Superradiance transition and Transport through nanostructures

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What is Superradiance?

- Discrete quantum systems coupled with environment described by a continuum of states.
- Effect of opening: Energy Shift and Finite Lifetime.
- Generically, at weak coupling, all internal states are similarly affected by the opening: lifetime decreases as the coupling increases.
- At a critical value: sharp restructuring of the system.
- Beyond this critical value, a few states become short-lived states, leaving all other (long-lived) states effectively decoupled from the environment. Analogy with Dicke super-radiance in quantum optics.

OUTLINE

- MOTIVATION
- PARADIGMATIC MODEL OF COHERENT QUANTUM TRANSPORT
- NON HERMITIAN HAMILTONIAN APPROACH
- SUPERRADIANCE TRANSITION: SYMMETRIC AND ASYMMETRIC COUPLING
- START WITH SIMPLE PARADIGMATIC MODEL THEN GENERALIZE: DISORDERED CASE, MULTIDIMENSIONAL CASE, STAR GRAPH

G.L.C. and L. Kaplan, PRB **79**, 155108 (2009),
G.L.C. et all., archive: 1007.5247, submitted to PRB

MOTIVATION

REALISTIC MODEL, SIMPLE AND INTERESTING

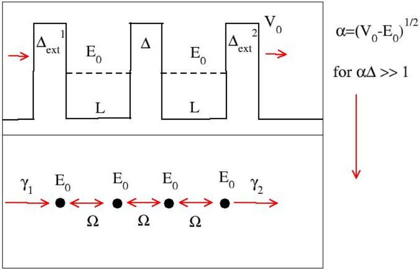
Random Matrix Theory (H. A. Weidenmüller, Y. V. Fyodorov, G.L.C., F. M. Izrailev, V. G. Zelevinsky), nuclei (A. Volya and V. Zelevinsky), billiards (I. Rotter, E. Persson,...), electron waveguides (L. E. Reichl)

Sequence of Potential Barrier. Simple: energy dependence can be neglected!

Interesting. Paradigmatic Model for Coherent Qauntum Transport: superlattices, sequence of Qdots, molecular wires. Basic Energy Science: coherent quantum transport found in photosynthetic systems.

WE CAN INCREASE THE COUPLING TO THE EXTERNAL WORLD UP TO THE POINT WHERE SR TRANSITION OCCURS? HOW SR EFFECTS TRANSPORT?

Sequence of Potential Wells



following S.A. Gurvitz (PRA **38**, 1747 (1988))

$$\gamma_{1,2} = \frac{8\alpha^3 E_0 k}{V_0^2(1+\alpha L/2)} \exp\left(-2\alpha\Delta_{ext}\right);\ \delta_{1,2} = \frac{k^2-\alpha^2}{4\alpha k}\gamma_{1,2};$$

$$\Omega = \frac{2\alpha^2 E_0}{V_0(1+\alpha L/2)} \exp\left(-\alpha\Delta\right)$$

where $\alpha = \sqrt{\frac{2m}{\hbar^2}(V_0 - E_0)}$, and $k = \sqrt{\frac{2m}{\hbar^2}E_0}$

Modelling with Non Hermitian Hamiltonian

$$H_{eff} = \begin{pmatrix} E_0 + \delta_1 - i/2\gamma_1 & \Omega & 0 \\ \Omega & E_0 & \Omega \\ 0 & \Omega & E_0 + \delta_2 - i/2\gamma_2 \end{pmatrix}$$

Note that $\delta_{1,2} = 0$ for $E_0 = V_0/2$.
And the transmission can be obtained from:

$$T(E) = \left| \frac{\sqrt{\gamma_1 \gamma_2}\,\Omega^{N-1}}{\prod_k^N (E - \mathcal{E}_k)} \right|^2 \qquad \textit{Agreement for} \quad \alpha\Delta \gg 1$$

$$\textit{with} \quad \mathcal{E}_k = E_k - \frac{i}{2}\Gamma_k$$

Small Coupling Limit and Superradiance Transition

N states coupled by tunneling Ω

$$w_q = E_0 - 2\Omega \cos(\frac{q\pi}{N+1}) \text{ with } q = 1, ..., N$$

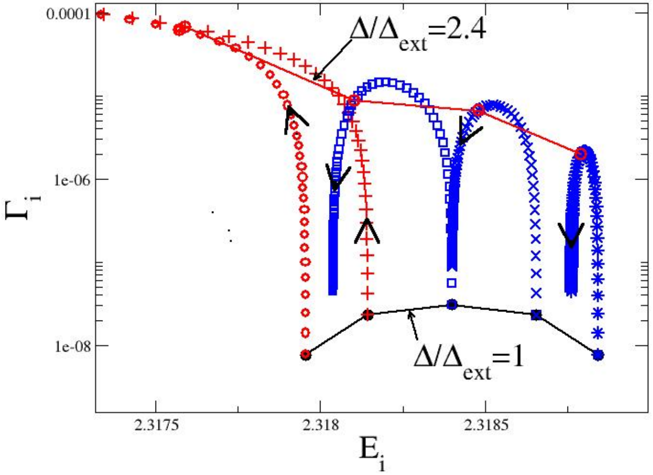
for large N:

$D = 4\Omega/N$, and for small γ: $\langle\Gamma\rangle = 2\gamma/N$,

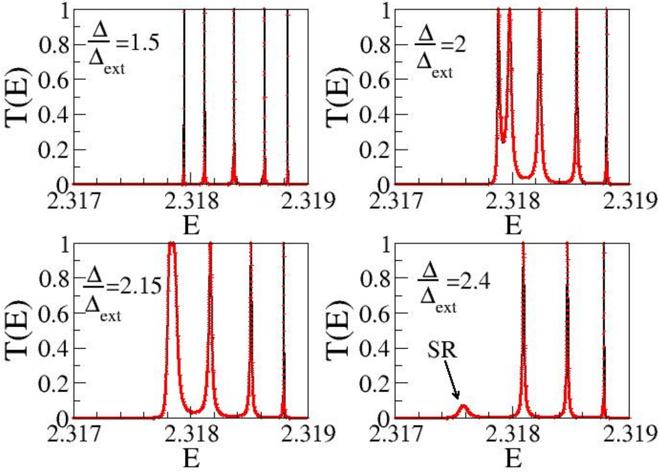
SUPERRADIANCE:

$$\frac{\langle\Gamma_q\rangle}{D} = \frac{\gamma}{2\Omega} \approx 1 \quad \textit{which is} \quad \frac{\Delta}{\Delta_{ext}} = 2$$

Symmetric Coupling: Superradiant and Trapped States



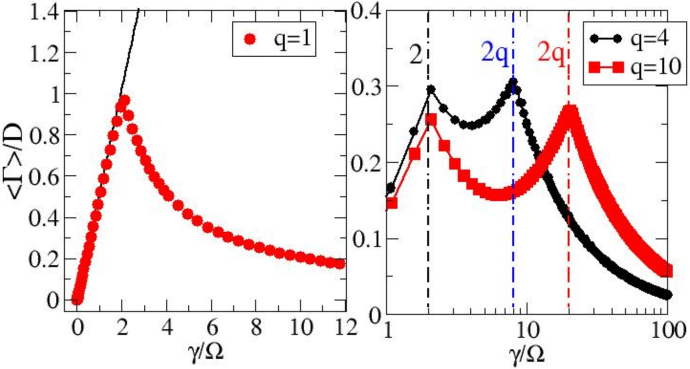
Superradiant Transition: Resonance structure



Asymmetric Coupling: Double Superradiant Transition

Average of N-2 smallest resonance widths

Asymmetry parameter q: $\gamma_1 = \gamma$ $\gamma_2 = \gamma / q$



Double SR transition at $\gamma/\Omega=2$ and $\gamma/\Omega=2q$

Narrow Resonances: N -> N-1 -> N-2

Asymmetric Coupling: Transmission

- **N=1**

$$T_1 = \frac{4q}{(q+1)^2} \quad NEVER \;\; PERFECT!$$

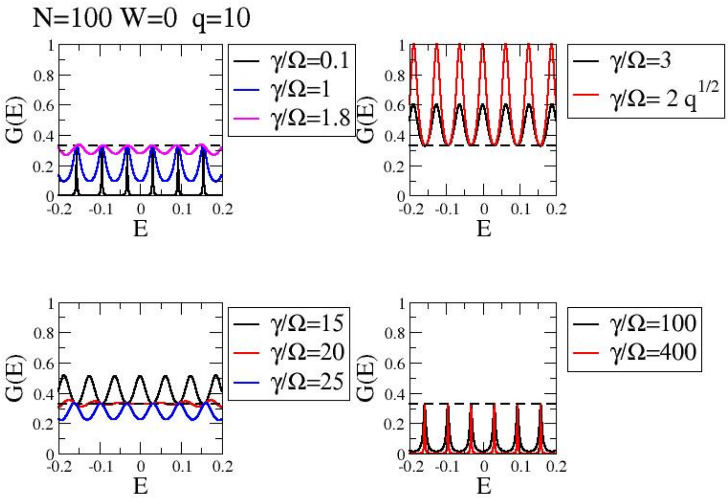
- **N=2**

$$T_2 = |\frac{\gamma/\Omega}{\sqrt{q}((\gamma/\Omega)^2/4q+1)}|^2 \;\; T_2 = 1 \;\; (\frac{\gamma}{\Omega})_{cr} = 2\sqrt{q}$$

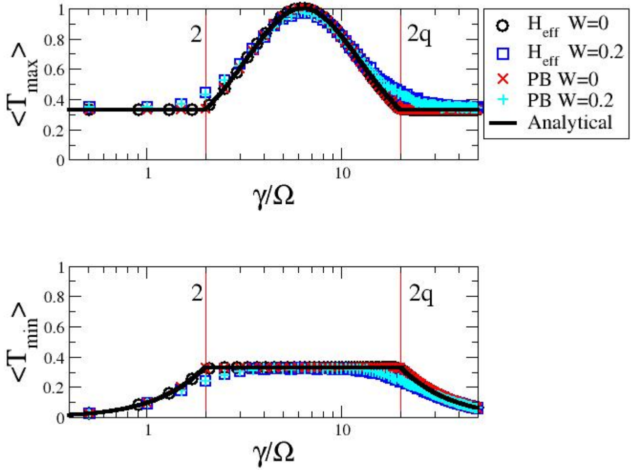
- **N STATES**

$$S = \frac{2\pi\gamma/\Omega}{(q+1)((\gamma/\Omega)^2/4q+1)} \quad Valid \;\; for \;\; Every \;\; N!$$

Structure of Resonances



Double Superradiant Transition: Maximal and Minimal Transmission



Introduce On-Site Disorder

Random variation $\delta L \in [-WL/4E_0, +WL/4E_0]$ in well lengths $\Rightarrow$ random variation in $\delta E_0 \in [-W/2, +W/2]$ in site energies

Weak Disorder

- $W < 2\pi\Omega$
- $D \approx 2\pi\Omega/N$
- Maximum Transmission $\gamma/\Omega = 2\sqrt{q}$ as in the clean case.

Strong Disorder

- $W > 2\pi\Omega$
- Localized Transport Regime: T log-normally distributed
- Mean Level spacing $D \approx W/N$
- Maximum Transmission at $< \Gamma > /D \approx 1 \Longleftrightarrow \gamma/N \sim W/N \Longleftrightarrow \gamma \sim W$

Weak Disorder

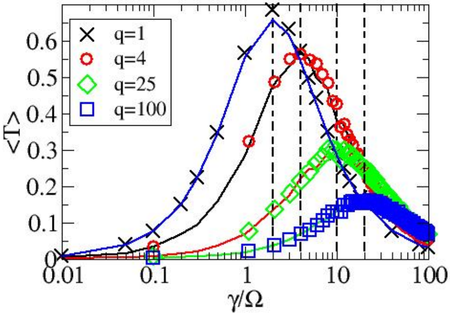
Average Transmission near the center of the energy band

Symbols: Effective Hamiltonian   Curves: Sequence of Potential Wells

For $W < 4\Omega \quad (\gamma/\Omega)_{cr} = 2\, q^{1/2}$

N=100

Disorder $W/\Omega = 0.5$

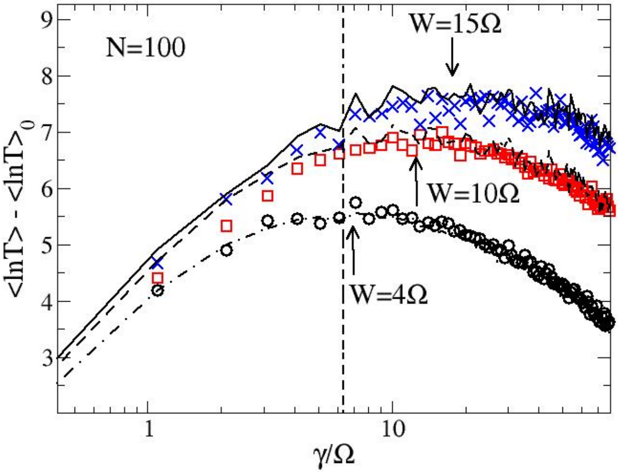


Notice: maximum of each curve intersects q=1 (symmetryc) curve

at critical coupling left and right probability are equal!

Strong Disorder

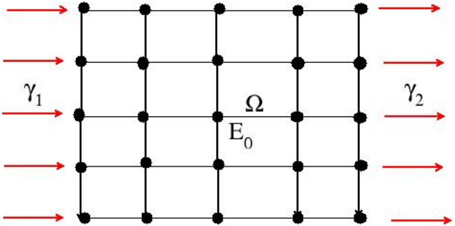
For W >> 4Ω $(\gamma/\Omega)_{cr}$ increases with W



Multidimensional Case

2D ANDERSON MODEL

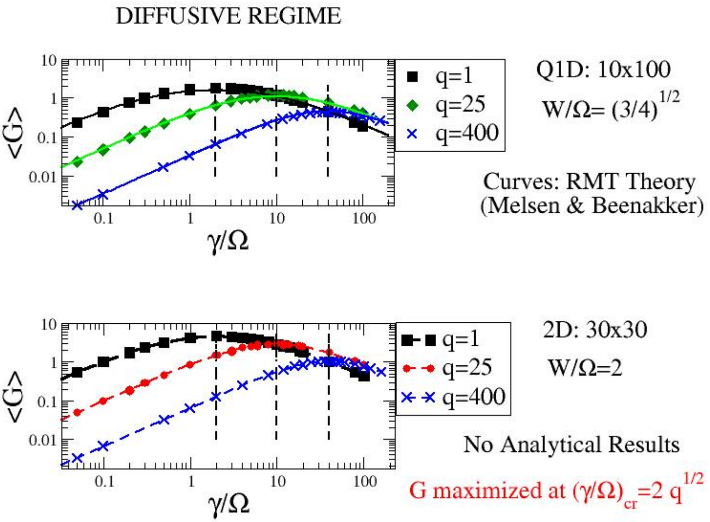
l: MFP   Ballistic: $l > L$ Diffusive: $l < L < L_{loc}$ Localized: $L > L_{loc}$



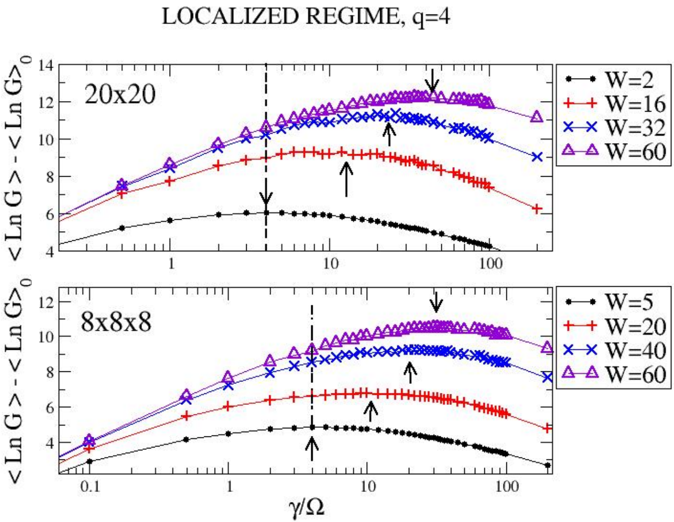
For small W: $\gamma_{cr} = 2q^{1/2}$ For large W: γ_{cr} increases with W

$$G(E) = \Sigma_{a=1,..,M} \Sigma_{b=M+1,..,2M} T(E)^{ab}$$

Conductance as a function of the coupling Strength

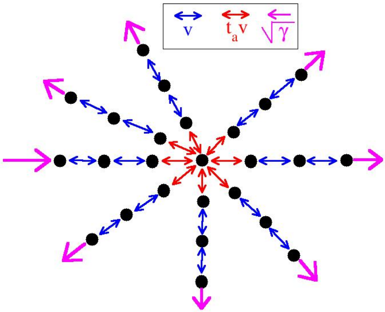


Strong Disorder: Localized Regime



G maximized at $(\gamma/\Omega)_{cr} \sim W$

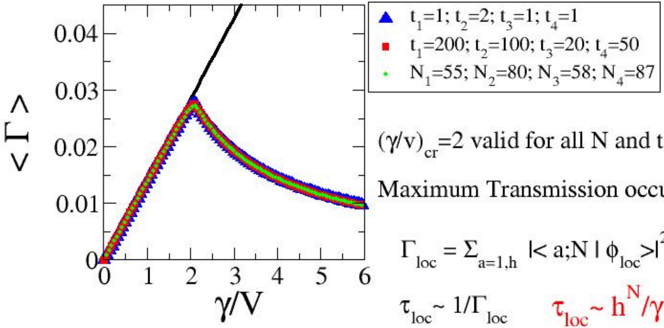
STAR GRAPH



Localized States: $E = \pm \left(\frac{T_{tot}}{\sqrt{T_{tot} - 1}} \right) v$, $T_{tot} = \sum_{a=1}^{h} t_a^2$

Localization Length: $\xi = \frac{1}{\ln(T_{tot} - 1)}$, $T_{tot} \geq 2$

STAR GRAPH II



$(\gamma/v)_{cr}=2$ valid for all N and t

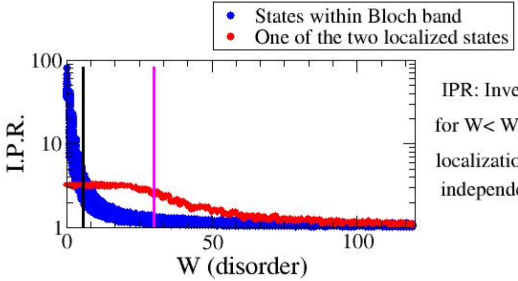
Maximum Transmission occurs at SR

$\Gamma_{loc} = \Sigma_{a=1,h} \, |< a;N \, | \, \phi_{loc} >|^2 \, \gamma$

$\tau_{loc} \sim 1/\Gamma_{loc}$ $\quad \tau_{loc} \sim h^N/\gamma$

Maximum Transmission: $T^{ab}(E=E_{loc}) = (4\, t_a^{\,2}\, t_b^{\,2})/T_{tot}^{\;2}$

STAR GRAPH III

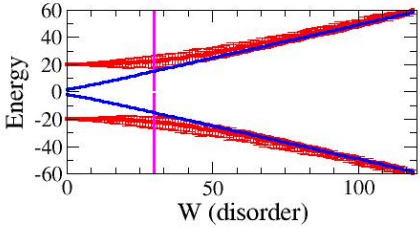


IPR: Inverse partecipation ration

for $W < W_{cr}$

localization length :

independent of disorder



for $W < W_{cr}$

Energy of the localized states:

independent of disorder

Conclusions and Perspectives

- Non Hermitian Hamiltonian approach powerful tool to study coherent quantum transport in mesoscopic systems.
- We have demostrated that Superradiant Transition does occur in a realistic and paradigmatic model of quantum transport (Sequence of Potential Barriers).
- Transport properties strongly depend on degree of opening. Maximal transmission and change of structure of resonances at SR.
- Coherent Quantum Transport is important in nanotechnologies and Basic energy sciences.

REFERENCES

- *Transmission through nanostructure with asymmetric coupling.*, G.L.C., M. Smith, S. Sorathia, V. Zelevinsky, R. A. Sen'kov and L. Kaplan, archive: 1007.5247, submitted to PRB.
- *Superradiance Transition and Transport Through Nanosystems* G.L.C. and L. Kaplan, Phys. Rev. B 79, 155108 (2009).
- *Internal chaos in an open quantum system: From Ericson to conductance fluctuations.* S. Sorathia, F. M. Izrailev, G. L. C., V. G. Zelevinsky and G. P. Berman, EuroPhys. Lett. **88** 27003, (2009).
- *Transition from isolated to overlapping resonances in the open system of interacting fermions.* G.L.C., F.M. Izrailev, V.G. Zelevinsky and G.P.Berman, Phys. Lett B, **659**, 170 (2008).
- *Continuum Shell Model: Comparison with Ericson fluctuations.* G.L.C., V. Zelevinsky, F. Izrailev and G.P.Berman, Phys. Rev. E„ **76** 031119 (2007).

Integrated Transmission

It is useful to consider the integrated transmission over one Miniband:

$$S = \int T(E)dE$$

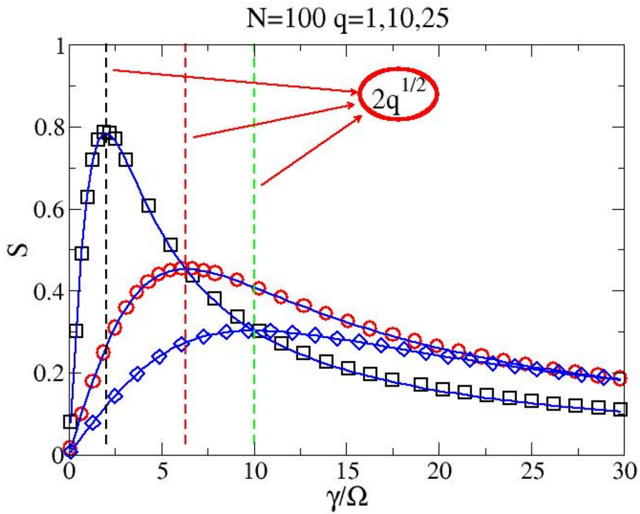
$$S \approx N2\langle\Gamma\rangle = \frac{4N}{N+1}\gamma \text{ independent of } N\,!$$

We can estimate the maximum S_{max} at the superradiant transmission, $\gamma = 2\Omega$, so that

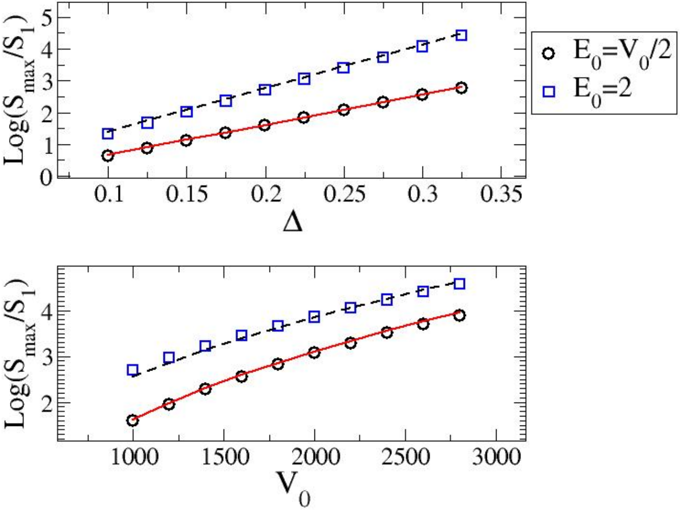
$$\frac{S_{max}}{S_1} \approx \frac{\Omega}{\gamma_1} = \frac{V_0}{4\alpha\sqrt{E_0}} \exp\left(\alpha\Delta\right)$$

with $\alpha = \sqrt{(2m/\hbar^2(V_0 - E_0))}$

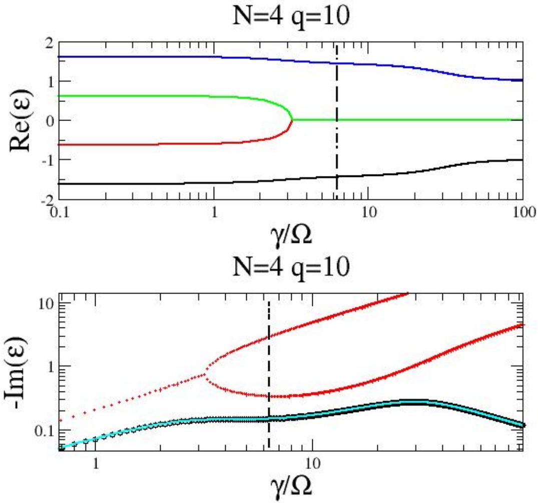
Integrated Transmission



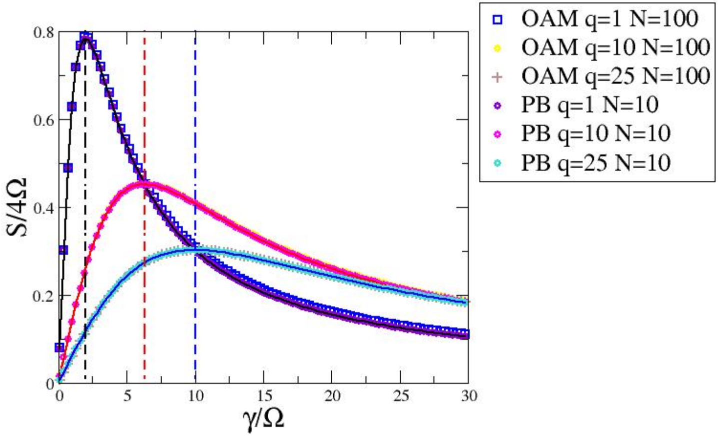
Transmission Gain



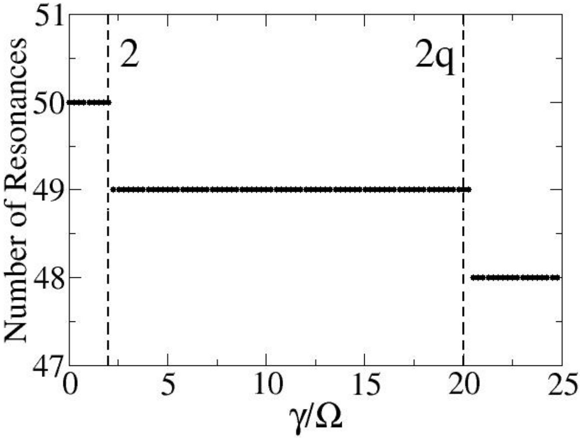
Double Superradiant Transition



Integrated Transmission



Structure of Resonances



Effective Hamiltonian

N intrinsic states, $|i\rangle$, coupled to M open channels, $|c, E\rangle$ with transition amplitude $A_i^c(E) = \langle i|\mathcal{H}|c, E\rangle$, can be described by an effective non-hermitian Hamiltonian:

$$H_{eff}(E) = H + \Delta(E) - i/2W(E)$$

with,

$$W_{i,j}(E) = 2\pi \sum_{c(open)} A_i^c(E) A_j^c(E)$$

and

$$\Delta_{i,j}(E) = \sum_c P.v. \int dE' \frac{A_i^c(E')(A_j^c(E'))^*}{E - E'}$$

Effective Hamiltonian

The Effective Hamiltonian determines the transmission:

$$T^{a,b}(E) = |Z^{a,b}(E)|^2$$

with,

$$Z^{a,b}(E) = \sum_{i,j}^{N} A_i^a \frac{1}{E - H_{eff}} A_j^b$$

Complex eigenvalues of H_{eff} coincides with the poles of $Z(E)$

$$\mathcal{E}_i = E_i - \frac{i}{2}\Gamma_i$$

Superradiance Transition

Simplified H_{eff}:

$$H_{\text{eff}} = H_0 - \frac{i}{2}\gamma W$$

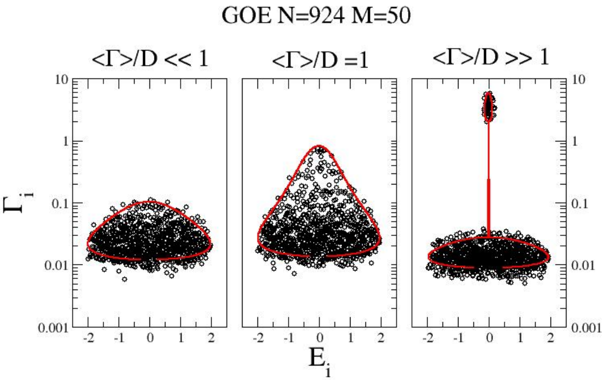
For $\gamma \ll 1$: $\mathcal{E}_i = E_0^i - \frac{i}{2}\gamma W_{ii}$.
For $\gamma \gg 1$: H_0 can be viewed as a perturbation acting on W.
Only M states will have a decay width in the limit of large coupling.
Roughly, the transition occurs when

$$\langle \Gamma \rangle / D \approx 1,$$

where D is the mean level spacing of H_0.

Superradiance is a general phenomenon!

Widths Segregation in RMT



Analytical Curve: F. Haake, et al., Z. Phys. B 88 (1992) 359.

Disordered Case: Localization Length

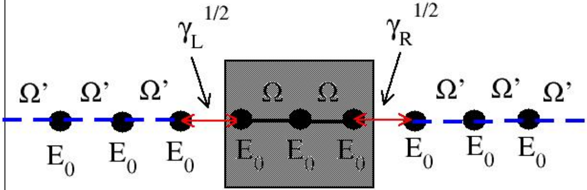
We consider random variations of the diagonal energies, $E_0 + \delta E_0$, where δE_0 is uniformly distributed in the interval $[-W/2, +W/2]$, and W is the disorder parameter. For $\alpha\Delta \gg 1$, a random variation of δE_0 in the interval $[-W/2, +W/2]$ corresponds to a random variation of the well width δL in $[-WL/4E_0, +WL/4E_0]$.

$$L_{\rm loc} \approx 105.2 \left(\frac{W}{\Omega}\right)^{-2} .$$

$$\gamma_{cr} \propto D \quad since \;\; max \;\; for \;\; \frac{\langle \Gamma \rangle}{D} \approx 1$$

SUPERRADIANCE CRITERIUM WORKS ALSO IN THE DISORDERD CASE!

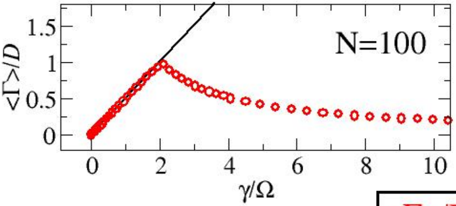
Energy Dependence: another model for the continuum



$$H_{nm}^{eff} = H_{nm} + 2(\varepsilon/2 - i/2(1-\varepsilon^2)^{1/2})(\gamma_L \delta_{n,1} + \gamma_R \delta_{n,N})\delta_{nm}$$

with $\varepsilon = (E - E_0)/2\Omega'$

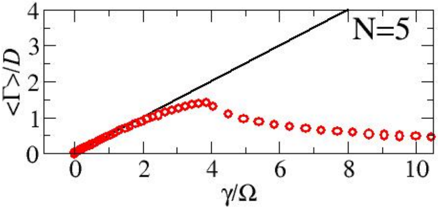
Superradiance Transition



1) $D = 4\,\Omega / N$

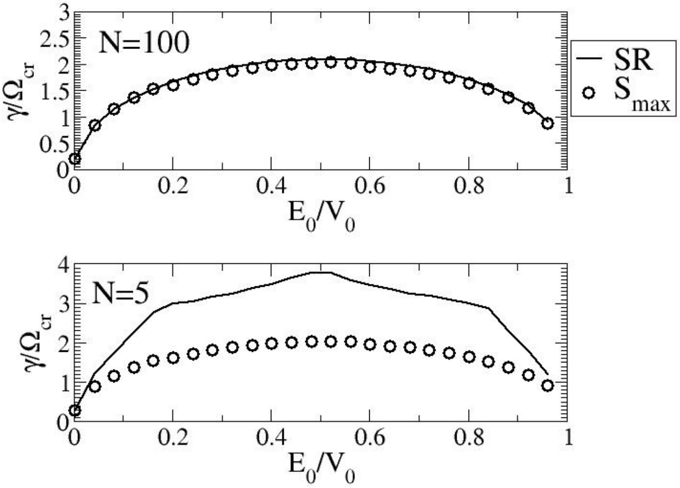
2) $\langle\Gamma\rangle = 2\gamma/N$

$\langle\Gamma\rangle/D = 1 \Rightarrow \gamma/\Omega_{cr} = 2$



$\Delta_{ext} = \Delta/2$

Maximal Transmission and Superradiant Transition



Analytic Results for Transmission: General N

- $$T(E) = \frac{4\gamma_1\gamma_2 P_-^2}{\left[1+\gamma_1\gamma_2\left(P_-^2 - P_+^2\right)\right]^2 + (\gamma_1+\gamma_2)^2 P_+^2}.$$

$$E_n = 2\Omega \cos\left(\frac{\pi n}{N+1}\right) \quad P_\pm(E) = \frac{1}{N+1}\sum_{n=1}^{N}(\pm 1)^n \frac{1}{E-E_n} \sin^2\left(\frac{\pi n}{N+1}\right)$$

- For $|E| \ll \Omega$ $D = 2\pi\Omega/N$, let $E = E_n + rD$,

$$T(E) \approx \frac{16\gamma_1\gamma_2}{(4\Omega+\gamma_1\gamma_2/\Omega)^2 \sin^2(\pi r) + 4(\gamma_1+\gamma_2)^2\cos^2(\pi r)}$$

for r integer $T = T_1$, for r half integer $T = T_2$

$$T_1 \leq T_2 \quad \textit{for} \quad 2 \leq \gamma/\Omega \leq 2q$$