Talking Through the Continuum: A Mesoscopic Multi-State Fano Resonance

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Introduction: The Fano Resonance ... the Archetypal Open System

Effects of Configuration Interaction on Intensities and Phase Shifts*

U. Fano
National Bureau of Standards, Washington, D. C.

\[
\sigma = \frac{(\epsilon + q)^2}{\epsilon^2 + 1}, \quad \epsilon = \frac{2(E - E_0)}{\Gamma}
\]

\[
q = \frac{\langle \Phi \mid T \mid i \rangle}{\pi V_E \langle \psi \mid T \mid i \rangle}
\]
MULTIPLE Demonstrations in Which a QUANTUM DOT is Used to Provide the DISCRETE State
5. A NUMBER OF DISCRETE STATES AND ONE CONTINUUM

Consider now the situation where a set of discrete states $\varphi_1, \cdots, \varphi_n, \cdots$ experiences configuration interaction with a set of states $\psi_{E'}$, belonging to one continuous spectrum. The energy submatrix which we want to diagonalize is

\[
(\varphi_m | H | \varphi_n) = E_n \delta_{mn}, \tag{45a}
\]

\[
(\psi_{E'} | H | \varphi_n) = V_{E'\ n}, \tag{45b}
\]

\[
(\psi_{E''} | H | \psi_{E'}) = E' \delta(E'' - E'). \tag{45c}
\]

Equation (45a) implies that the smaller submatrix
FEW Examples: \textit{q}-REVERSAL due to LEVEL OVERLAP in Rydberg Atoms Most Prominent
Here We Discuss a **MESOSCOPIC** Realization of a Multi-State Fano Resonance That Exploits the Unique Behavior of **QUANTUM POINT CONTACTS**

A Critical Aspect of These Devices For This Study is Their Ability to Act as a TUNABLE BARRIER to Carriers ALONG their Direction of Flow

Barrier Allows Electron Density at QPC Center to Be LOWERED – Even PINCHED-OFF Completely
Magnetic impurity formation in quantum point contacts

Tomaž Rejec¹ & Yigal Meir¹,²

**BOUND STATE Thought to SPONTANEOUSLY Form in These Structures at PINCH-OFF**


# Quantum Point Contacts Near Pinch-Off

... Suggested by Numerous **MANY-BODY** Theories

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<td>Local Spin-Density Approximation</td>
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<td><strong>Nature</strong> <strong>442</strong>, 900 (2006)</td>
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<td>• Kondo effect from dynamically-fluctuating spin</td>
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<td>A.D. Güçlü et al.</td>
<td>Variational &amp; Diffusion Quantum Monte Carlo</td>
<td>• Inhomogeneous wire with low-density region where interactions dominant</td>
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<td><strong>PRB</strong> <strong>80</strong>, 201302 RC (2009)</td>
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<td>• <strong>Single-electron localization</strong> at low density</td>
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<td>E. Welander et al.</td>
<td>Local Spin-Density Approximation</td>
<td>• Inhomogeneous wire with low-density region where interactions dominant</td>
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<td><strong>PRB</strong> <strong>82</strong>, 073307 BR (2010)</td>
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<td>• Electron <strong>bound state</strong> arises from Coulomb interactions &amp; evolves sensitively with density</td>
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<td>J.H. Hsiao, T.M. Hong</td>
<td>Non-Equilibrium Green Functions with Spin-Orbit (Rashba) + Electron Interactions</td>
<td>• Spin-orbit interactions induce <strong>local-moment</strong> when QPC symmetry broken by source bias</td>
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<td><strong>PRB</strong> <strong>82</strong>, 115309 (2010)</td>
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<td>• Resulting spin polarization enhanced by role of Coulomb interaction</td>
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<td>T. Song, K.H. Ahn</td>
<td>Exact Diagonalization</td>
<td>• Scattering resonances due to 1D-2D transition</td>
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<td><strong>PRL</strong> <strong>106</strong>, 057203 (2011)</td>
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<td>• Resonant levels used to compute eigenstates of interacting electrons</td>
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<td>• Dependent on filling both <strong>Kondo effect</strong> &amp; <strong>ferromagnetic</strong> character obtained!</td>
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Localization of electrons and formation of two-dimensional Wigner spin lattices in a special cylindrical semiconductor stripe

E. Welander,* I. I. Yakimenko, and K.-F. Berggren
Department of Physics, Chemistry and Biology, Linköping University, S-58183 Linköping, Sweden

The barrier is effectively rectangular for $s$ in the range of 4 or higher but at lower values of $s$ the edges become smoothed and eventually the barrier becomes more like a conventional QPC saddle.

In the present case spatial and spin distributions are obtained from the self-consistent solution of the Kohn-Sham (LSDA) equations for the occupied electron orbitals $\psi_k^\sigma (\sigma = \pm \frac{1}{2})$, $H^\sigma : V_{conf} + \text{KINETIC ENERGY OPERATOR} + \text{COULOMB, EXCHANGE & CORRELATION TERMS}$
Bound-State Formation in QPCs: A Possible Scenario

**SINGLE Electron LOCALIZED** on the QPC Near Pinch-Off Due to Charge **PILEUP** at its **ENDS**

Localization **ELECTROSTATIC** in Origin: **COULOMB BLOCKADE** Keeps Other Electrons **OFF** QPC
We Use Fano Resonances in COUPLED QPCs to DETECT the Presence of BOUND STATES

DISCRETE LEVEL: Bound State in SWEPT QPC
CONTINUUM: DETECTOR QPC & Intervening 2DEG
Fano Resonance Due to Bound-State Formation in Coupled QPCs

Probing the Microscopic Structure of Bound States in Quantum Point Contacts

Y. Yoon,¹ L. Mourokh,²,³ T. Morimoto,⁴ N. Aoki,⁵ Y. Ochiai,⁴,⁵ J. L. Reno,⁶ and J. P. Bird¹

See Also:
V. Puller et al., PRL 92, 096802 (2004)
Y. Yoon et al., PRB 79, 121304(R) (2009)
Y. Yoon et al., APL 94, 213103 (2009)
Prior Work Showed Multi-State Fano Resonances for Interacting Levels of the SAME Atom or Dot

We Form REMOTE States on SEPARATE QPCs & Allow Interaction via a COMMON CONTINUUM
Detector Exhibits **TWO** Fano Resonances Due to the **TWO** Different Bound States

Resonance Positions **CONTROLLED** by Respective QPC Gate Voltages ($V_s$ & $V_c$)
Simultaneous TUNING of $V_s$ & $V_c$ Yields an AVOIDED CROSSING of the Resonances

A Mesoscopic Multi-State Fano Resonance

Anti-Crossing is UNUSUAL – MISSING Branch!
A Mesoscopic Multi-State Fano Resonance

Missing Branch – Detector Resonance With FAR QPC When Near One is PINCHED-OFF

Missing Branch Can be RECOVERED Using Pinched-Off QPC as the DETECTOR
A Mesoscopic Multi-State Fano Resonance

Missing Branch of the Spectrum is “RECOVERED” by Using the PINCHED-OFF QPC as DETECTOR

Reveals a PRONOUNCED (meV) Avoided Crossing!
To Describe this Problem THEORETICALLY we Start From the Following HAMILTONIAN

\[
\mathcal{H} = \sum_{p\sigma} \varepsilon_{p\sigma} n_{p\sigma} + \sum_{q\sigma} \varepsilon_{q\sigma} n_{q\sigma} + \sum_{k\sigma} E_k n_{k\sigma} \\
+ \sum_{i\sigma} \left( \varepsilon_{i\sigma} + U_i n_{i\bar{\sigma}} / 2 \right) n_{i\sigma} + \sum_{k\in(2)\sigma} \nu_{k\sigma_2} c_{k\sigma_2}^\dagger d_{2\sigma} \\
+ \sum_{k\in(1)\sigma} \nu_{k\sigma_1} c_{k\sigma_1}^\dagger d_{i\sigma} + \sum_{k\in(2)p\sigma} t_{kp\sigma} c_{k\sigma}^\dagger a_{p\sigma} \\
+ \sum_{kq\sigma} t_{kq\sigma} c_{k\sigma}^\dagger a_{q\sigma} + H.c.,
\]

**Occupation of Bound States 1 & 2**

**Coupling of Two Bound States and Detector to Intervening Regions of 2DEG**

For the **SUBSYTEM** Formed by the Two Bound States & Their Intervening Continuum ...

\[ H_{BS} = \sum_{k \in (1)\sigma} E_k n_k \delta + \sum_{i\sigma} (\varepsilon_{i\sigma} + U_i n_{i\sigma}/2)n_{i\sigma} \]

\[ + \sum_{k\sigma i} (v_{k\sigma i} c_{k\sigma}^{\dagger} d_{i\sigma} + H.c.). \]

We Obtain the Effective **INTERACTION POTENTIAL** Between the Two Bound States

\[ W_{ij} = \sum_k u^{*}_{k\sigma i} u_{k\sigma j} \left( \frac{1}{E_k - \varepsilon_{i\sigma}} - \frac{1}{E_k - \varepsilon_{j\sigma}} \right), \]

**SUMMATION RUNS OVER ALL STATES OF THE INTERVENING 2D CONTINUUM!**
Calculations Based on This Model **REPRODUCE**
the Unusual Avoided-Crossing of Experiment
A Mesoscopic Multi-State Fano Resonance

The implication of our analysis is that we can essentially replace the set-up consisting of the two BSs and their intervening 2DEG (Fig. 10(a)) with an effective model that more closely resembles the double-well potential characteristic of quantum-dot molecules (Fig. 10(b)). In this representation, the two BSs can be considered to effectively be directly coupled to each other, by a potential barrier that is actually lower than the barriers that couple the BSs to the 2DEG. With this coupling...
We Have Demonstrated a Multi-State Fano Resonance in Which Two Spatially-Remote Discrete States are Each Coupled to a Common Continuum

This Continuum Supports a Highly-Robust Effective Interaction Between the Two States Due to the Fact That it is Mediated by a Large Number of Degenerate Continuum States

While the Continuum is Often Viewed as a Source of Decoherence Our Work Suggests its Use to Engineer the Interactions of Mesoscopic Structures