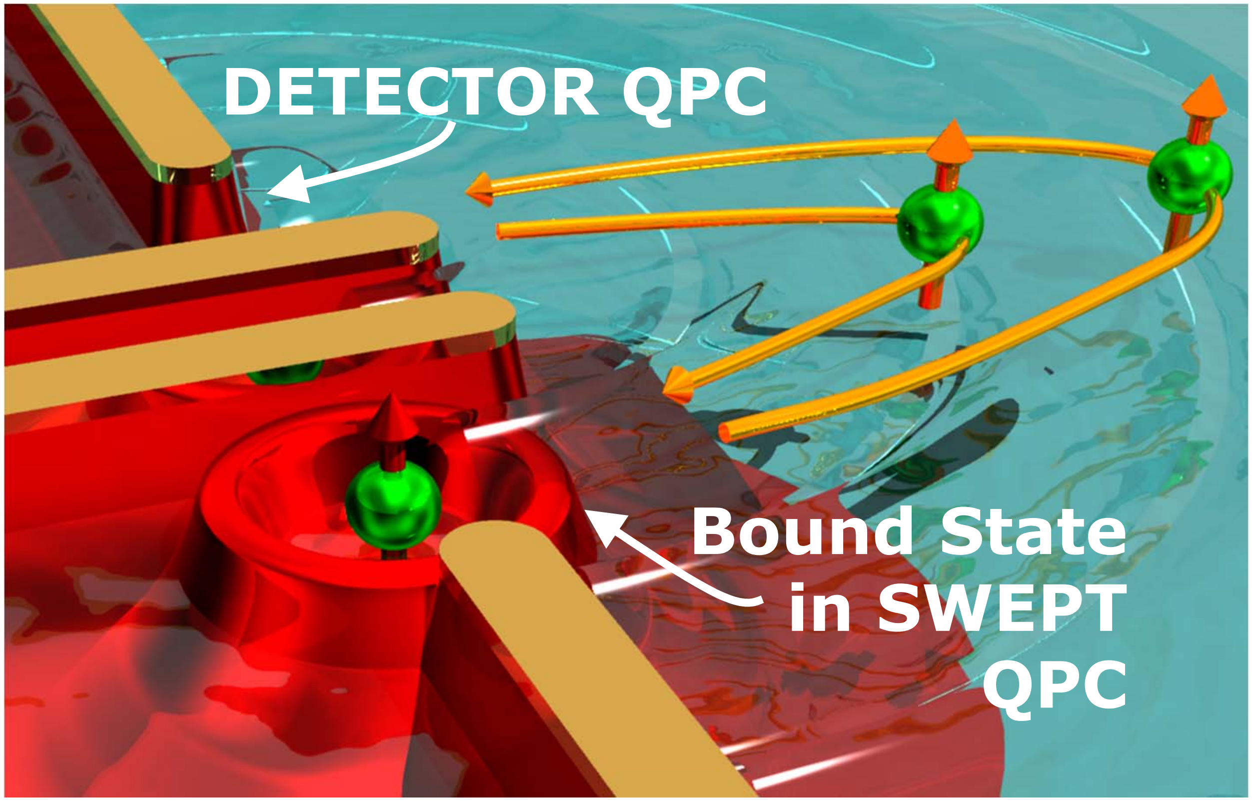
SINGLE Electron LOCALIZED on the QPC Near Pinch-Off Due to Charge PILEUP at its ENDS





Localization ELECTROSTATIC in Origin: COULOMB BLOCKADE Keeps Other Electrons OFF QPC

We Use Fano Resonances in COUPLED QPCs to DETECT the Presence of BOUND STATES



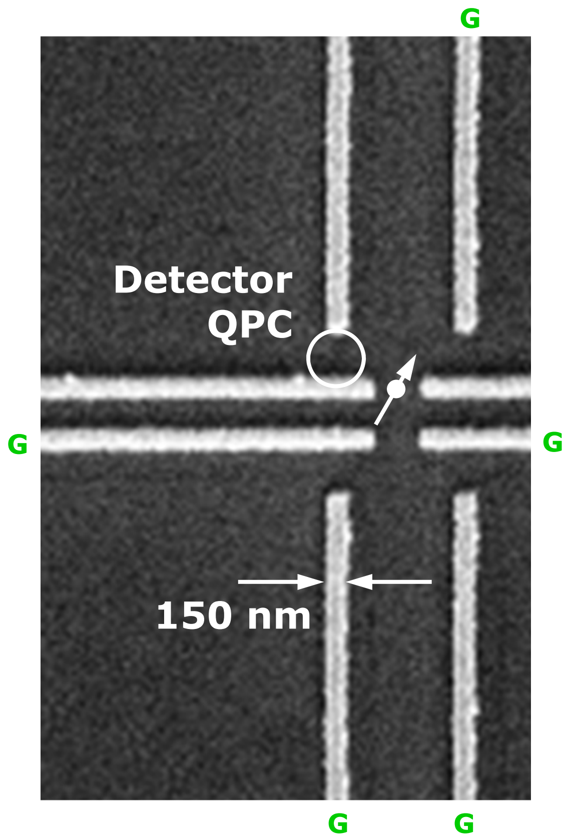
DISCRETE LEVEL: Bound State in SWEPT QPC
CONTINUUM: DETECTOR QPC & Intervening 2DEG

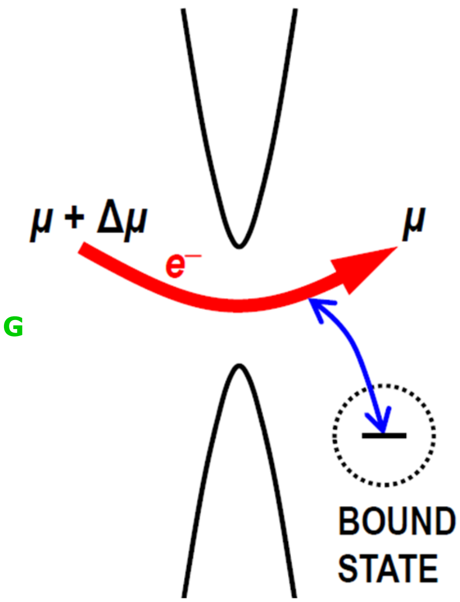
Fano Resonance Due to Bound-State Formation in Coupled QPCs

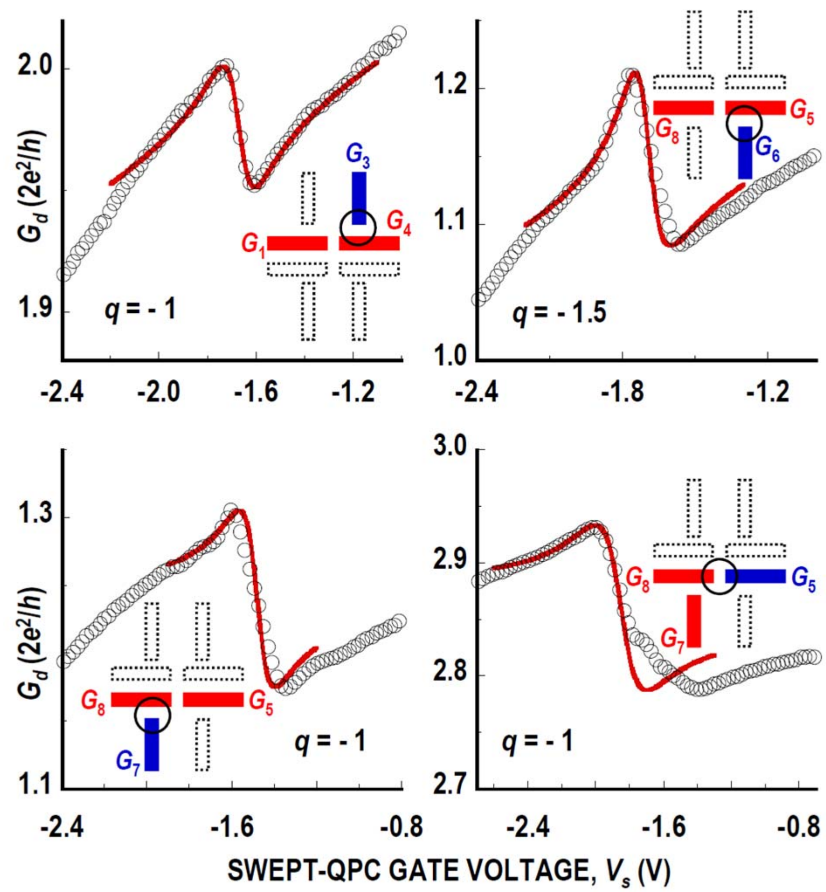
PRL **99,** 136805 (2007) PHYSICAL REVIEW LETTERS week ending 28 SEPTEMBER 2007

Probing the Microscopic Structure of Bound States in Quantum Point Contacts

Y. Yoon,[1] L. Mourokh,[2,3] T. Morimoto,[4] N. Aoki,[5] Y. Ochiai,[4,5] J. L. Reno,[6] and J. P. Bird[1]

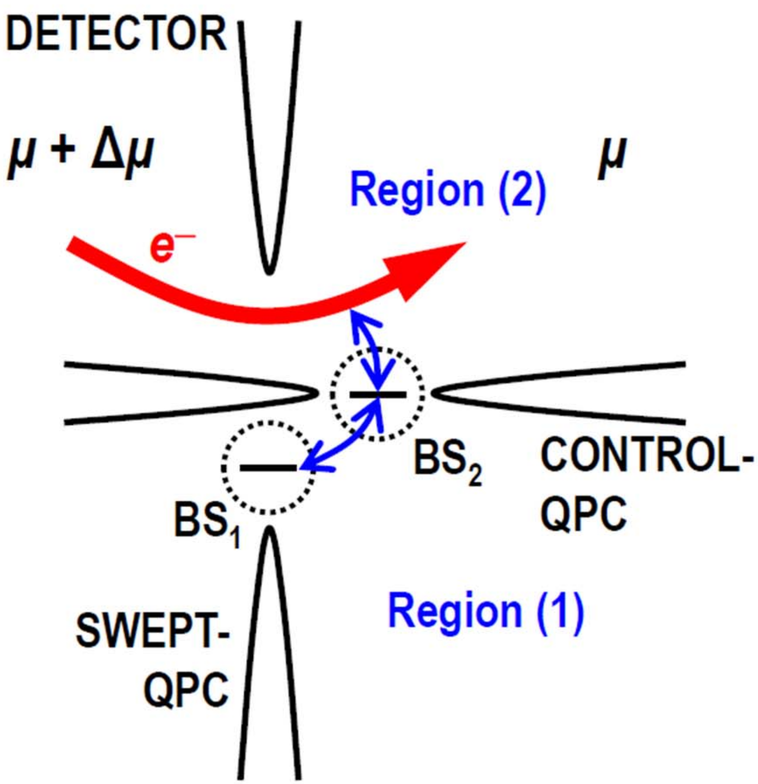


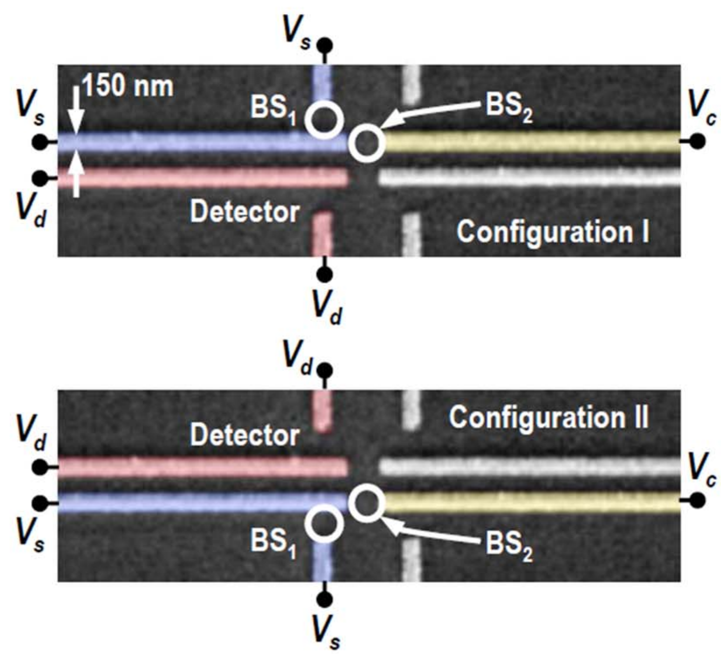




See Also:
V. Puller et al., PRL 92, 096802 (2004)
Y. Yoon et al., PRB 79, 121304(R) (2009)
Y. Yoon et al., APL 94, 213103 (2009)

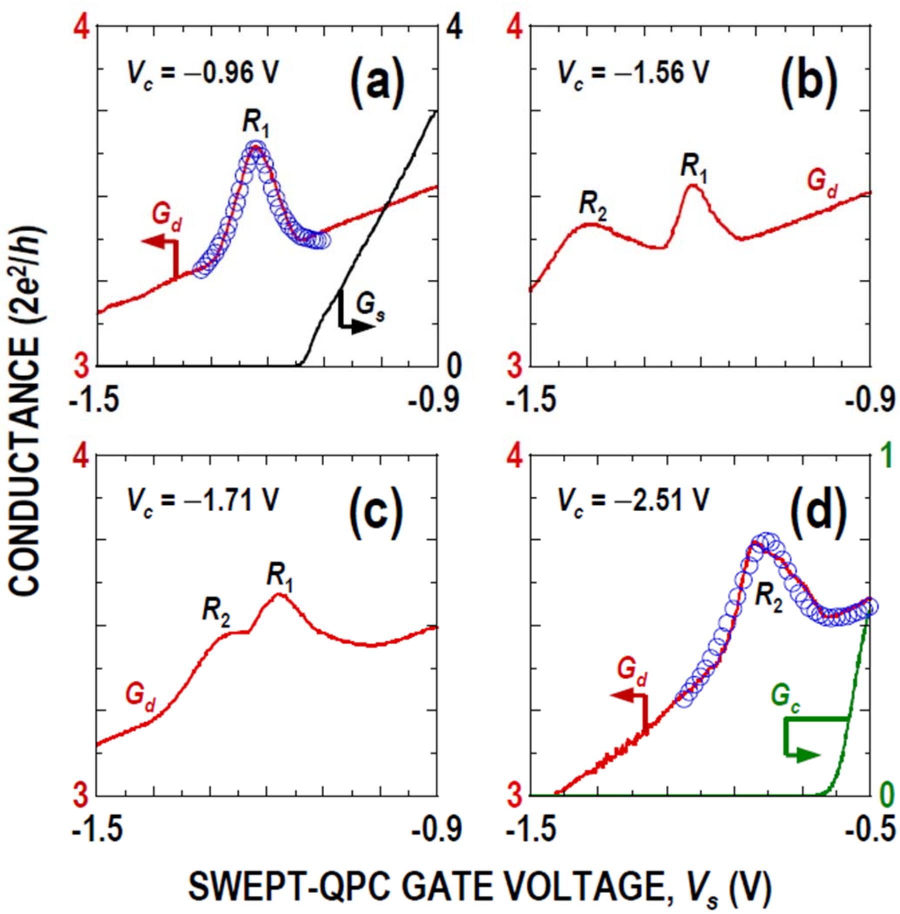
Prior Work Showed Multi-State Fano Resonances for Interacting Levels of the SAME Atom or Dot

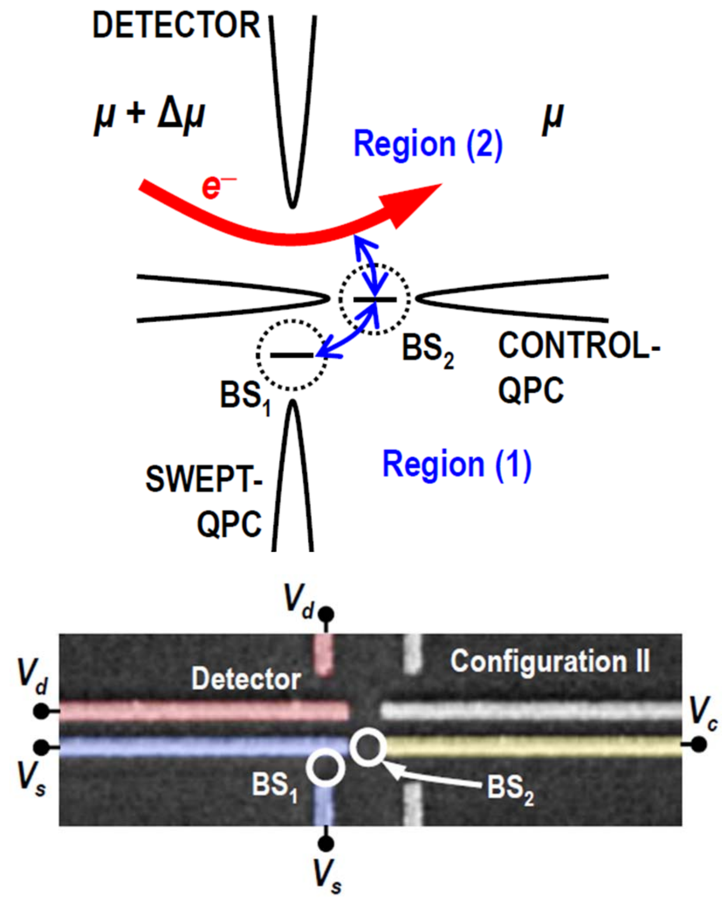




We Form REMOTE States on SEPARATE QPCs & Allow Interaction via a COMMON CONTINUUM

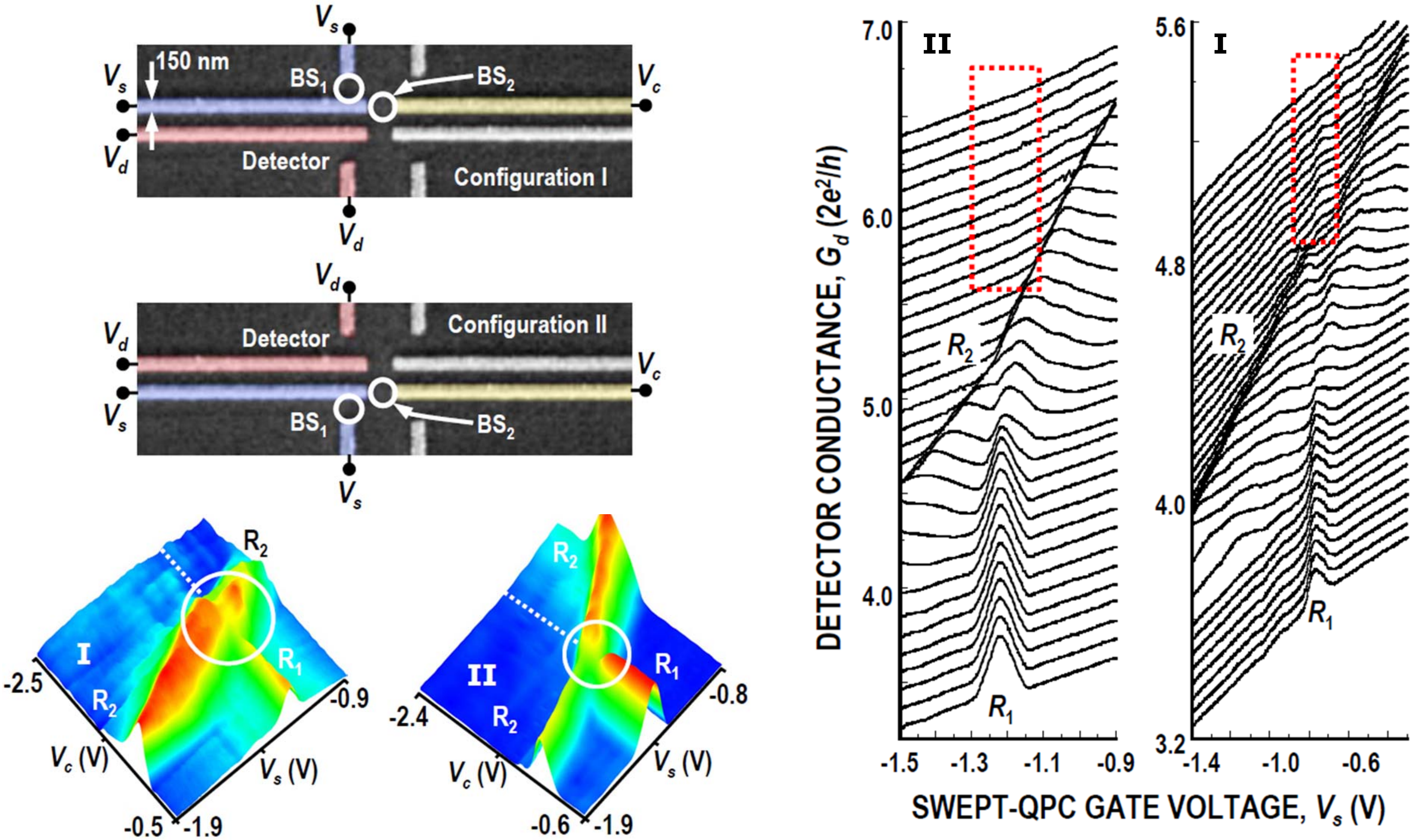
Detector Exhibits TWO Fano Resonances Due to the TWO Different Bound States





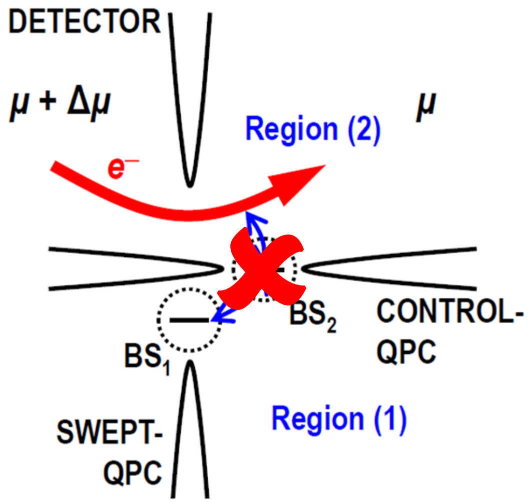
Resonance Positions CONTROLED by Respective QPC Gate Voltages (V_s & V_c)

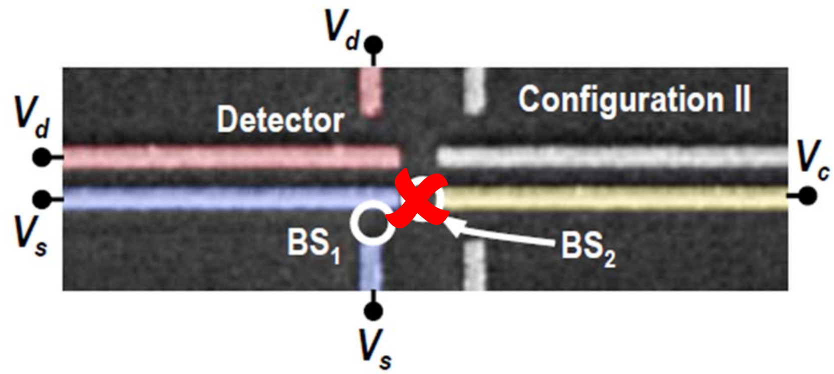
Simultaneous **TUNING** of V_s & V_c Yields an **AVOIDED CROSSING** of the Resonances



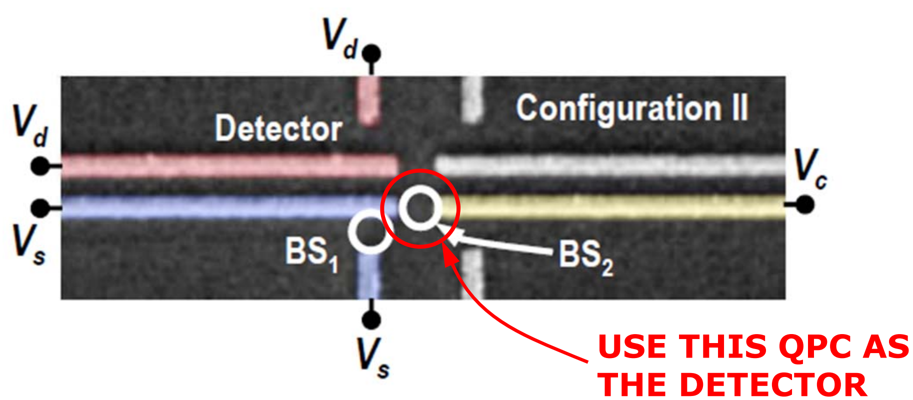
Anti-Crossing is **UNUSUAL** – **MISSING** Branch!

Missing Branch – Detector Resonance With FAR QPC When Near One is PINCHED-OFF

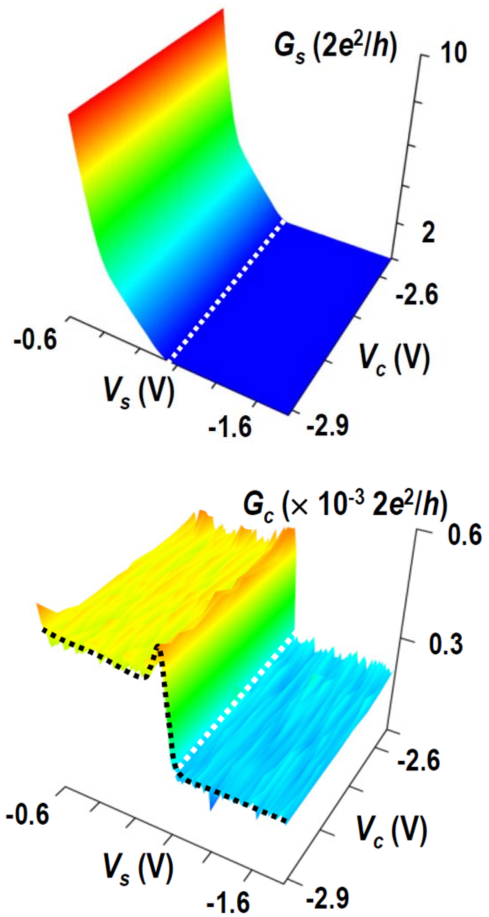


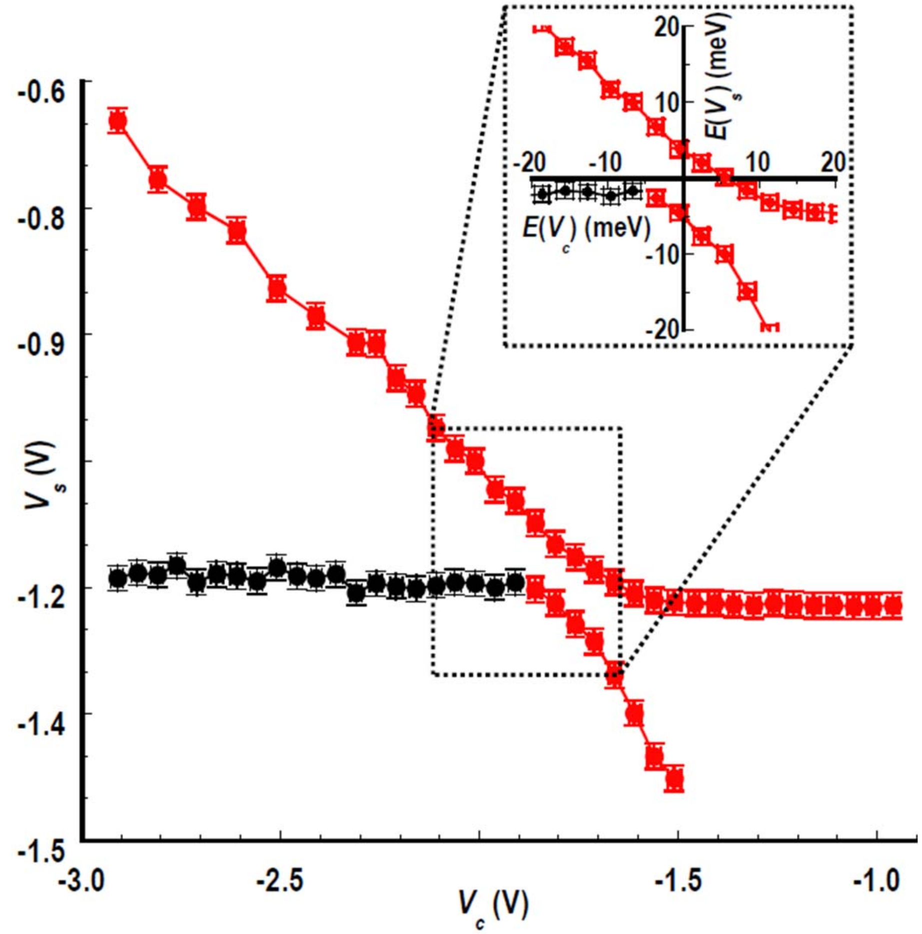


Missing Branch Can be RECOVERED Using Pinched-Off QPC as the DETECTOR



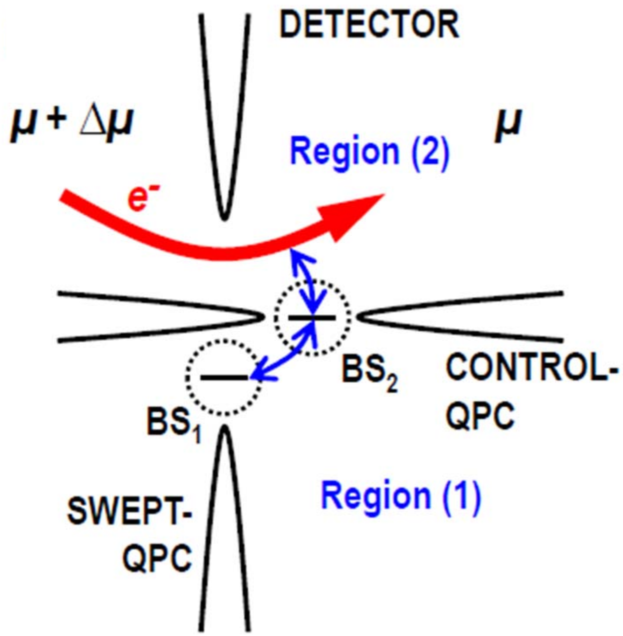
Missing Branch of the Spectrum is "RECOVERED" by Using the PINCHED-OFF QPC as DETECTOR





Reveals a PRONOUNCED (meV) Avoided Crossing!

To Describe this Problem THEORETICALLY we Start From the Following HAMILTONIAN



DETECTOR 1D MODES — CONTROL-QPC 1D MODES — 2D-CONTINUUM STATES

$$\mathcal{H} = \sum_{\mathbf{p}\sigma} \varepsilon_{\mathbf{p}\sigma} n_{\mathbf{p}\sigma} + \sum_{\mathbf{q}\sigma} \varepsilon_{\mathbf{q}\sigma} n_{\mathbf{q}\sigma} + \sum_{\mathbf{k}\sigma} E_{\mathbf{k}} n_{\mathbf{k}\sigma}$$

OCCUPATION OF BOUND STATES 1 & 2

$$+ \sum_{i\sigma} (\varepsilon_{i\sigma} + U_i n_{i\bar{\sigma}}/2) n_{i\sigma} + \left(\sum_{\mathbf{k}\in(2)\sigma} v_{\mathbf{k}\sigma 2} c^{\dagger}_{\mathbf{k}\sigma} d_{2\sigma} \right.$$

$$+ \sum_{\mathbf{k}\in(1)\sigma} v_{\mathbf{k}\sigma 1} c^{\dagger}_{\mathbf{k}\sigma} d_{i\sigma} + \sum_{\mathbf{k}\in(2)\mathbf{p}\sigma} t_{\mathbf{k}\mathbf{p}\sigma} c^{\dagger}_{\mathbf{k}\sigma} a_{\mathbf{p}\sigma}$$

$$\left. + \sum_{\mathbf{k}\mathbf{q}\sigma} t_{\mathbf{k}\mathbf{q}\sigma} c^{\dagger}_{\mathbf{k}\sigma} a_{\mathbf{q}\sigma} + H.c. \right),$$

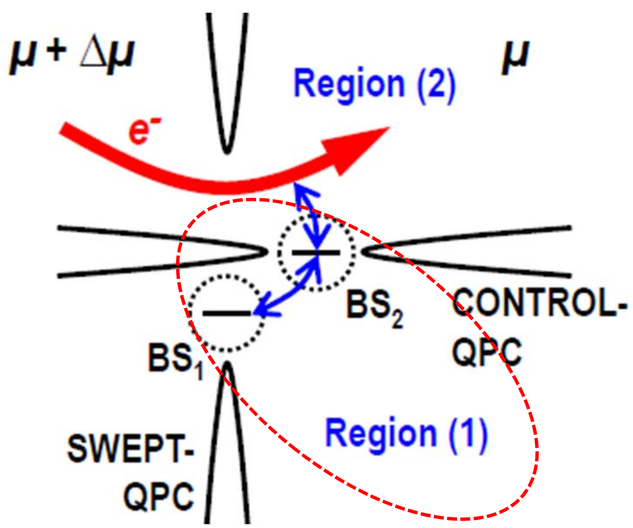
COUPLING OF TWO BOUND STATES AND DETECTOR TO INTERVENING REGIONS OF 2DEG

EXTENSION OF: V. Puller et al., PRL <u>92</u>, 096802 (2004). DESCRIBING SINGLE BOUND-STATE/ DETECTOR INTERACTION

For the SUBSYTEM Formed by the Two Bound States & Their Intervening Continuum …

$$\mathcal{H}_{BS} = \sum_{\mathbf{k}\in(1)\sigma} E_{\mathbf{k}} n_{\mathbf{k}\sigma} + \sum_{i\sigma} (\varepsilon_{i\sigma} + U_i n_{i\bar{\sigma}}/2) n_{i\sigma} + \sum_{\mathbf{k}\sigma i} (v_{\mathbf{k}\sigma i} c^{\dagger}_{\mathbf{k}\sigma} d_{i\sigma} + H.c.).$$

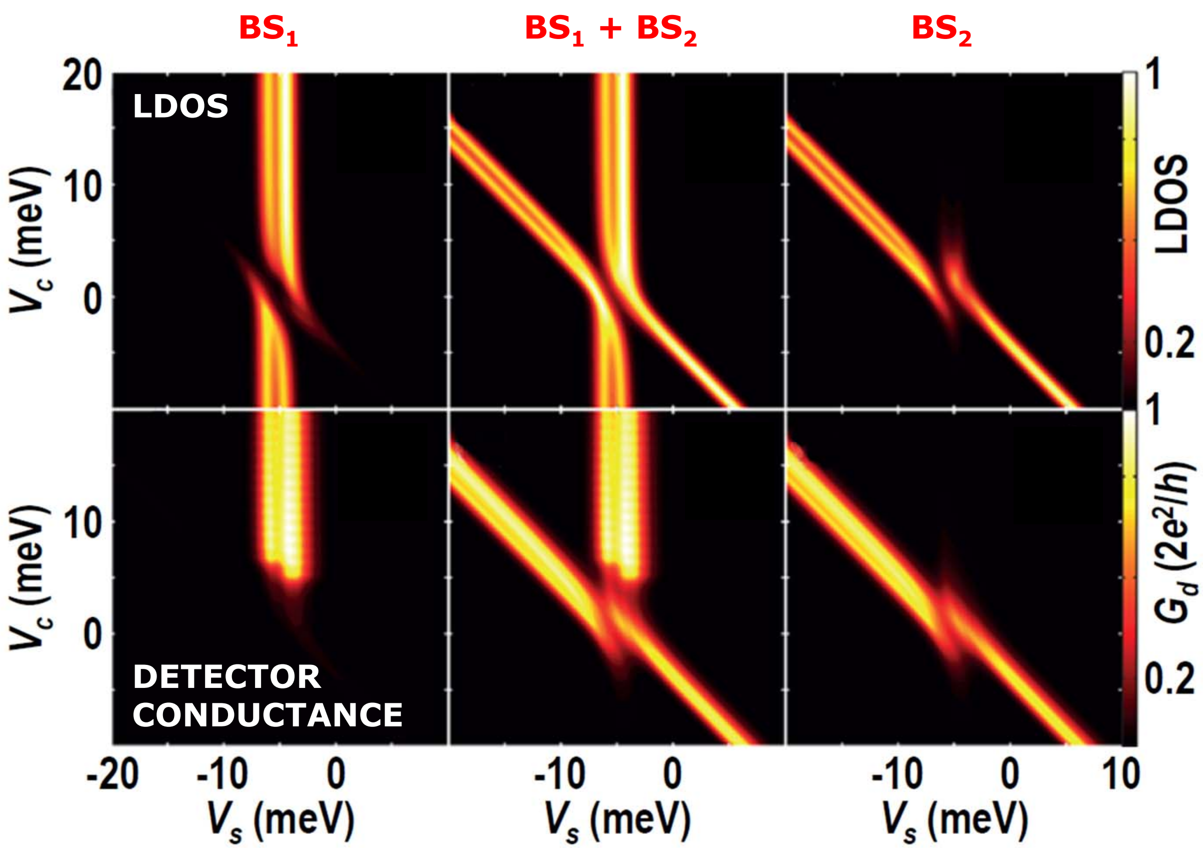
We Obtain the Effective INTERACTION POTENTIAL Between the Two Bound States

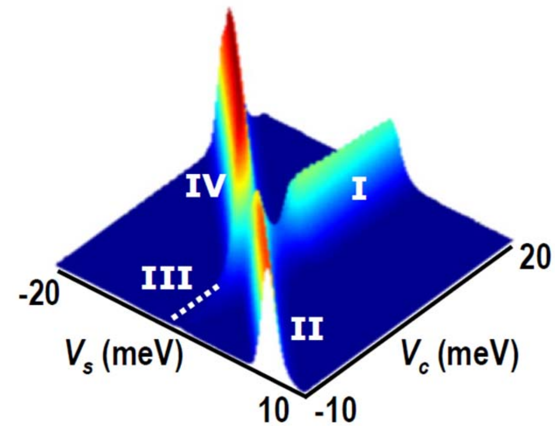


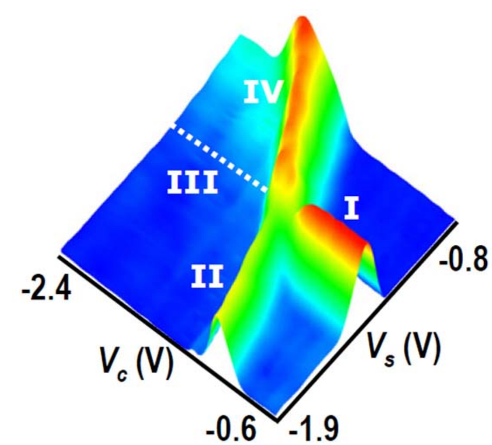
$$W_{ij} = \sum_{\mathbf{k}} v^{*}_{\mathbf{k}\sigma i} v_{\mathbf{k}\sigma j} \left(\frac{1}{E_{\mathbf{k}} - \varepsilon_{i\sigma}} - \frac{1}{E_{\mathbf{k}} - \varepsilon_{j\sigma}} \right),$$

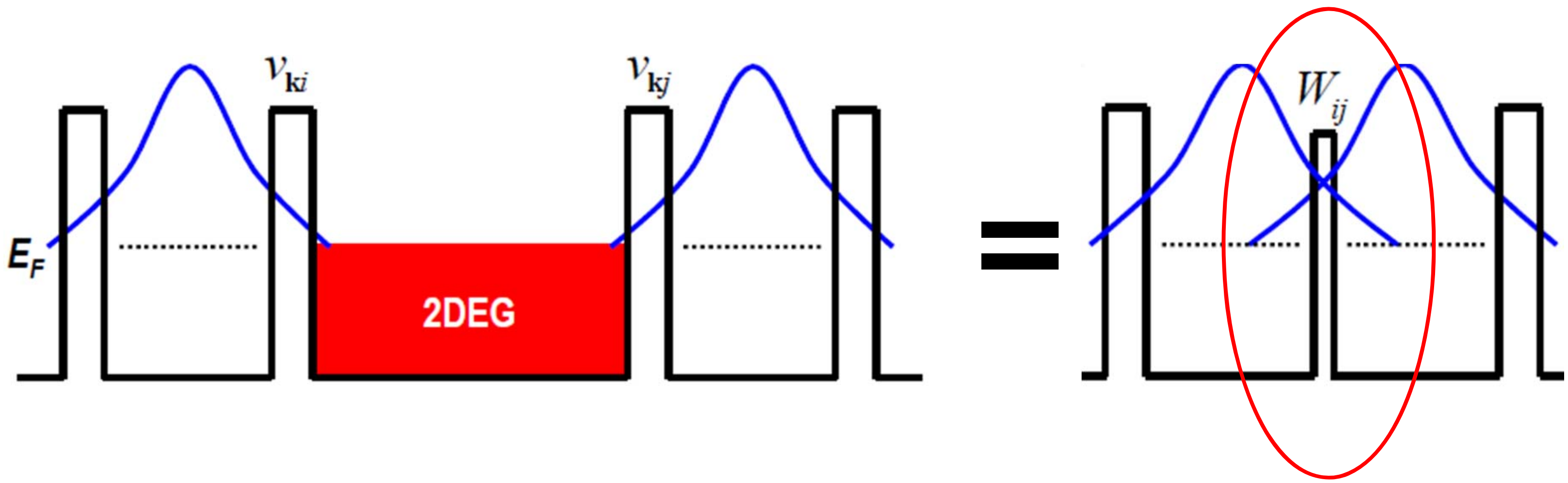
SUMMATION RUNS OVER ALL STATES OF THE INTERVENING 2D CONTINUUM!

Calculations Based on This Model REPRODUCE the Unusual Avoided-Crossing of Experiment









pled to. The implication of our analysis is that we can essentially replace the set-up consisting of the two BSs and their intervening 2DEG (Fig. 10(a)) with an effective model that more closely resembles the double-well potential characteristic of quantum-dot molecules (Fig. 10(b)). In this representation, the two BSs can be considered to effectively be directly coupled to each other, by a potential barrier that is actually lower than the barriers that couple the BSs to the 2DEG. With this coupling

Conclusions

➊ **We Have Demonstrated a Multi-State Fano Resonance in Which Two Spatially-Remote Discrete States are Each Coupled to a Common Continuum**

➋ **This Continuum Supports a Highly-Robust Effective Interaction Between the Two States Due to the Fact That it is Mediated by a Large Number of Degenerate Continuum States**

➌ **While the Continuum is Often Viewed as a Source of Decoherence Our Work Suggests its Use to Engineer the Interactions of Mesoscopic Structures**