New Zealand INSTITUTE for Advanced Study

Centre for Theoretical Chemistry and Physics (CTCP)

Non-hermitian scattering and non-linear bifurcations with Bose-Einstein condensates

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Massey University
COLLEGE OF SCIENCES

Quantum Physics with Non-Hermitian Operators, 15-25 June 2011, MPIPKS Dresden

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Quantum gases at Massey

Few-particles, full quantum dynamics

Soliton dynamics

Low dimensional BEC Quantum fluctuations and strong correlations

Superfluid Fermi gases: BEC-BCS crossover

Macroscopic quantum superpositions in strongly-interacting systems

Josephson Junction analogs, Vortex tunneling

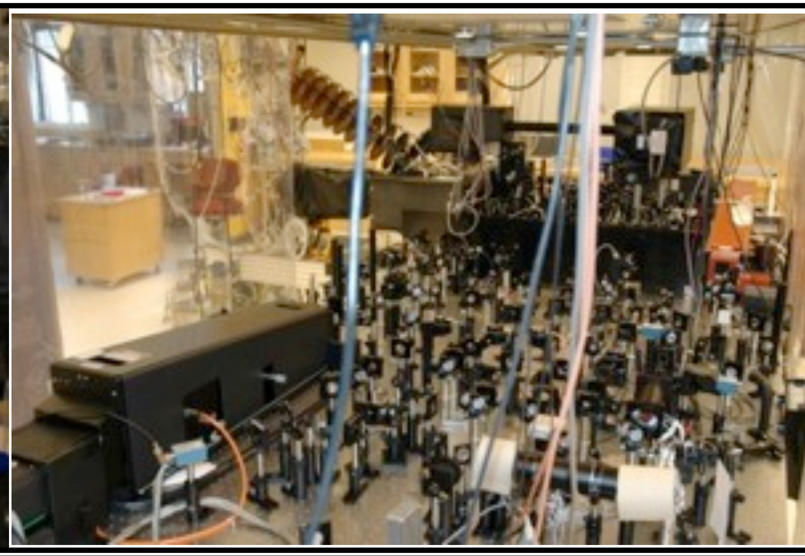
MARSDEN FUND
TE PŪTEA RANGAHAU A MARSDEN

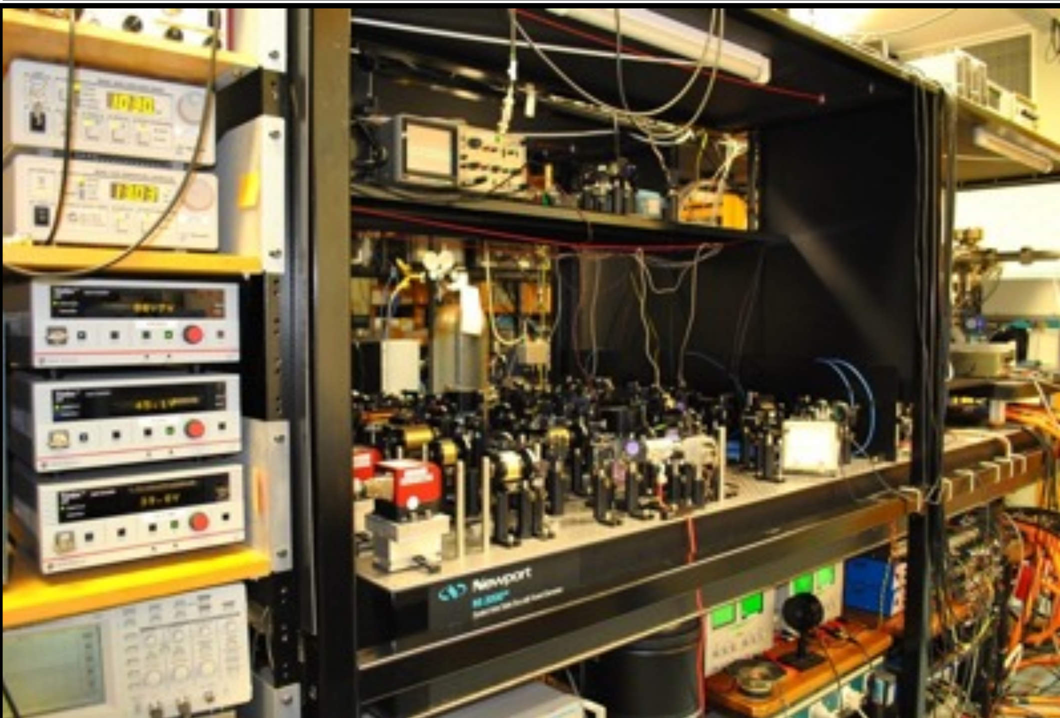
Outline

- Introduction
- Non-Hermitian physics in quasiparticle scattering
  - Levinson's theorem
  - Fano Resonance
- Bifurcations and instabilities
  - fluxon-dark soliton bifurcation as exceptional point

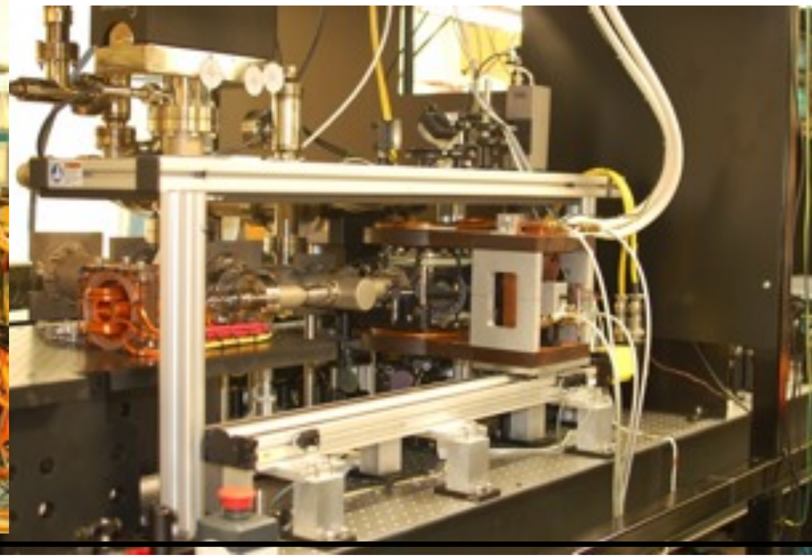
Ultra-cold atom experiments

Lene Hau
Harvard
Na BEC

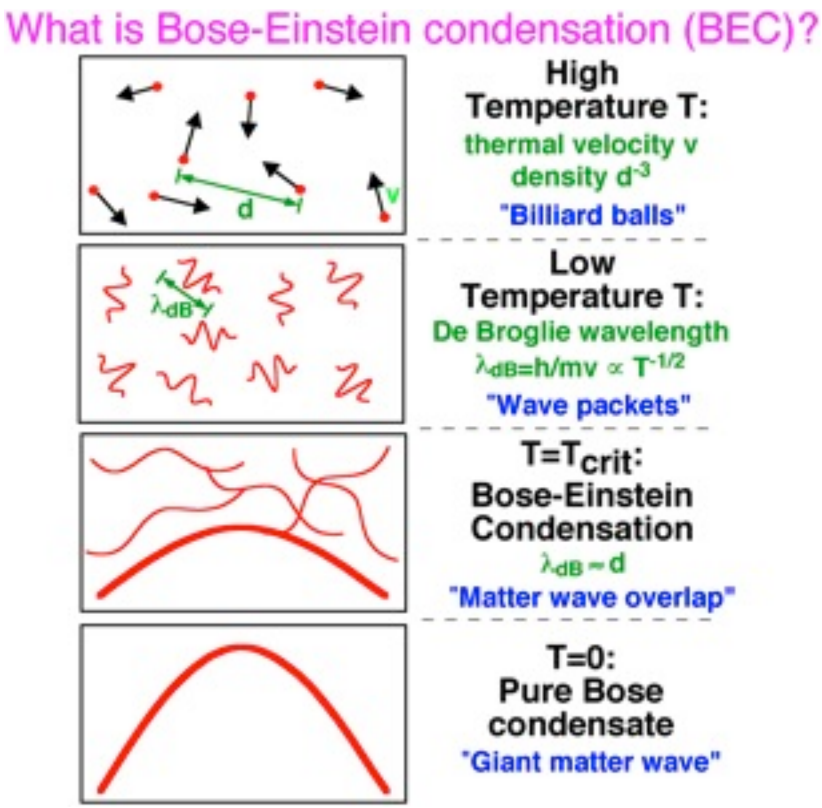




University of Otago (New Zealand)
Andrew Wilson - Nils Kaergard
Rb - K experiment

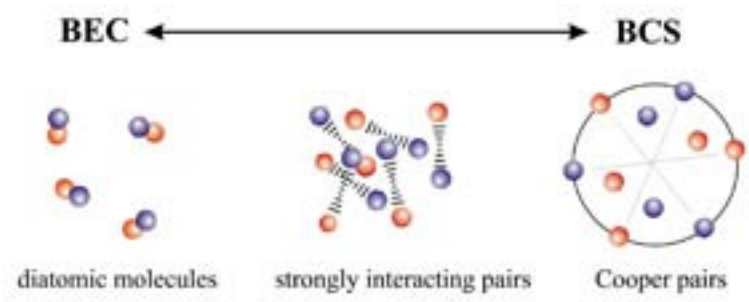


The many-body physics



Picture credits: Ketterle group

Superfluid Fermi gas



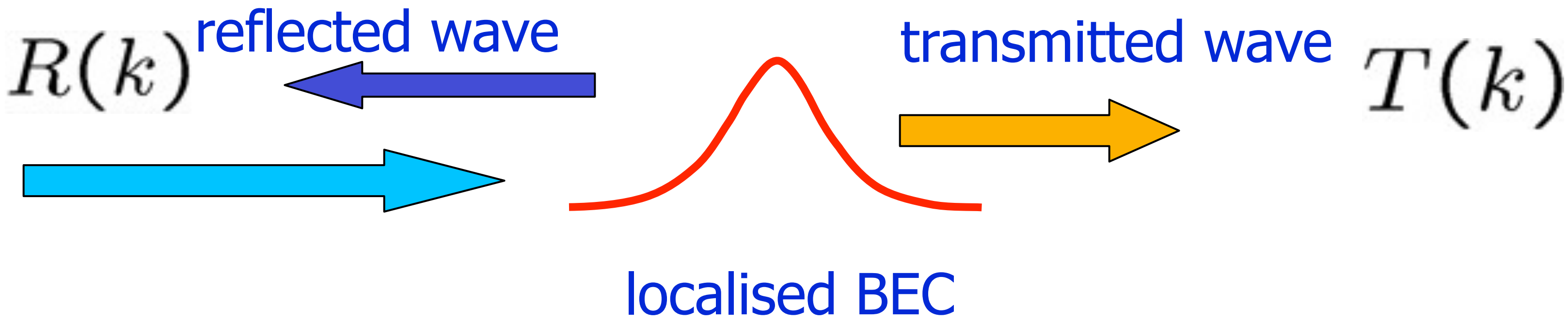
Picture credits: Jin group

Interactions are typically short range (van-der Waals) and ***can be tuned*** by exploiting magnetic-field dependent Fano-Feshbach resonances.

Topics:

- Strongly correlated (quantum) phases in optical lattice potentials
- Artificial gauge potentials
- Few-particle (Efimov) physics
- Dipolar (long-range, anistropic) interactions
- Macroscopic quantum phenomena
- Nonlinear waves

Scattering to probe BECs?



What can we learn from scattering atoms identical to the BEC atoms?
Are there interesting coherent effects?

BECs are not created at zero temperature. Thermal atoms (at low density) would scatter from BEC.

Is quasiparticle scattering different from single-particle QM scattering theory?

Theory: Bose-Einstein Condensate (BEC)

- Bose gas in an external potential

$$i\hbar\frac{\partial}{\partial t}\hat{\Psi}(\boldsymbol{r},t)=\left[-\frac{\hbar^2}{2m}\nabla^2+V_{\text{ext}}(\boldsymbol{r},t)+\int\hat{\Psi}^\dagger(\boldsymbol{r}',t)V(\boldsymbol{r}'-\boldsymbol{r})\hat{\Psi}(\boldsymbol{r}',t)\mathrm{d}\boldsymbol{r}'\right]\hat{\Psi}(\boldsymbol{r},t)$$

For BECs we may use the classical or mean field (Hartree) approximation:

Interaction becomes a tunable parameter

Gross-Pitaevskii (GP) equation

$$i\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{r},t)=\left[-\frac{\hbar^2}{2m}\nabla^2+V_{\text{ext}}(\boldsymbol{r},t)+\frac{4\pi a_s}{m}|\psi(\boldsymbol{r},t)|^2\right]\psi(\boldsymbol{r},t)$$

a_s s-wave scattering length

The GP equation is a nonlinear Schrödinger equation
it supports dynamically stable and unstable solutions,
soliton solutions, bifurcations, etc.

Elementary Excitations of the BEC

Time-dependent **Gross-Pitaevskii** equation

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \left\{-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{trap}} + \frac{4\pi a_{\mathrm{S}} N}{m}|\psi|^2\right\}\psi$$

Linearize the time-dependent equation around the stationary mean field:

$$\psi(\vec{r},t) = \phi(\vec{r})e^{i\mu t/\hbar} + \delta\psi(\vec{r},t)$$

$$\delta\psi(\vec{r},t) = u(\vec{r})e^{i\varepsilon t/\hbar} + v(\vec{r})e^{-i\varepsilon t/\hbar}$$

This yields the Bogoliubov (RPA) equations

Scattering of quasiparticles

Bogoliubov equations:

$$
\begin{aligned}
(T + V_d)u(r) + V_o v(r) &= (\varepsilon + \mu)u(r) \\
(T + V_d)v(r) + V_o^* u(r) &= (-\varepsilon + \mu)v(r)
\end{aligned}
$$

with the interactions

$$V_d(r) = V_{\text{ext}}(r) + 2\frac{4\pi a_S N}{m}|\phi(r)|^2 \qquad V_o(r) = \frac{4\pi a_S N}{m}\phi(r)^2$$

This is a two-channel scattering problem –

Resonances, **Levinson Theorem?**

Bogoliubov equations

Need to solve this non-Hermitian eigenvalue problem:

$$\begin{pmatrix} A & B \\ -B^* & -A \end{pmatrix} \begin{pmatrix} u(\mathbf{r}) \\ v(\mathbf{r}) \end{pmatrix} = \epsilon \begin{pmatrix} u(\mathbf{r}) \\ v(\mathbf{r}) \end{pmatrix}$$

with

$$A = -\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(\mathbf{r}) - \mu + 2g|\psi(\mathbf{r})|^2 \qquad B = g\psi(\mathbf{r})^2$$

where $\psi(\mathbf{r})$ is a solution of the stationary GP equation

$$\mu\psi(\mathbf{r}) = [-\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(\mathbf{r}) + g|\psi(\mathbf{r})|^2]\psi(\mathbf{r})$$

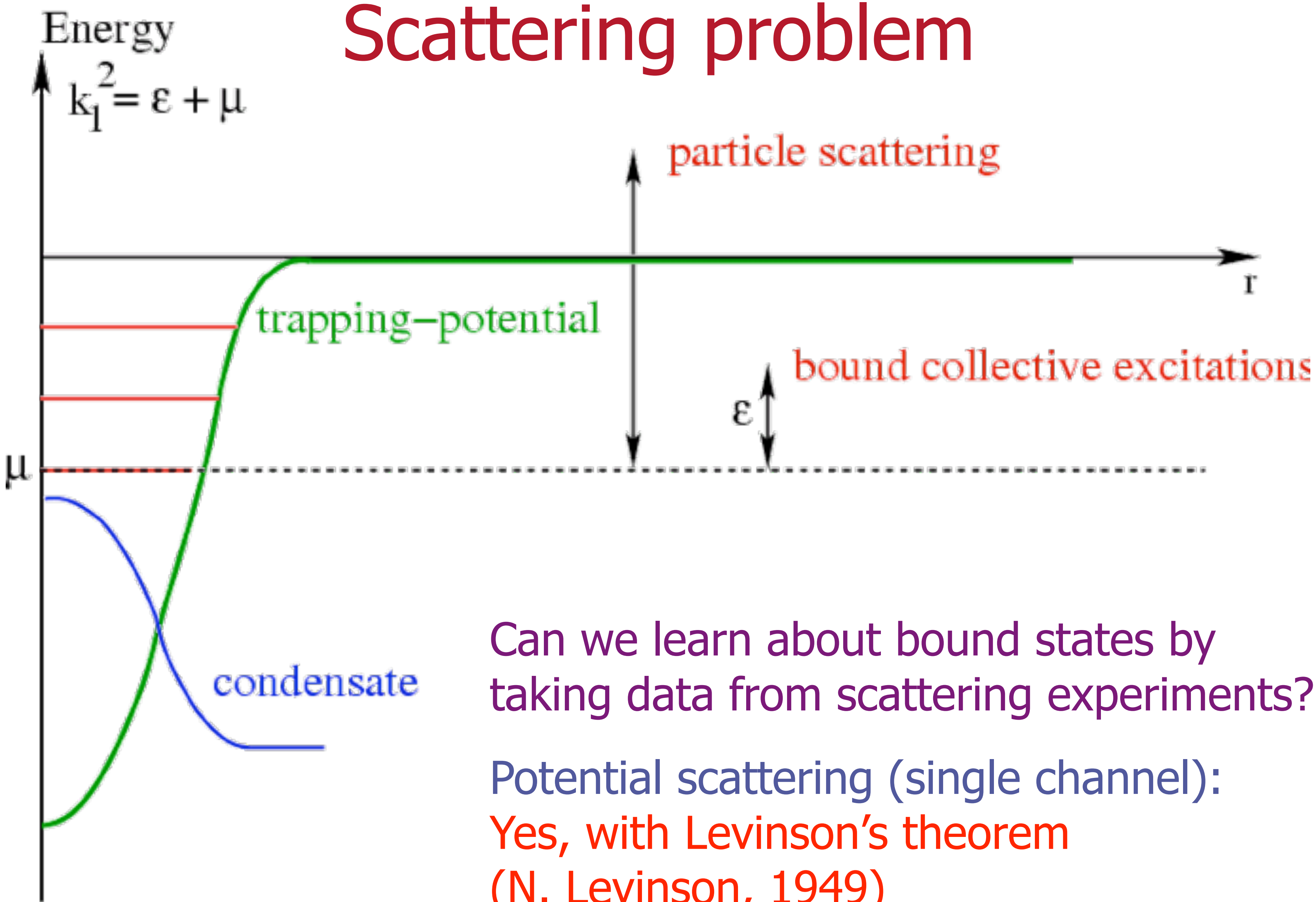
Properties:

$$A^\dagger = A$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} A & B \\ -B^* & -A \end{pmatrix}^\dagger \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} A & B \\ -B^* & -A \end{pmatrix}$$

Eigenvalues come in pairs, either real or imaginary

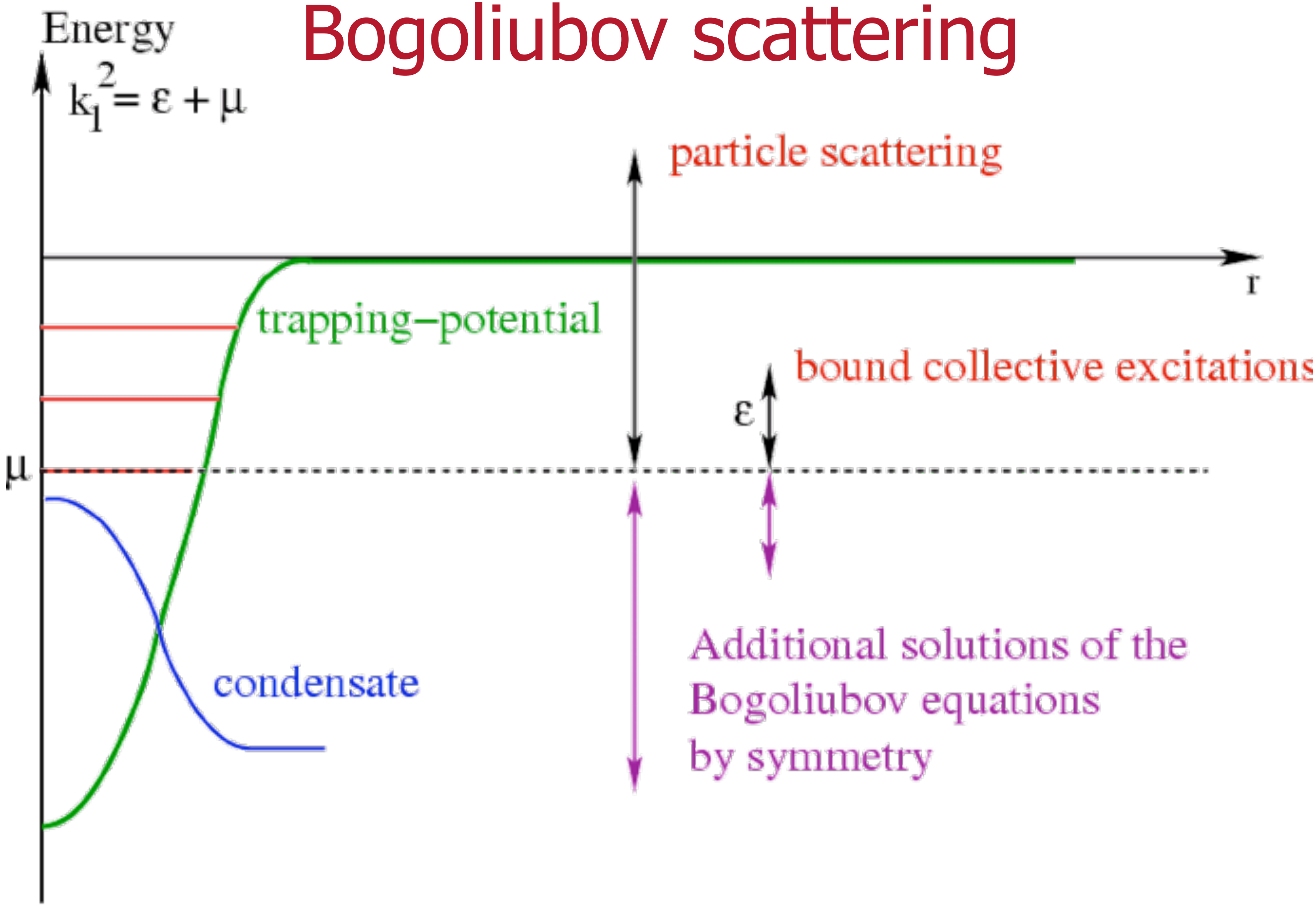
Scattering problem



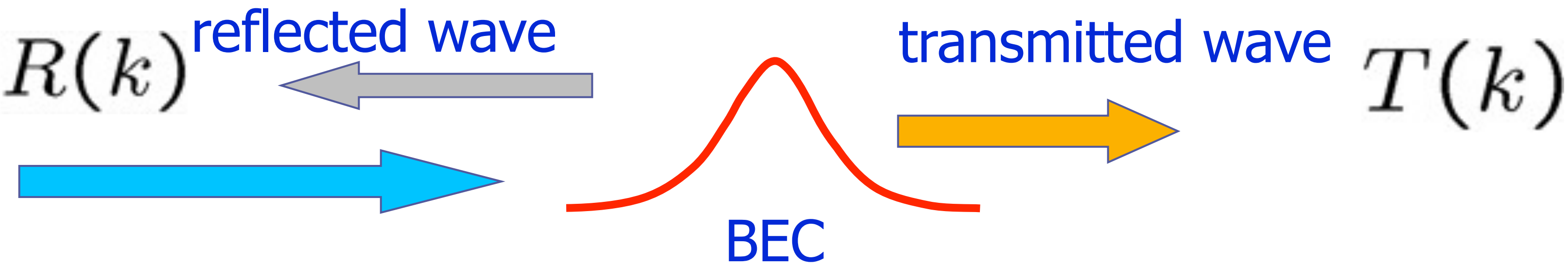
Can we learn about bound states by taking data from scattering experiments?

Potential scattering (single channel):
Yes, with Levinson's theorem
(N. Levinson, 1949)

Bogoliubov scattering



Levinson‘s theorem for BECs



The transmission amplitude T(k) carries a phase δ(k)

$$T(k) = e^{-2i\delta(k)}|T(k)|$$

Levinson‘s theorem for BECs:

$$\delta(k=0) = (n_b - 1/2)\pi$$

the phase shift at zero momentum is related to the number of discrete (localised) collective excitations n_b

JB, I Häring, and J-M Rost, PRL **91** (2003) 070403

REPORTS

SCIENCE **296** (2002) 1290

Formation of a Matter-Wave Bright Soliton

L. Khaykovich,[1] F. Schreck,[1] G. Ferrari,[1,2] T. Bourdel,[1] J. Cubizolles,[1] L. D. Carr,[1] Y. Castin,[1] C. Salomon[1*]

We report the production of matter-wave solitons in an ultracold lithium-7 gas.

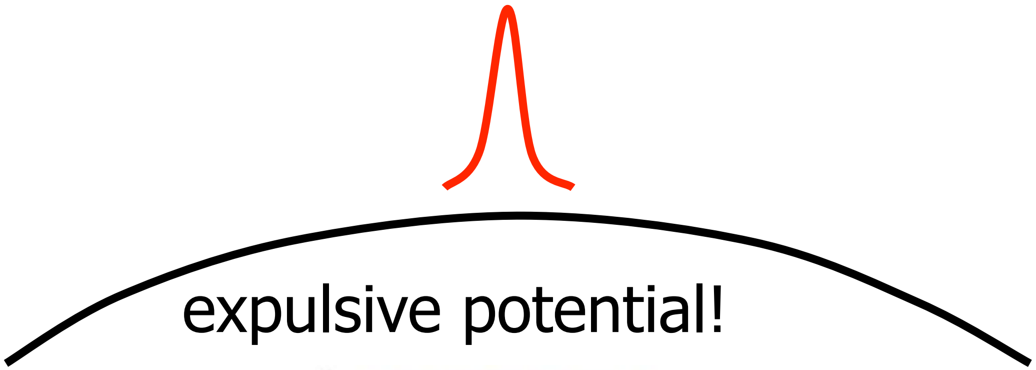
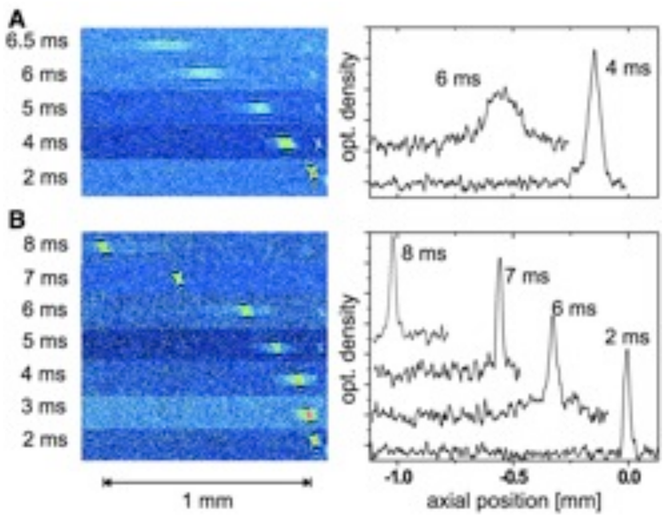


Fig. 3. Absorption images at variable delays after switching off the vertical trapping beam. Propagation of an ideal BEC gas (**A**) and of a soliton (**B**) in the horizontal 1D waveguide in the presence of an expulsive potential. Propagation without dispersion over 1.1 mm is a clear signature of a soliton. Corresponding axial profiles are integrated over the vertical direction.



Bright Solitons

are supported in BECs with

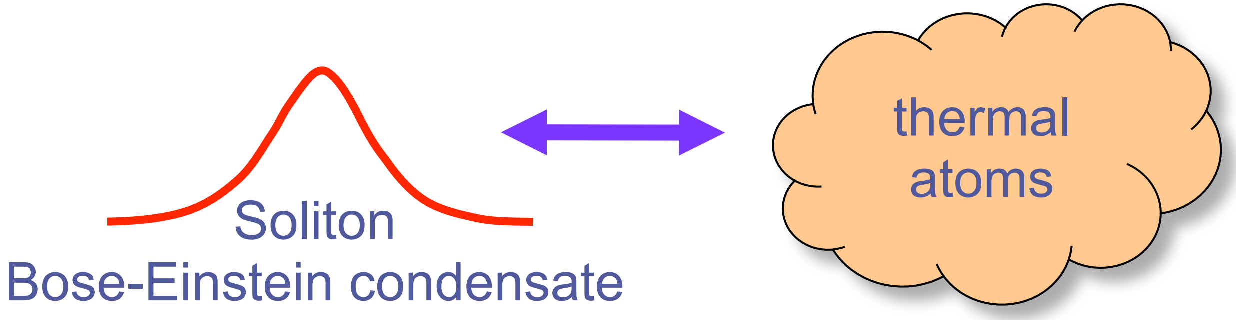
- attractive interactions (g<0)
- tight wave-guide geometry: quasi-1D system

A single nonspreading wave packet (soliton) is observed

Similar experiments: R. Hulet (2002) and M. Oberthaler (2004)

Scattering problem with solitons

How robust are solitons?



Will solitons decay by interacting with single thermal particles?

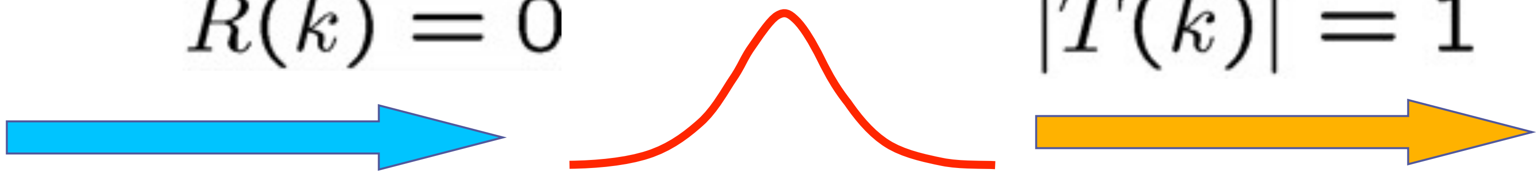
What happens in the scattering process?

Solitons are quantum objects with the coldest temperatures

ideal for atom optics, interferometry,

gravitational and inertial sensors

Quasiparticle scattering on soliton



Bogoliubov scattering from a bright soliton in the cubic NLS

is reflectionless ! Solitons are transparent

$$\phi(x) = \sqrt{N/(2b)}\,\mathrm{sech}^2(x/b)$$

$$\begin{aligned} u_k &= A(k)[\mathrm{sech}(x/b^2) + ikb\tanh(x/b) + (kb)^2 - 1]e^{ikx} \\ v_k &= A(k)\mathrm{sech}(x/b^2)e^{ikx} \end{aligned}$$

$$T(k) = \left(\frac{kb+i}{kb-i}\right)^2 \quad |T(k)| = 1$$

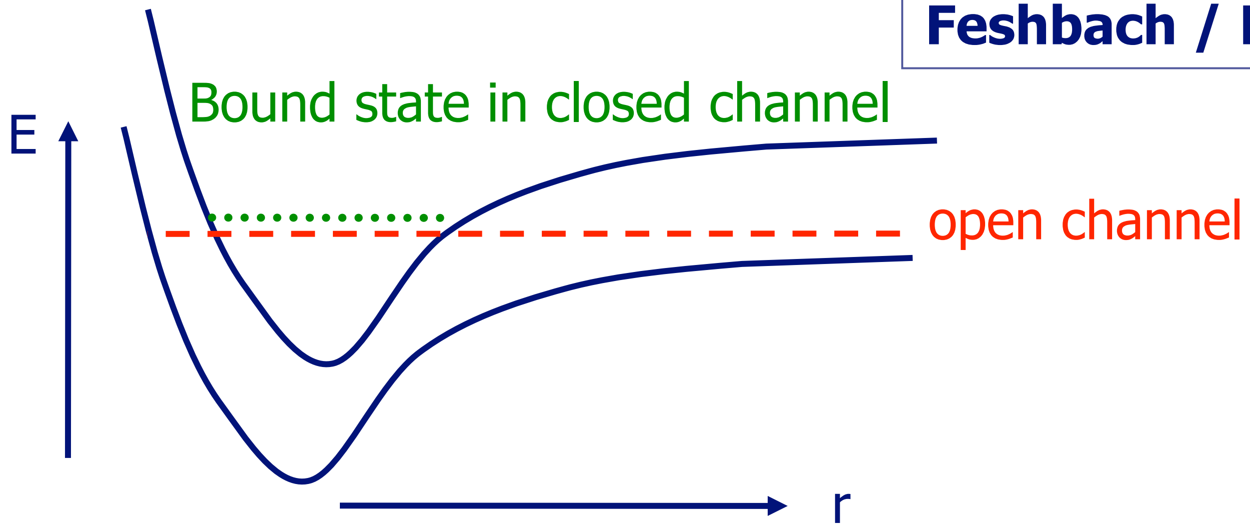
Realistic 3D situation: finite reflection leads to dissipative motion of soliton in thermal cloud.

S Sinha, AYu Cherny, D Kovrizhin, JB PRL 96, 030406 (2006)

Resonant scattering

- In two-channel scattering we sometimes find resonances of the Fano / Feshbach type, e.g. in atom-atom scattering

Feshbach / Fano resonance

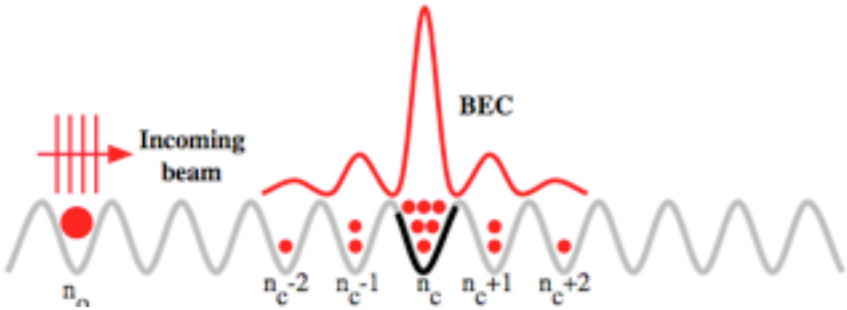


Can we find similar resonances for quasiparticles scattering on BECs?

Let's see. Go lattice …

Lattice with nonlinear defect

Consider a BEC in an optical lattice confined by attractive (or repulsive) interactions that are *only present within a single well*.



The appropriate version of the GP equation is the **discrete nonlinear Schrödinger equation (DNLS)**

$$i\frac{\partial \Psi_n}{\partial t} = -(\Psi_{n+1} + \Psi_{n-1}) - \gamma|\Psi_{n_c}|^2\Psi_{n_c}\delta_{n,n_c}$$

We can find solutions analytically!

Tsironis, Molina, Hennig PRE **50**, 2365 (1994)

Solutions of the defect model

There are two types of solutions:

•**Extended (linear) waves** exist far away from defect

$$\Psi_n = \Psi_0 \exp(ikn) \exp(-iE_k t)$$

They form a band with dispersion

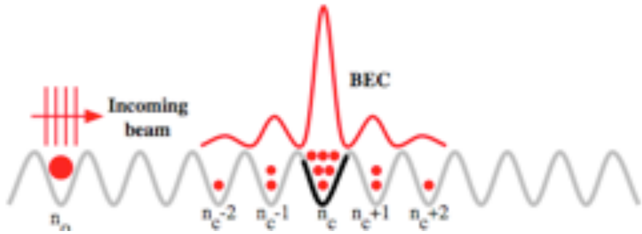
$$E_k \equiv -2\cos k,$$

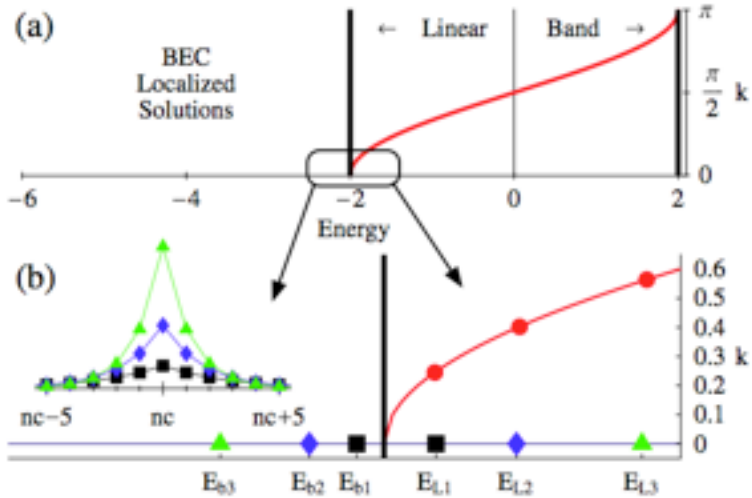
•**Localized solutions** decay exponentially

$$\Psi_n(t) = b_n(t) = b_{n_c} x^{|n-n_c|} \exp(-iE_b t)$$

The energies lie outside the linear band

$$E_b = -\sqrt{4+g^2} \qquad g \equiv \gamma b_{n_c}^2 \;\; (g > 0)$$





Scattering problem

Approach the scattering problem:

•Expand DNLS around localised solution

$$\Psi_n(t) = \phi_n(t) + b_n(t)$$

$$\phi_n(t) = u_n e^{-iEt} + v_n^* e^{-i(2E_b - E)t}$$

•Derive discrete Bogoliubov equations

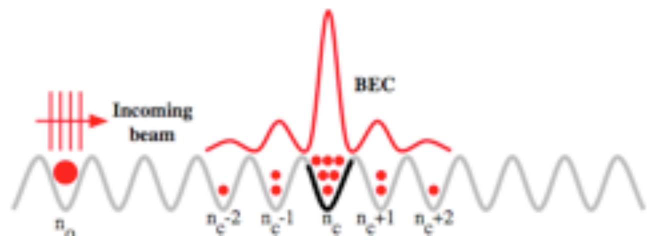
$$E u_n = -(u_{n+1} + u_{n-1}) - g(2u_{n_c} + v_{n_c})\delta_{n,n_c}$$
$$(2E_b - E)v_n = -(v_{n+1} + v_{n-1}) - g(2v_{n_c} + u_{n_c})\delta_{n,n_c}$$

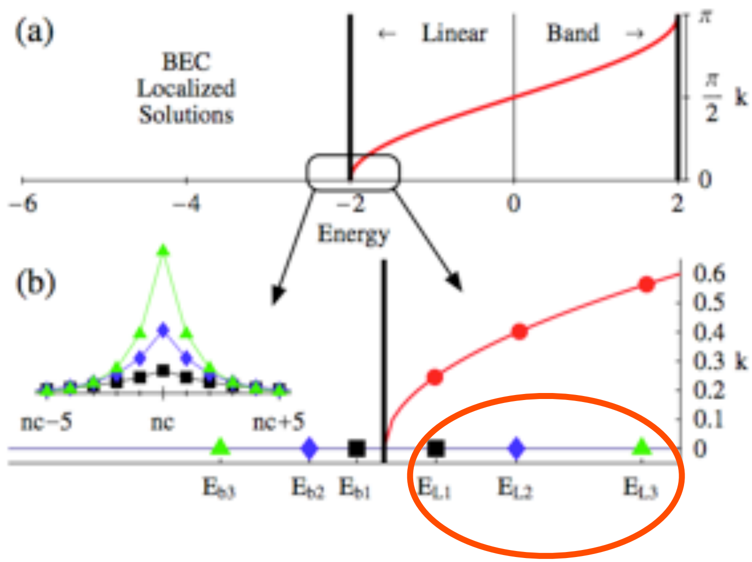
•Note that there is a **localized solution** in the decoupled v channel ($g\, u_{n_c} = 0$):

$$v_n = v_{n_c}\, w^{|n-n_c|} \qquad w = -g + \sqrt{1+g^2}$$

With energy **within the linear band!**

$$\boxed{E = E_L \equiv 2(E_b + \sqrt{1+g^2})}$$





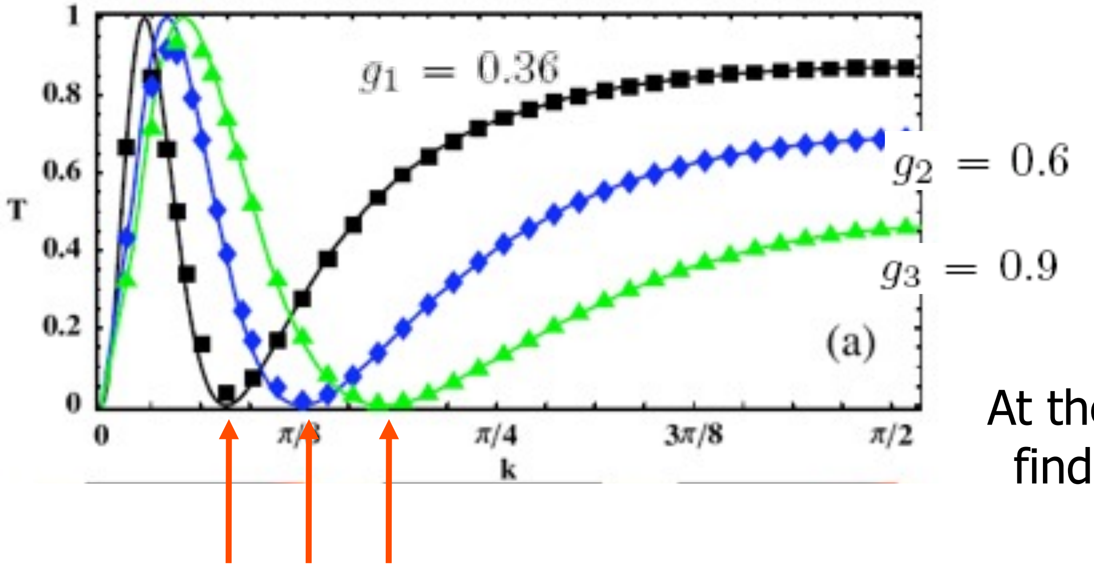
Can we expect a Fano resonance at the local mode energy?

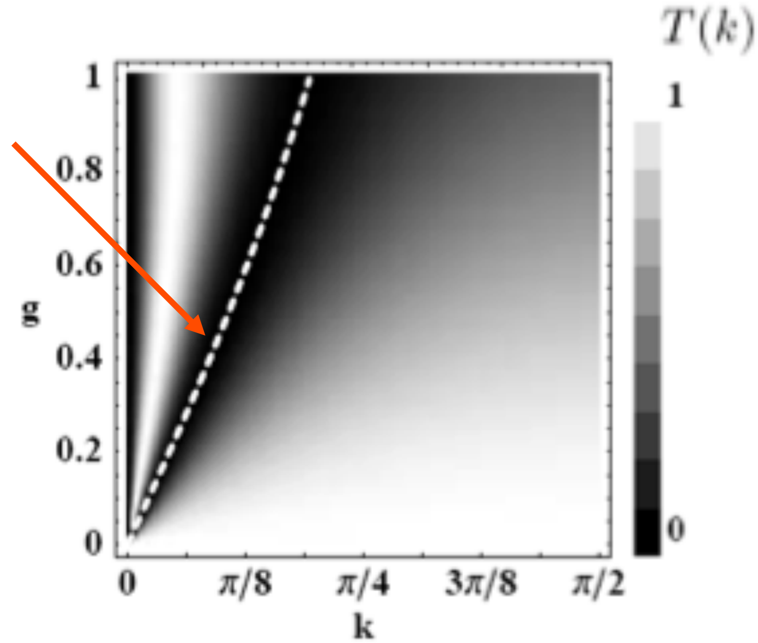
R Vicencio, JB, S Flach, PRL **98**, 184102 (2007)

Fano Blockade

The scattering problem can be solved exactly:

$$T(k) = \frac{4\sin^2 k}{4\sin^2 k + \left(2g + \frac{g^2}{\sqrt{(E_k - 2E_b)^2 - 4} - 2g}\right)^2}$$



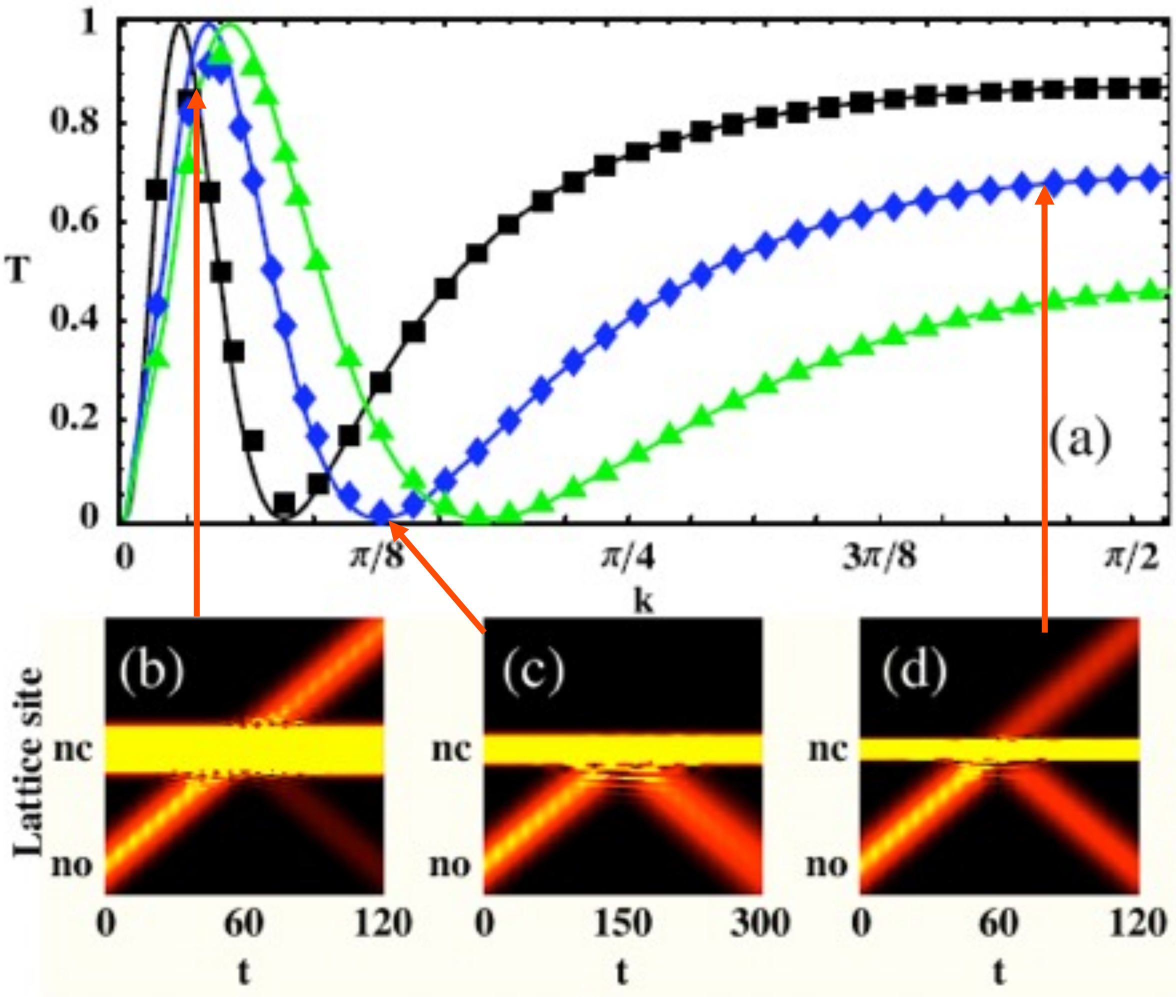


At the position of the local mode we find a reflection (Fano) resonance

R Vicencio, JB, S Flach, PRL **98**, 184102 (2007)

Fano Blockade

The reflection and transmission resonances may be useful for switching atom beams!

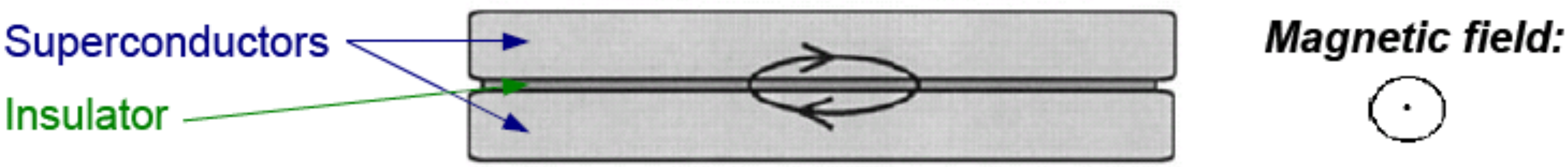


Numerical simulations with wavepackets confirm the results of stationary scattering theory

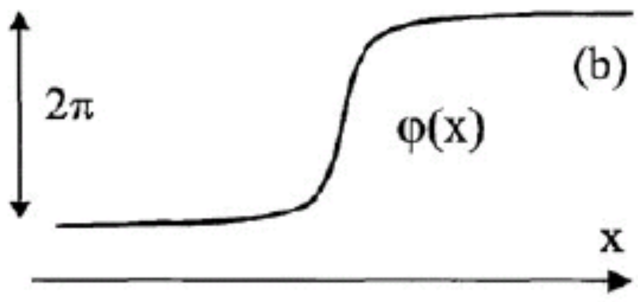
R Vicencio, JB, S Flach, PRL **98**, 184102 (2007)

Topological - non-topological soliton bifurcation

Josephson vortex (fluxon) in superconductor



- ***Josephson vortex:*** identified by a soliton in the relative phase



- One quantum of magnetic flux

A. V. Ustinov, Physica D, ***123*** *(1998)*

Nature **425**,155 (2005)

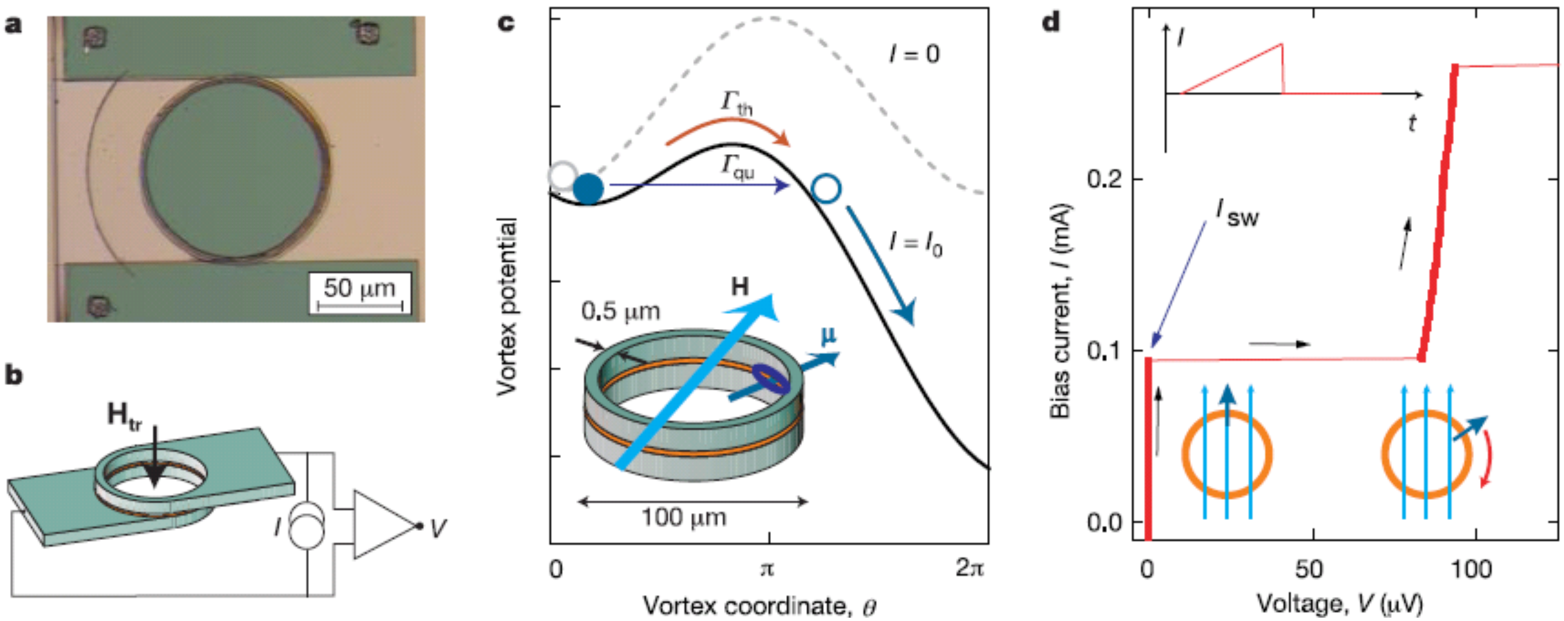
Quantum dynamics of a single vortex

A. Wallraff*, A. Lukashenko, J. Lisenfeld, A. Kemp, M. V. Fistul, Y. Koval & A. V. Ustinov

Physikalisches Institut III, Universität Erlangen-Nürnberg, D-91058 Erlangen, Germany

* Present address: Department of Applied Physics, Yale University, New Haven, Connecticut 06520, USA

letters to nature



Quantum tunneling of vortices

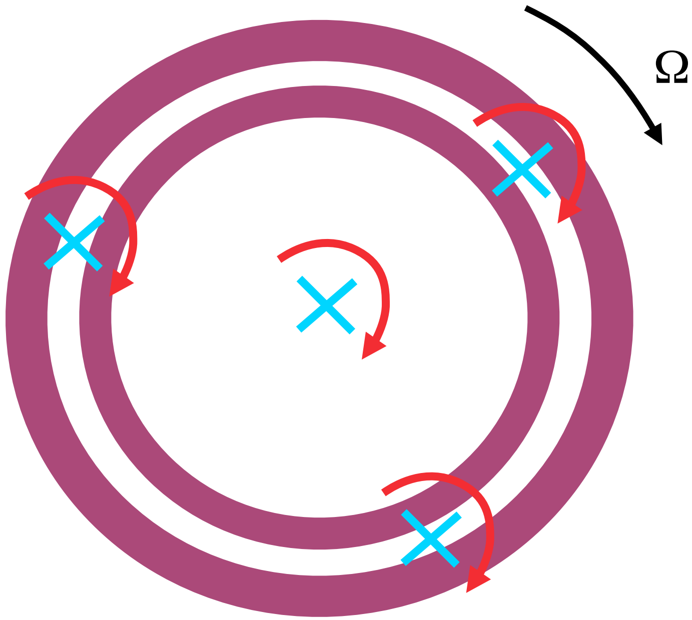
- A vortex in a 2D BEC can tunnel between two pinning potentials on observable time scales.

O Fialko, AS Bradley, JB, arXiv:1105.5869 (2011)

- Robust superposition state between vortices can be created in 1D strongly interacting Bose (Tonks-Girardeau) gas.

DW Hallwood, T Ernst, JB, PRA **82**, 062623 (2010)

Double ring under rotation



Vortices can enter the tunnel barrier between the two rings.

In analogy to long Josephson junctions we call these “**Josephson vortices**” or “**rotational fluxons**”

Competing effects of

External rotation: favours different circulation between rings

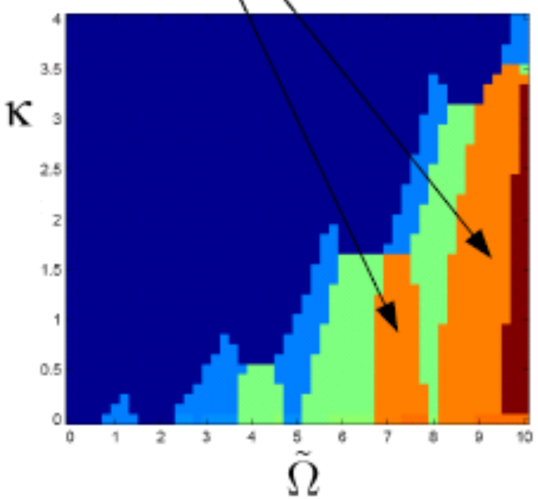
Tunnel coupling: favours equal phase across the junction

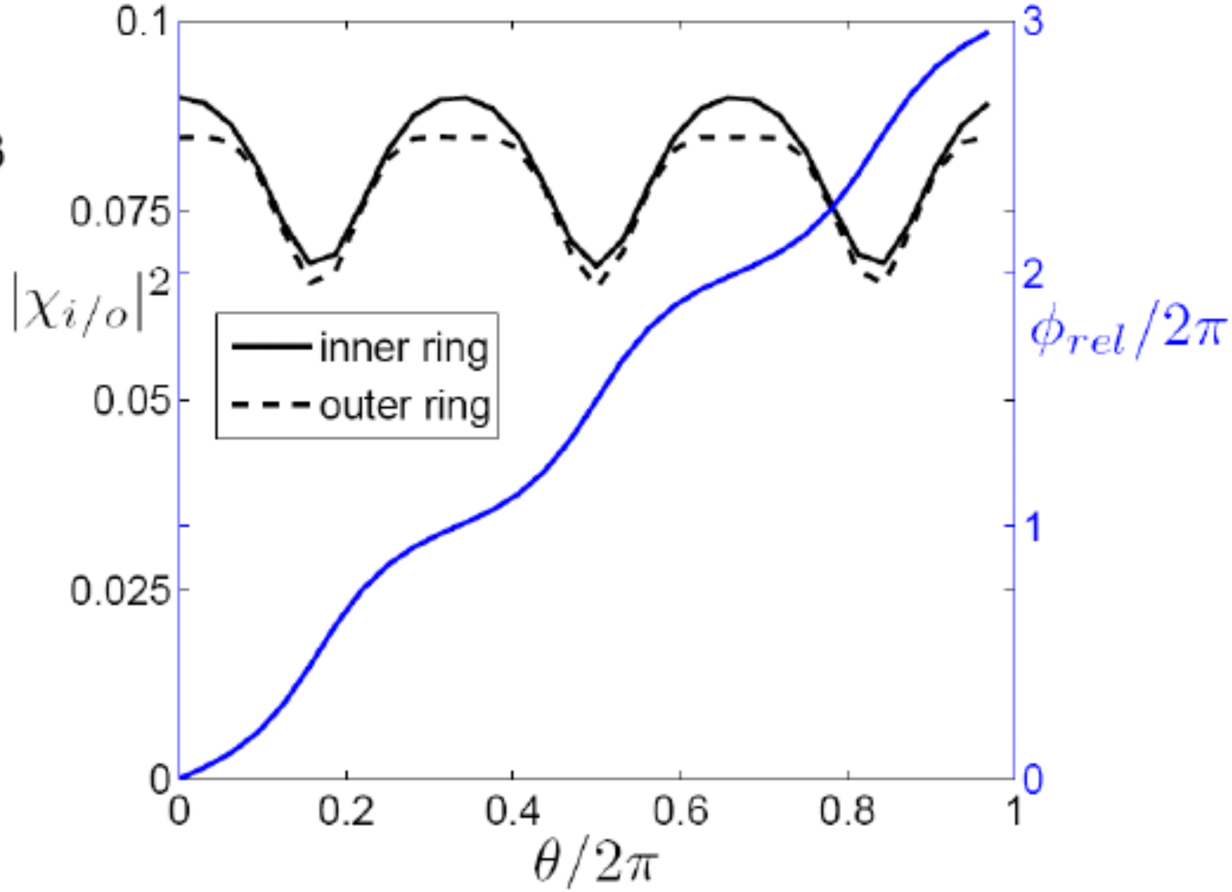
JB, TJ Haigh, U Zülicke, PRA **80**, 011602(R) (2009)

Multiple Josephson Vortex Solutions

When $\langle L_z \rangle_\mathrm{o}$ - $\langle L_z \rangle_\mathrm{i}$ > 1 , ground state is ***lattice*** of Josephson vortices

e.g. $\langle L_z \rangle_\mathrm{o}$ - $\langle L_z \rangle_\mathrm{i}$ ~ 3





JB, TJ Haigh, U Zülicke, PRA **80**, 011602(R) (2009)

Single fluxon in linear geometry

Two linearly coupled GP equations

$$i\partial_t\psi_1 = (-\frac{1}{2}\partial_{xx} + |\psi_1|^2 - \mu)\psi_1 - k\psi_2$$

$$i\partial_t\psi_2 = (-\frac{1}{2}\partial_{xx} + |\psi_2|^2 - \mu)\psi_2 - k\psi_1$$

have fluxon solutions for 0 < k < 1/3

$$\psi_{1/2}^{\text{fl}} = \sqrt{1+k}\tanh(2\sqrt{k}x) \pm i\sqrt{1-3k}\,\text{sech}(2\sqrt{k}x)$$

or dark soliton solution for k > 1/3

$$\psi_1^{\text{ds}} = \psi_2^{\text{ds}} = \sqrt{1+k}\tanh(\sqrt{1+k}\,x)$$

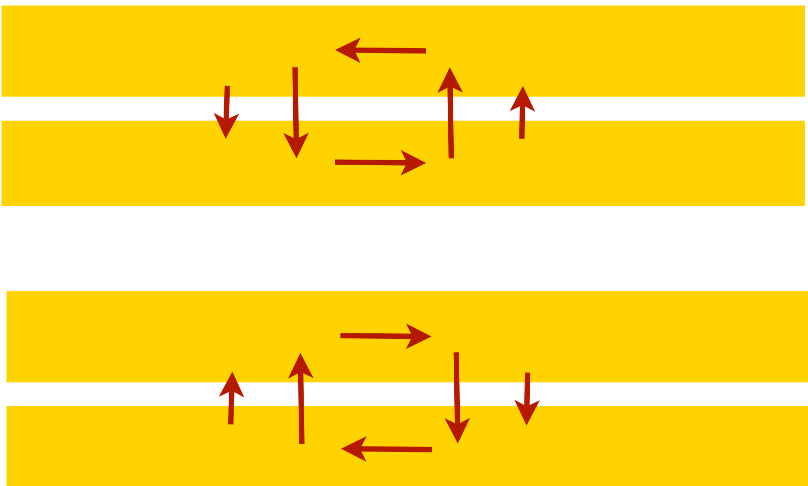
Kaurov, Kuklov, PRA (2005, 2006)

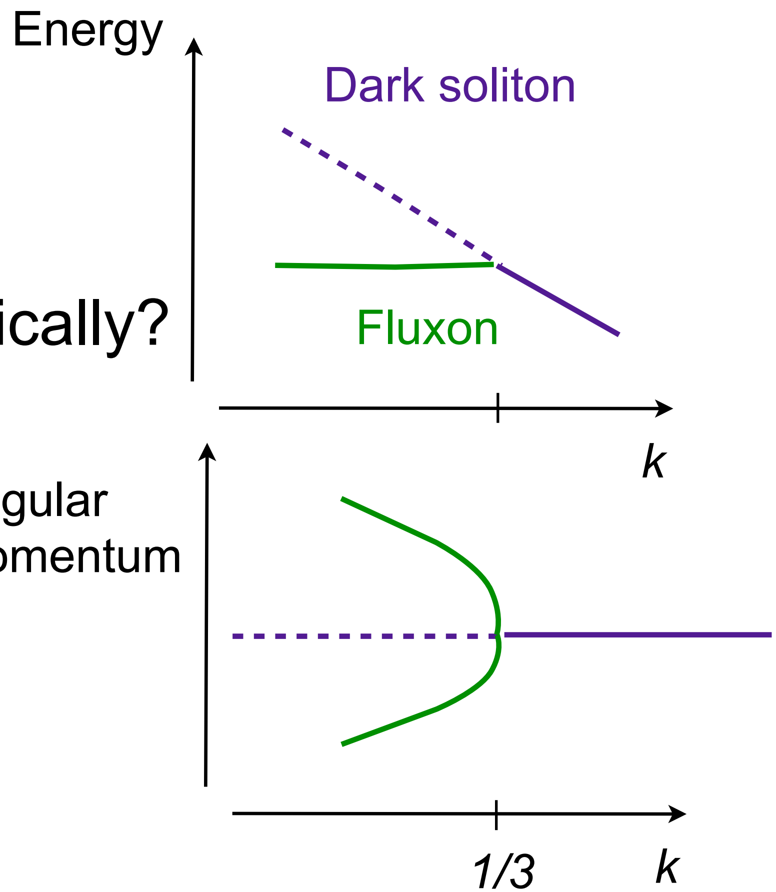
Qadir, Susanto, Matthews, arXiv (2011)

Fluxon - dark soliton bifurcation

Is the bifurcation point an exceptional point?

- square root branch point at $k = 1/3$
- encircling the branch point in the complex plane interchanges fluxon with anti-fluxon
- Can this be achieved physically?





Thanks!

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Sergej Flach
Tania Haigh
Ivo Häring
Dima Kovrizhin
Jan-Michael Rost
Subhasis Sinha
Rodrigo Vicencio
Ulrich Zülicke

The end!