EPs in Microwave Billiards: Eigenvectors and the Full Hamiltonian for T-invariant and T-noninvariant Systems

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Dresden 2011

- Precision experiment with microwave billiard
  → extraction of full EP Hamiltonian from scattering matrix
- Properties of eigenvalues and eigenvectors at and close to an EP
- EPs in systems with violated T invariance
- Encircling the EP: geometric phases and amplitudes
- PT symmetry of the EP Hamiltonian

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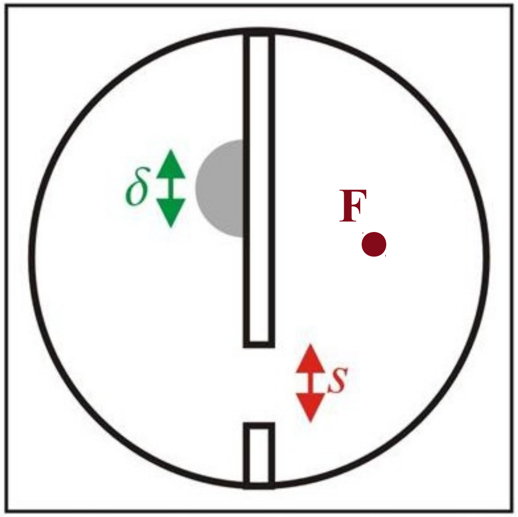
Microwave Resonator for the Observation of Exceptional Points

- Divide a circular microwave billiard into two approximately equal parts
- The coalescence at the EP is accomplished by the variation of two parameters



- The opening **s** controls the coupling of the eigenmodes of the two billiard parts
- The position **δ** of the Teflon disk mainly effects the resonance frequencies of the left part
- Insert a ferrite **F** and magnetize it with an exterior magnetic field B to induce T violation

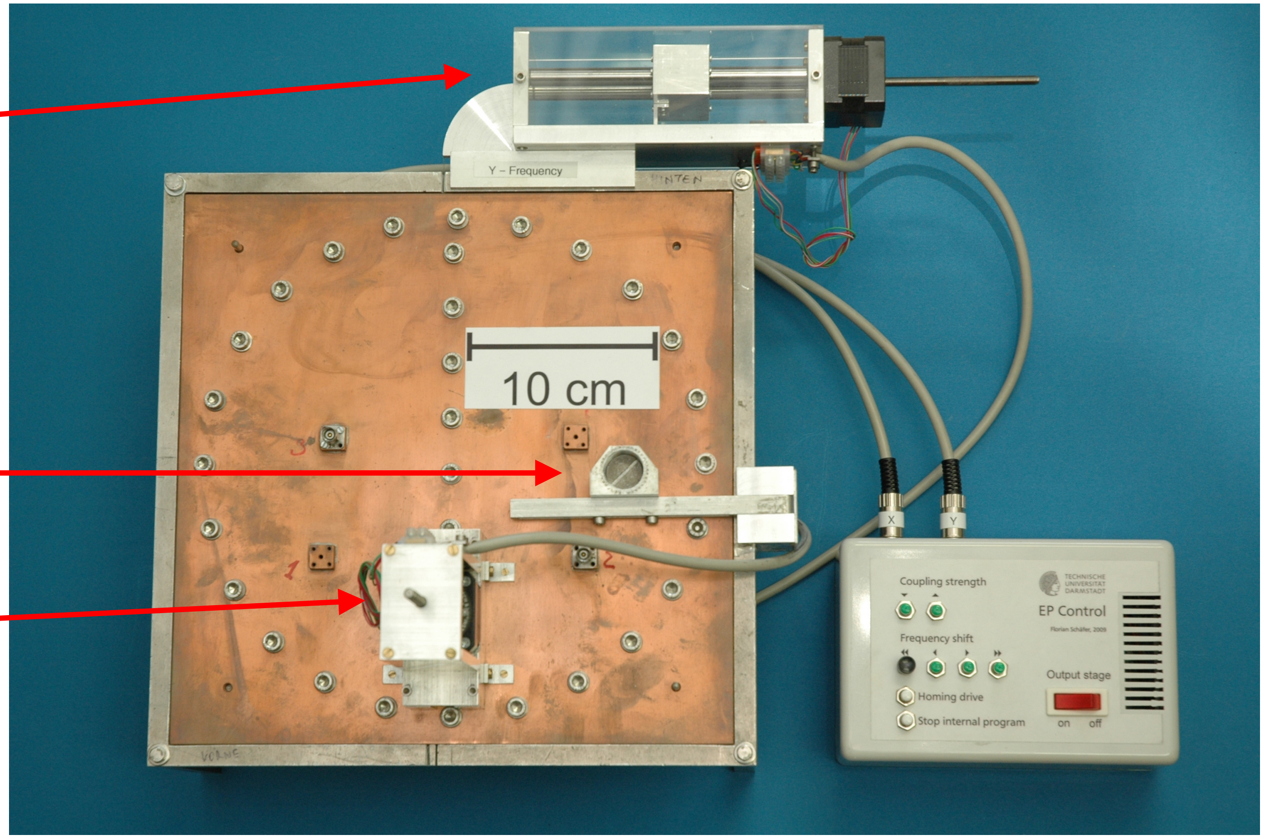
Experimental setup

(B. Dietz et al., Phys. Rev. Lett. **106**, 150403 (2011))

variation of δ

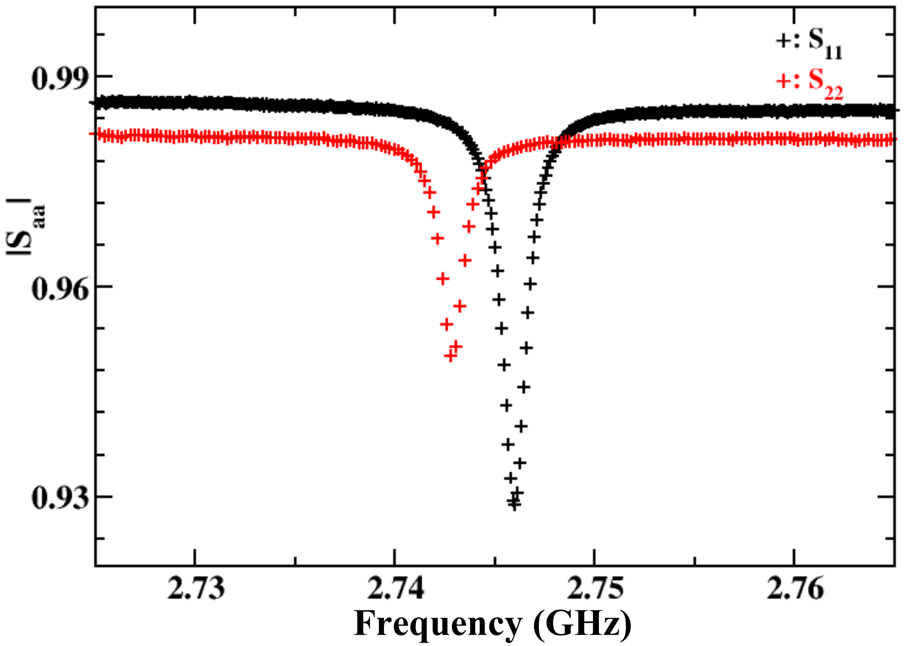
variation of B

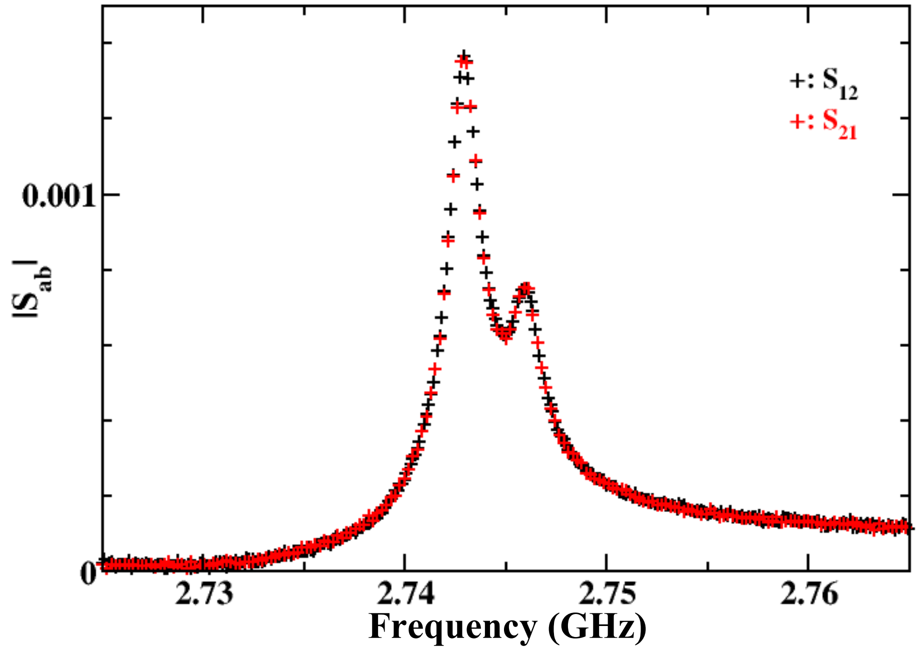
variation of s



- Parameter plane (s,δ) is scanned on a very fine grid

Resonance Spectra Close to an EP (B=0)





- Scattering matrix: $\hat{S} = \hat{I} - 2\pi i \hat{W}^T (E - \hat{H}_{eff})^{-1} \hat{W}$
- $\hat{H}_{eff}$: two-state Hamiltonian including dissipation and coupling to the exterior
- $\hat{\mathbf{W}}$: coupling of the resonator modes to the antenna states

Two-State Matrix Model for the T-invariant Case

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- Determine the non-Hermitian complex symmetric 2× 2 matrix $\hat{H}_{eff}$ and its eigenvalues and eigenvectors for each (**s**,**δ**) from the measured $\hat{S}$ matrix

$$\hat{H}_{eff}(s,\delta) = \begin{pmatrix} E_1 & H_{12}^S \\ H_{12}^S & E_2 \end{pmatrix}$$

ries are functions of **δ** and **s**

- Eigenvalues:

$$e_{1,2} = \left(\frac{E_1 + E_2}{2}\right) \pm \Re$$

$$\Re = H_{12}^S \sqrt{Z^2 + 1}; \; Z = \frac{E_1 - E_2}{2H_{12}^S}$$

- EPs:

$$\Re = 0 : Z = \pm i \leftrightarrow \delta = \delta_{EP}, \; s = s_{EP}$$

Resonance Shape at the EP

- At the EP the $\hat{H}_{eff}$ is given in terms of a Jordan normal form

$$\hat{H}_{eff}(s_{EP},\delta_{EP}) = \begin{pmatrix} \lambda_0 & 1 \\ 0 & \lambda_0 \end{pmatrix} \quad \text{with} \quad \lambda_0 = \frac{E_1 + E_2}{2}$$

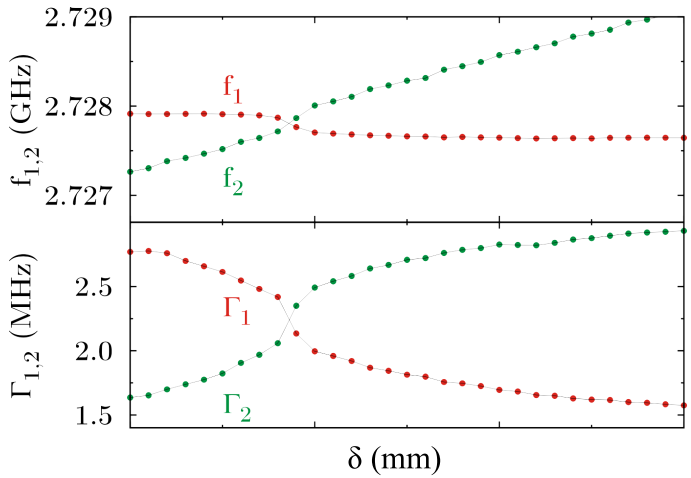
- S_{ab} has two poles of 1st order and one pole of 2nd order

$$S_{ab} = \delta_{ab} - V_{1a}\frac{1}{(f-\lambda_0)}V_{1b} - V_{2a}\frac{1}{(f-\lambda_0)}V_{2b} - \frac{H_{12}^{S}}{(f-\lambda_0)^2}\left\{ i\left(V_{1a}V_{1b} - V_{2a}V_{2b}\right) + V_{1a}V_{2b} + V_{2a}V_{1b}\right\}$$

→ at the EP the resonance shape is not described by a Breit-Wigner form

- Note: this lineshape leads to t^2-behavior (→ first talk)

Localization of an EP (B=0)



- Change of the real and the imaginary part of the eigenvalues $e_{1,2}=f_{1,2}+i\,\Gamma_{1,2}$. They cross at $s=s_{EP}=1.68$ mm and $\delta=\delta_{EP}=41.19$ mm.

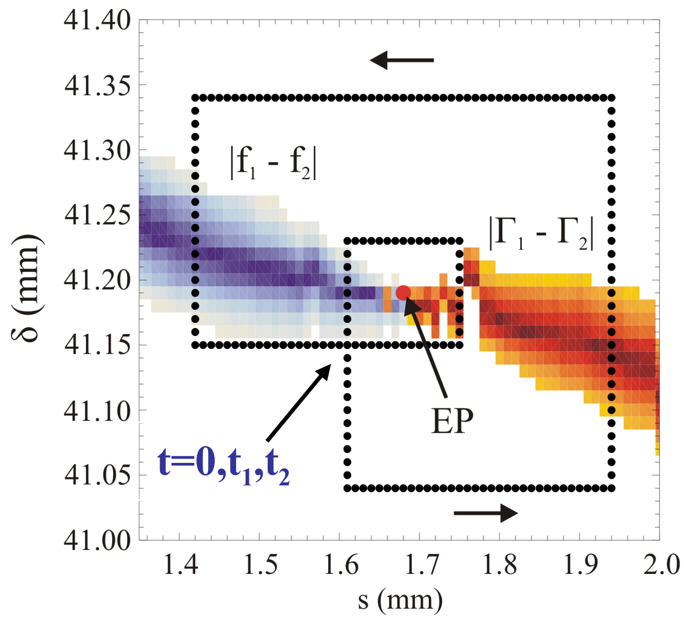
- Change of modulus and phase of the ratio of the components $r_{j,1}$, $r_{j,2}$ of the eigenvector $|r_j\rangle$

$$\nu_j = \frac{r_{j,1}}{r_{j,2}} = |\nu_j| e^{i\Phi_j}$$

- At (s_{EP},δ_{EP}) $\quad |r_j\rangle = \begin{pmatrix} r_{j,1} \\ r_{j,2} \end{pmatrix} \rightarrow \begin{pmatrix} i \\ 1 \end{pmatrix}$

s=1.68 mm

Eigenvalues and Ratios of Eigenvector Components in the Parameter Plane (B=0)



- S matrix is measured for each point of a grid with $\Delta s = \Delta\delta = 0.01$ mm
- Note the dark line, where e_1-e_2 is either real or purely imaginary.

  There, $\hat{H}_{eff} \rightarrow PT$-symmetric $\hat{H}$
- Parameterize the contour by the variable t with $t=0$ at start point, $t=t_1$ after one loop, $t=t_2$ after second one

Encircling the EP in the Parameter Plane ($\mathcal{T}$-invariant Case)

- The biorthonormalized eigenvectors $\langle l_j(t)|$ and $|r_j(t)\rangle$ with $\langle l_j(t)|\, r_j(t)\rangle = 1$ are defined up to a geometric factor $e^{\pm i\gamma_j(t)}$
- The geometric phases $\gamma_j(t)$ are fixed by the condition of parallel transport

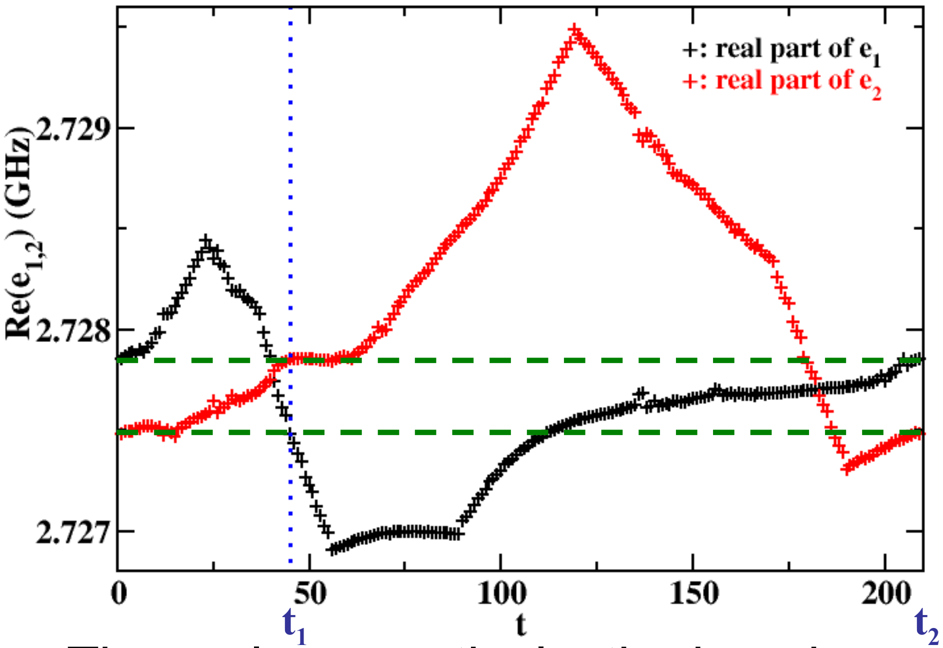
$$\left\langle l_j(t)e^{-i\gamma_j(t)} \middle| \frac{d}{dt} r_j(t)e^{i\gamma_j(t)} \right\rangle = 0 \qquad \Rightarrow \gamma_1(t) = \gamma_2(t) \equiv 0$$

- With $\tan\theta(t) = \sqrt{1+Z^2} - Z$ the eigenvectors are

$$|r_1(t)\rangle = \begin{pmatrix} \cos\theta(t) \\ \sin\theta(t) \end{pmatrix}, |r_2(t)\rangle = \begin{pmatrix} -\sin\theta(t) \\ \cos\theta(t) \end{pmatrix}$$

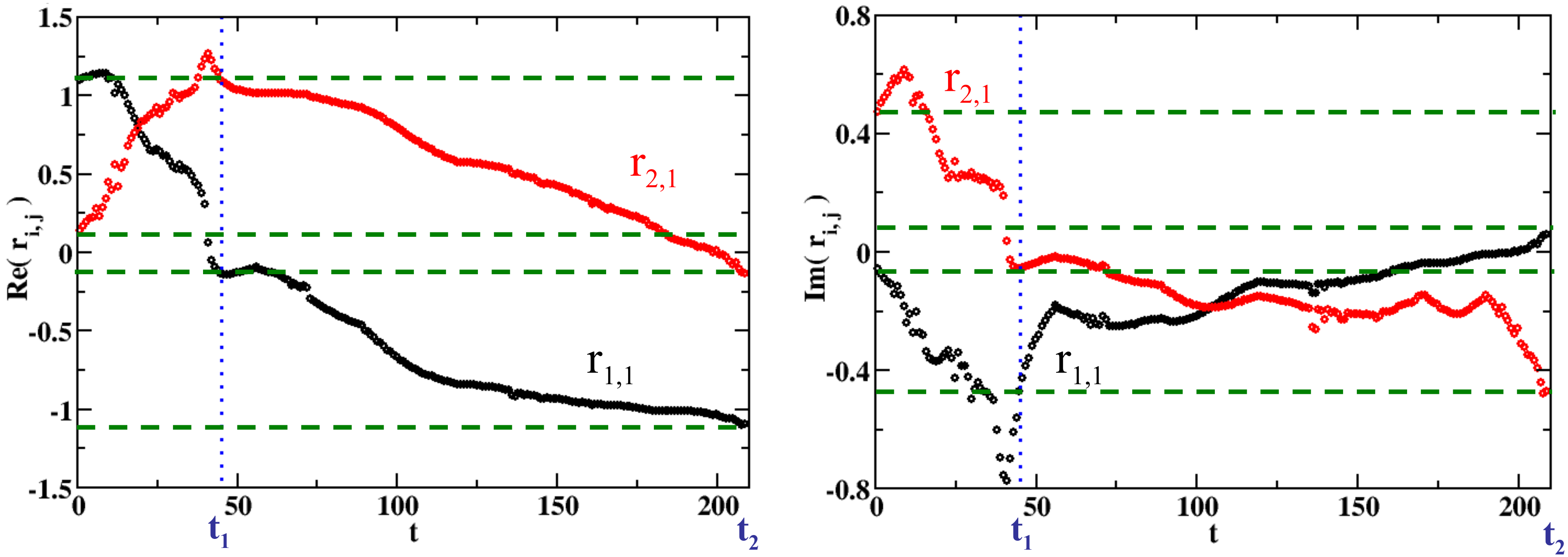
- Encircling the EP once: $\theta \to \theta + \frac{\pi}{2} \Rightarrow e_1 \leftrightarrow e_2$, $\begin{pmatrix} |r_1\rangle \\ |r_2\rangle \end{pmatrix} \to \begin{pmatrix} -|r_2\rangle \\ |r_1\rangle \end{pmatrix}$

Change of the Eigenvalues along the Contour (B=0)



- The real, respectively, the imaginary parts of the eigenvalues cross once during each encircling at different t
- The eigenvalues are interchanged $\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \rightarrow \begin{pmatrix} e_2 \\ e_1 \end{pmatrix}$
- Note: $\mathrm{Im}(e_1) + \mathrm{Im}(e_2) \approx \mathrm{const.} \longrightarrow$ dissipation depends weakly on (s,δ)

Evolution of the Eigenvector Components along the Contour (B=0)

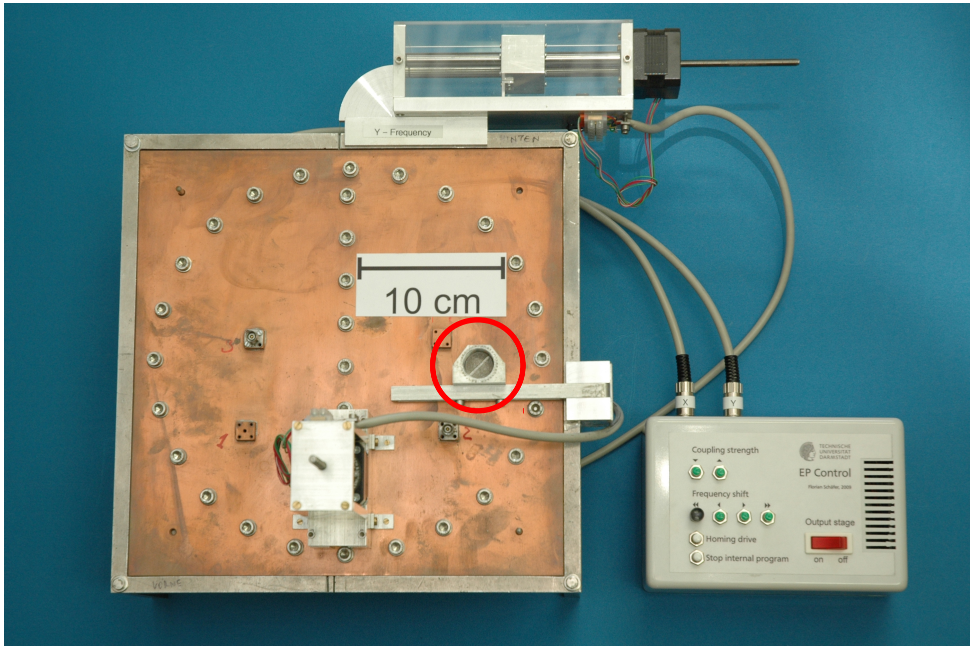


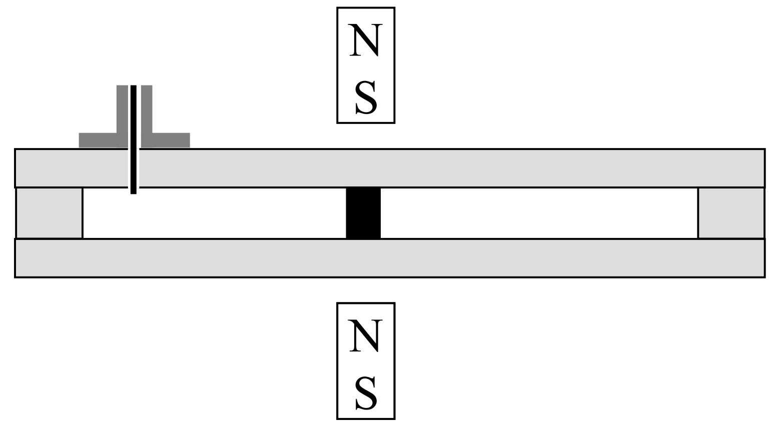
- Evolution of the first component $r_{j,1}$ of the eigenvector $|r_j\rangle$ as function of t
- After each loop the eigenvectors are interchanged and the first one picks up a geometric phase π $\begin{pmatrix}|r_1\rangle\\|r_2\rangle\end{pmatrix} \rightarrow \begin{pmatrix}-|r_2\rangle\\|r_1\rangle\end{pmatrix}$
- Phase does not depend on choice of circuit $\rightarrow$ topological phase

Summary for T-invariant case

- Full EP Hamiltonian extracted from measured scattering matrix
  → direct determination of its eigenvalues and eigenvectors possible
- Behavior of eigenvalues and eigenvectors for T-invariant case as expected
  → confirms validity of the procedure used for data analysis
- Next step: Investigation of the T-noninvariant case following the same procedure

Microwave Billiard for the Study of Induced T Violation

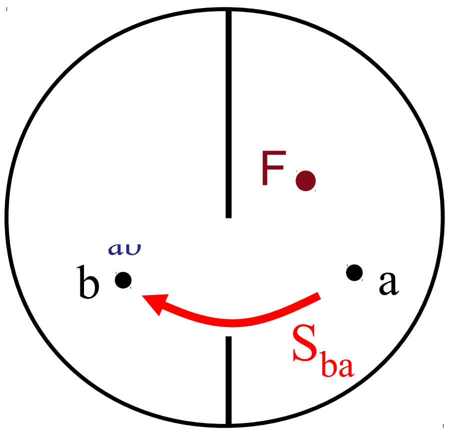




- A cylindrical ferrite is placed in the resonator
- An external magnetic field is applied perpendicular to the billiard plane
- The strength of the magnetic field is varied by changing the distance between the magnets

Induced Violation of *T* Invariance with a Ferrite

- Spins of magnetized ferrite precess collectively with their Larmor frequency about the external magnetic field (→ first talk)
- Coupling of rf magnetic field to the ferromagnetic resonance depends on the direction a ⇆ b



- T-invariant system

  → principle of reciprocity $S_{ab} = S_{ba}$

  → detailed balance $|S_{ab}|^2 = |S_{ba}|^2$

Test of Reciprocity

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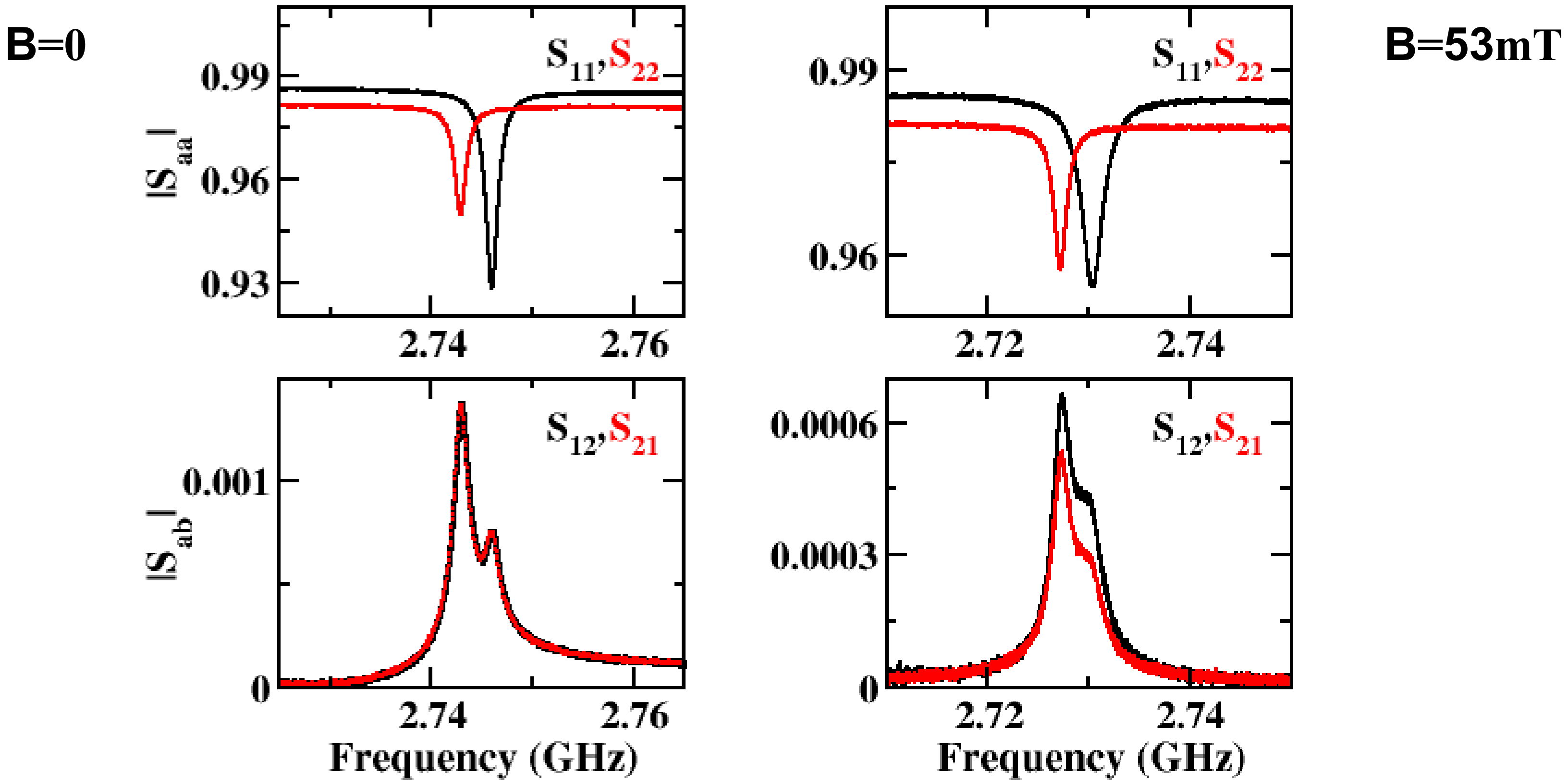


- Clear violation of the principle of reciprocity for nonzero magnetic field

Two-State Matrix Model for Broken *T* invariance

- $\hat{\mathrm{H}}_{\mathrm{eff}}$: non-Hermitian and non-symmetric complex 2×2 matrix (→ first talk)

$$\hat{\mathrm{H}}_{\mathrm{eff}}(s,\delta)=\begin{pmatrix} E_1 & H_{12}^{S} \\ H_{12}^{S} & E_2 \end{pmatrix}+i\begin{pmatrix} 0 & -H_{12}^{A} \\ H_{12}^{A} & 0 \end{pmatrix}$$

- H_{12}^{A} : *T*-breaking matrix element

- Eigenvalues: $$e_{1,2}=\left(\frac{E_1+E_2}{2}\right)\pm\Re$$

$$\Re=\sqrt{{H_{12}^{S}}^2+{H_{12}^{A}}^2}\sqrt{Z^2+1};\ Z=\frac{E_1-E_2}{2\sqrt{{H_{12}^{S}}^2+{H_{12}^{A}}^2}}$$

- EPs: $$\Re=0:\ Z=\pm i \leftrightarrow \delta=\delta_{EP},\ s=s_{EP}$$

T-Violation Parameter τ

- For each set of parameters (s,δ) $\hat{H}_{eff}$ is obtained from the measured $\hat{S}$ matrix

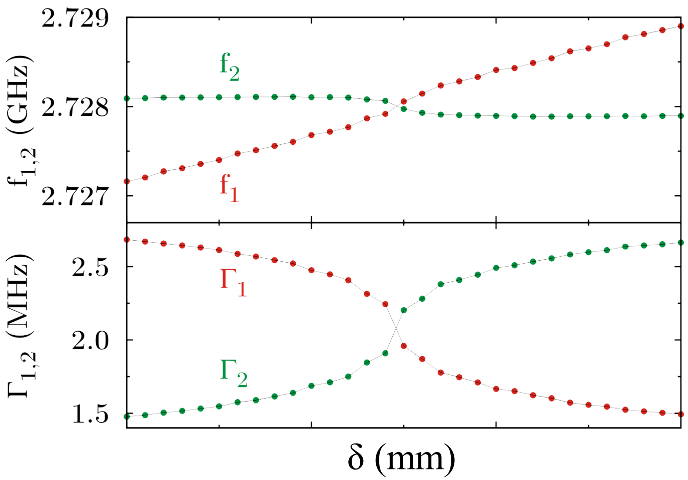
$$\hat{S} = \hat{I} - 2\pi i \hat{W}^T (E - \hat{H}_{eff})^{-1} \hat{W}$$

- $\hat{H}_{eff}$ and $\hat{W}$ are determined up to common real orthogonal transformations
- Choose real orthogonal transformation such that

$$\frac{(H_{12}^S + i H_{12}^A)}{(H_{12}^S - i H_{12}^A)} = e^{2i\tau} \quad \text{with } \tau \in [0,\pi[\text{ real}$$

- T violation is expressed by a real phase → usual practice in nuclear physics

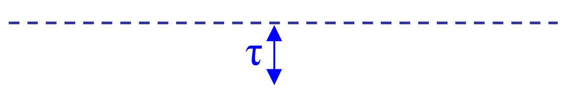
Localization of an EP (B=53mT)



- Change of the real and the imaginary part of the eigenvalues $e_{1,2}=f_{1,2}+i\,\Gamma_{1,2}$. They cross at $s=s_{EP}=1.66$ mm and $\delta=\delta_{EP}=41.25$ mm.

- Change of modulus and phase of the ratio of the components $r_{j,1}$, $r_{j,2}$ of the eigenvector $|r_j\rangle$

$$\nu_j = \frac{r_{j,1}}{r_{j,2}} = |\nu_j| e^{i\Phi_j}$$



s=1.66 mm

- At (s_{EP},δ_{EP}) $\quad |r_j\rangle = \begin{pmatrix} r_{j,1} \\ r_{j,2} \end{pmatrix} \rightarrow \begin{pmatrix} ie^{i\tau} \\ 1 \end{pmatrix}$

T-Violation Parameter τ at the EP

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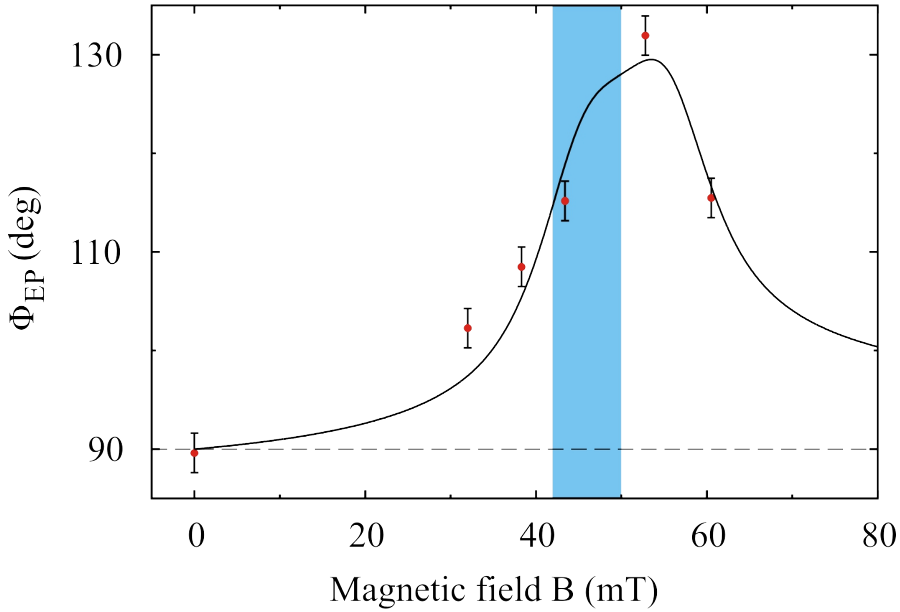
- $\Phi_{EP} = \pi/2 + \tau$



- Φ and also the T-violating matrix element shows resonance like structure

$$H_{12}^{A}(B) = \frac{\pi}{2} \cdot \lambda \quad \cdot \quad B \quad \cdot \quad T^{r} \quad \cdot \quad \frac{\omega_M^2}{\omega_0(B) - \overline{\omega} - i/T^{r}}$$

(→ first talk)

↑ coupling strength

↑ spin relaxation time

↑ magnetic susceptibility

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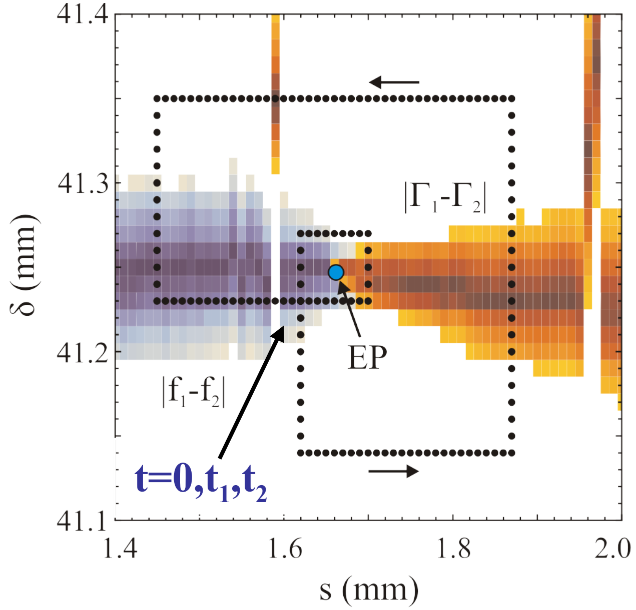
Eigenvalues and Ratios of Eigenvector Components in Parameter Plane (B=53mT)



- S matrix is measured for each point of a grid with $\Delta s = \Delta\delta = 0.01$ mm
- Note the dark line, where e_1-e_2 is either real or purely imaginary. There, $\hat{H}_{eff} \rightarrow PT$-symmetric $\hat{H}$
- Parameterize the contour by the variable t with $t=0$ at starting point, $t=t_1$ after one loop, $t=t_2$ after second one

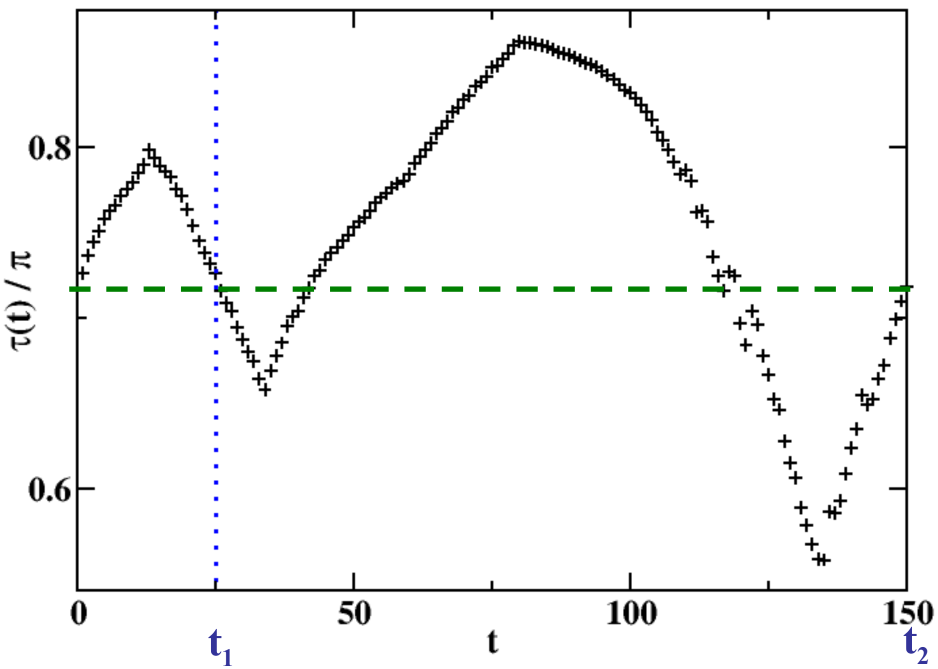
Encircling the EP in the Parameter Space (with T violation)

- Eigenvectors along contour: $\langle L_j(t)| = \langle l_j(t)| e^{-i\gamma_j(t)}, \quad |R_j(t)\rangle = |r_j(t)\rangle e^{i\gamma_j(t)}$
- Biorthonormality is defined up to a geometric factor $e^{\pm i\gamma_j(t)}$
- Condition of parallel transport $\left\langle L_j(t) \middle| \frac{d}{dt} R_j(t) \right\rangle = 0$ yields

$$\frac{d\gamma_1}{dt} = -\frac{d\gamma_2}{dt} \text{ and } \gamma_{1,2}(t) \neq 0 \text{ for } \frac{d\tau}{dt} \neq 0$$

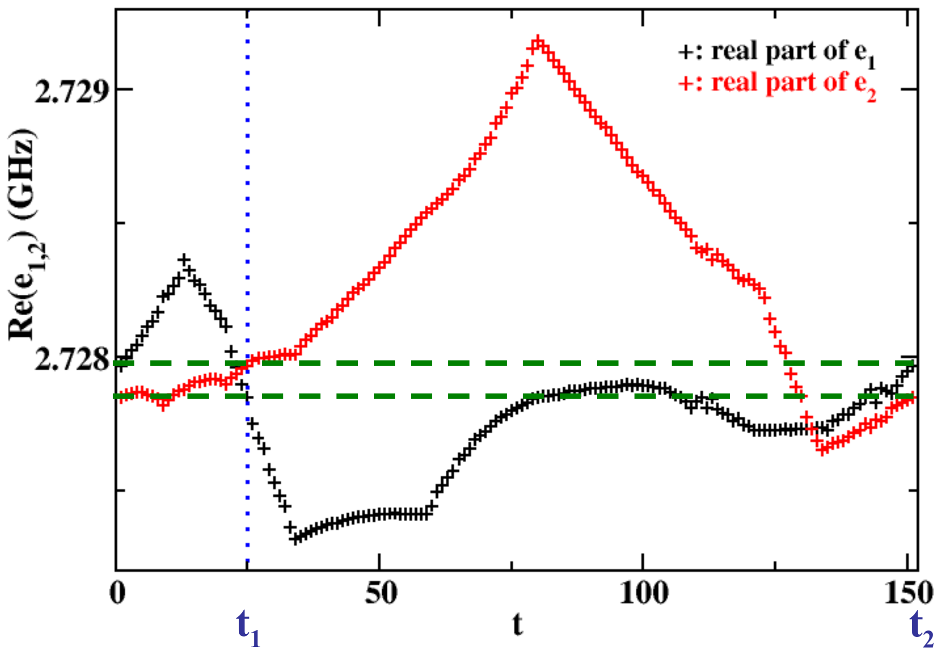
- Encircling EP once: $e_1 \leftrightarrow e_2, \begin{pmatrix} |r_1\rangle \\ |r_2\rangle \end{pmatrix} \rightarrow \begin{pmatrix} -|r_2\rangle \\ |r_1\rangle \end{pmatrix}$ → as in T-invariant case
- Additional geometric factor: $e^{i\gamma_j(t)} = e^{i\,\mathrm{Re}(\gamma_j(t))} e^{-\mathrm{Im}(\gamma_j(t))}$

$\mathcal{T}$-violation Parameter τ along Contour (B=53mT)



- $\mathcal{T}$-violation parameter τ varies along the contour even though B is fixed because the electromagnetic field at the ferrite changes
- τ increases (decreases) with increasing (decreasing) parameters s and δ
- τ returns after each loop around the EP to its initial value

Change of the Eigenvalues along the Contour (B=53mT)

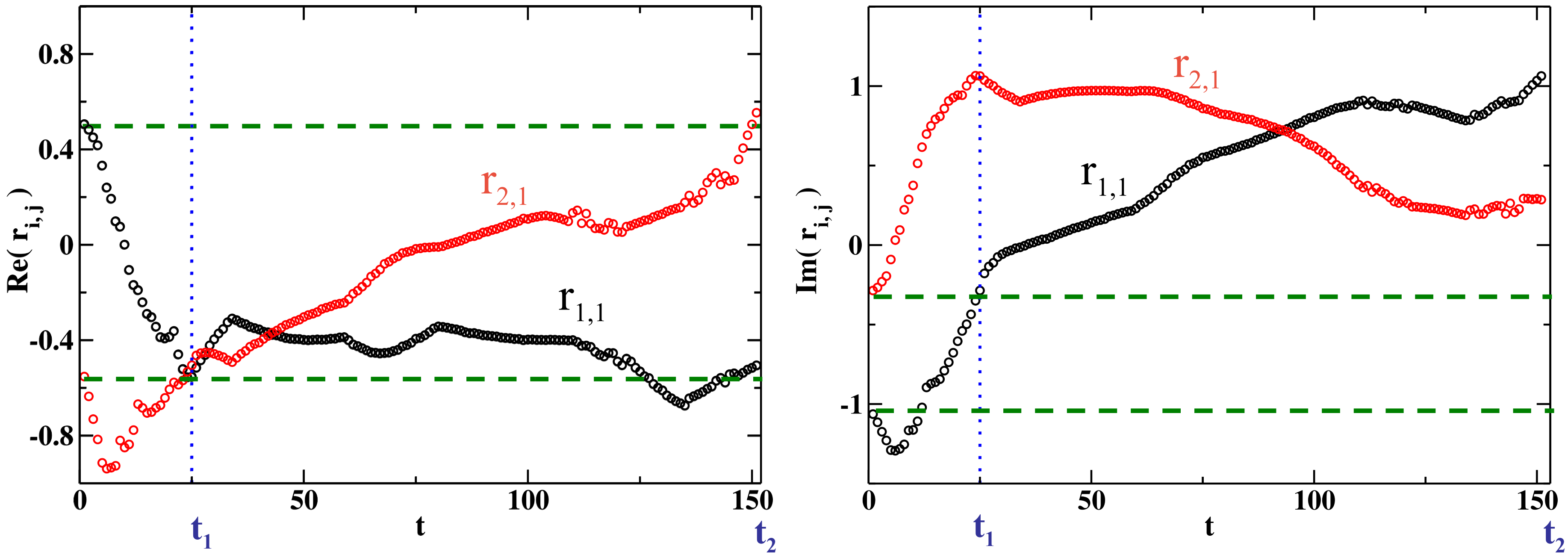


- The real and the imaginary parts of the eigenvalues cross once during each encircling at different values of t

→ the eigenvalues are interchanged

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \rightarrow \begin{pmatrix} e_2 \\ e_1 \end{pmatrix}$$

Evolution of the Eigenvector Components along the Contour (B=53mT)



- Measured transformation scheme:

$$\begin{pmatrix} |r_1\rangle \\ |r_2\rangle \end{pmatrix} \rightarrow \begin{pmatrix} -|r_2\rangle e^{i\gamma_1(t_1)} \\ |r_1\rangle e^{-i\gamma_1(t_1)} \end{pmatrix} \rightarrow \begin{pmatrix} -|r_1\rangle e^{i\gamma_1(t_2)} \\ -|r_2\rangle e^{-i\gamma_1(t_2)} \end{pmatrix}$$

- No general rule exists for the transformation scheme of the γ_j

Geometric Phase Re $(\gamma_j(t))$ Gathered along Two Loops

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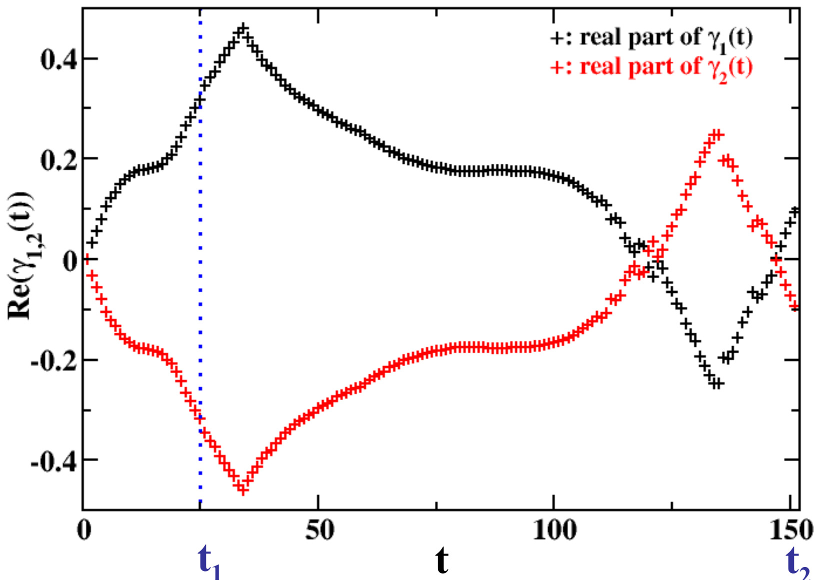
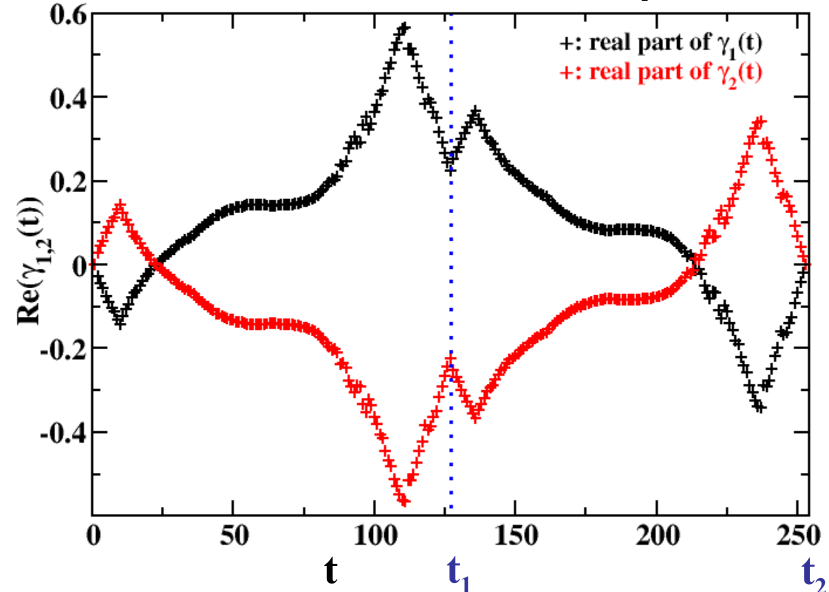
Two different loops



Twice the same loop



- Clearly visible that $\mathrm{Re}(\gamma_2(t)) = -\mathrm{Re}(\gamma_1(t))$

Two different loops:

$t = 0: \mathrm{Re}(\gamma_1(0)) = \mathrm{Re}(\gamma_2(0)) = 0$

$t = t_1: \mathrm{Re}(\gamma_1(t_1)) = -\mathrm{Re}(\gamma_2(t_1)) = 0.31778$

$t = t_2: \mathrm{Re}(\gamma_1(t_2)) = -\mathrm{Re}(\gamma_2(t_2)) = 0.0931$

Twice the same loop:

$t = 0: \mathrm{Re}(\gamma_1(0)) = \mathrm{Re}(\gamma_2(0)) = 0$

$t = t_1: \mathrm{Re}(\gamma_1(t_1)) = -\mathrm{Re}(\gamma_2(t_1)) = 0.22468$

$t = t_2: \mathrm{Re}(\gamma_1(t_2)) = -\mathrm{Re}(\gamma_2(t_2)) = -3.3\cdot 10^{-7}$

- Same behavior observed for the imaginary part of $\gamma_j(t)$

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Complex Phase $\gamma_1(t)$ Gathered along Two Loops

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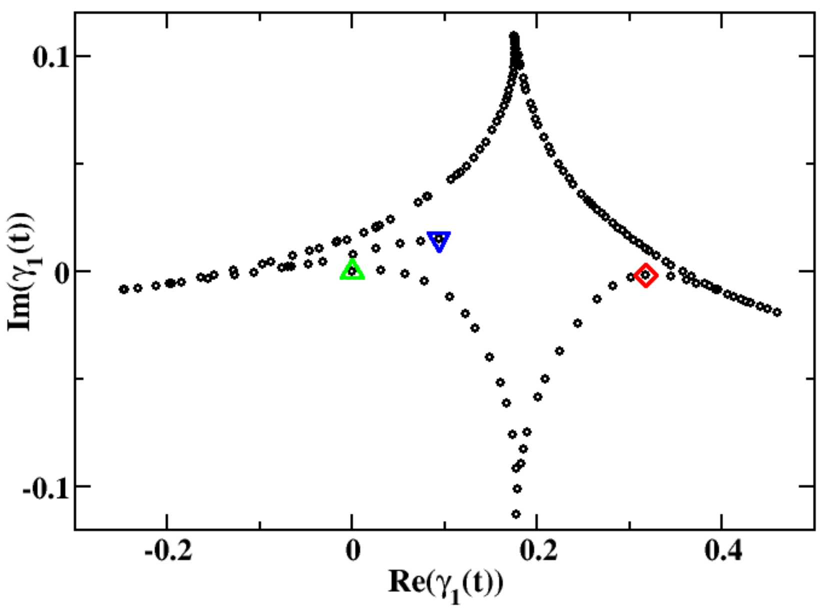
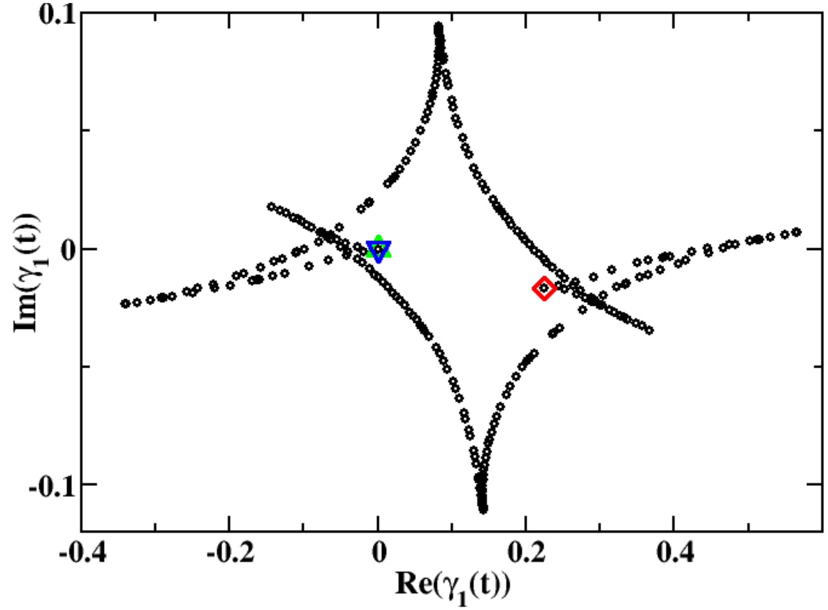
Two different loops



Twice the same loop



Δ : start point

$\Diamond$: $\gamma_1(t_1) \neq \gamma_1(0)$

∇ : $\gamma_1(t_2) \neq \gamma_1(0)$ if EP is encircled along different loops $\rightarrow |e^{i\gamma_1(t)}| \neq 1$ possible

$\rightarrow \gamma_1(t)$ depends on choice of path $\Rightarrow$ geometrical phase

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Complex Phases $\gamma_j(t)$ Gathered when Encircling EP Twice along Double Loop

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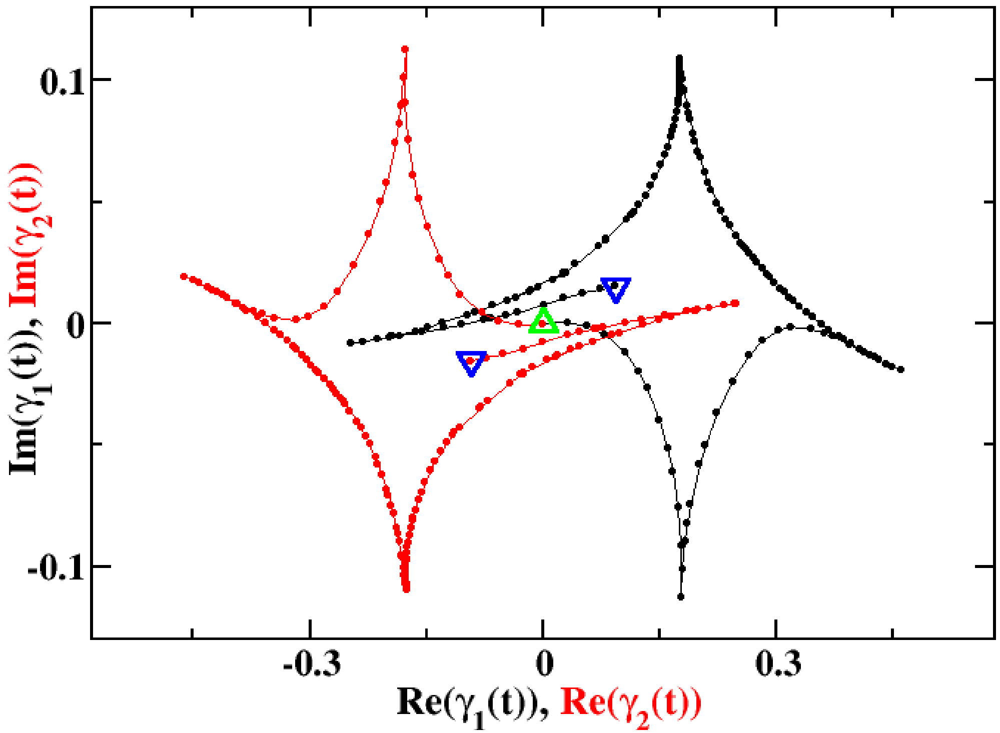

Δ : start point

∇ : $\gamma_1(nt_2) \neq \gamma_1(0)$, n=1,2,…

- Encircle the EP 4 times along the contour with two different loops
- In the complex plane $\gamma_{1,2}(t)$ drift away from the origin

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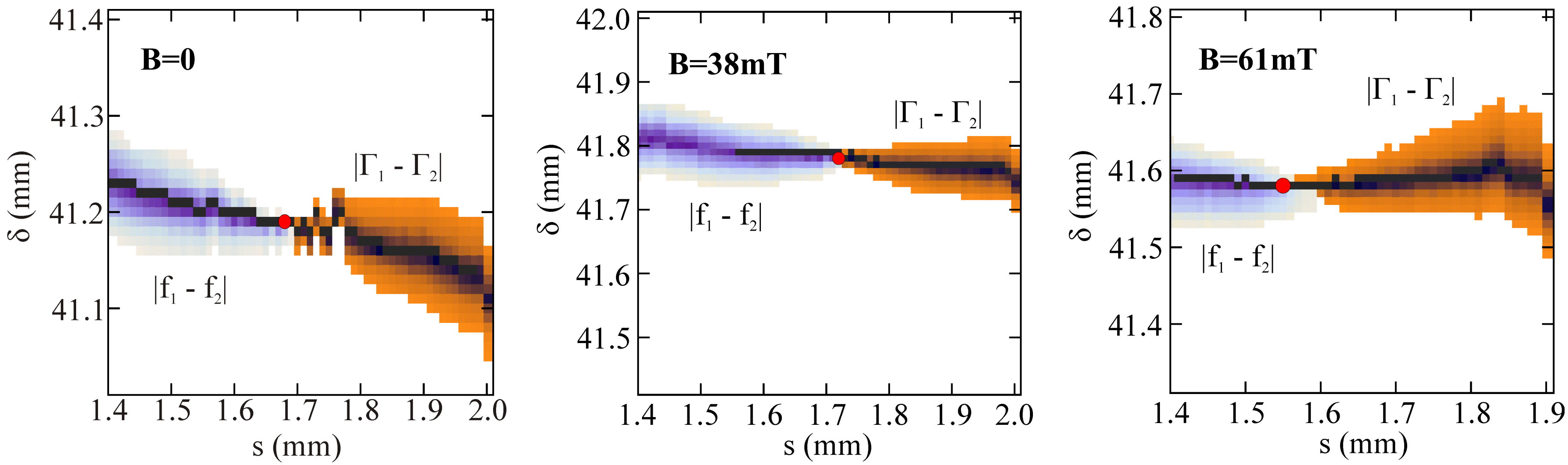
Difference of Eigenvalues in the Parameter Plane

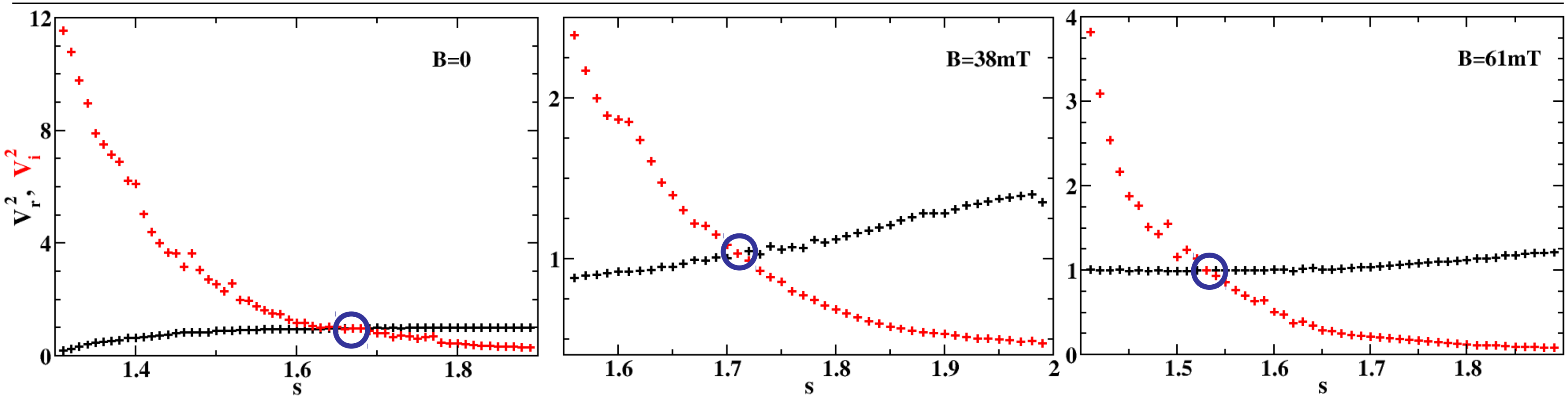


- Dark line: $|f_1\text{-}f_2|=0$ for $s<s_{EP}$, $|\Gamma_1\text{-}\Gamma_2|=0$ for $s>s_{EP}$

→ $(e_1\text{-}e_2) = (f_1\text{-}f_2)+i(\Gamma_1\text{-}\Gamma_2)$ is purely imaginary for $s<s_{EP}$ / purely real for $s>s_{EP}$

- $\pm\ (e_1\text{-}e_2)$ are the eigenvalues of $\hat{H}_{DL} = \hat{H}_{eff} - \frac{1}{2}\mathrm{Tr}\left(\hat{H}_{eff}\right)\hat{I}$

Eigenvalues of $\hat{H}_{DL}$ along Dark Line



- Eigenvalues of $\hat{H}_{DL}$: $\varepsilon_{\pm} = \pm\sqrt{V_r^2 - V_i^2 + 2iV_{ri}}$, V_r, V_i, V_{ri} real
- Dark Line: $V_{ri}=0 \rightarrow$ radicand is real
- V_r^2 and V_i^2 cross at the EP

$\boldsymbol{PT}$ symmetry of $\hat{\mathrm{H}}_{\mathrm{DL}}$ along Dark Line

- General form of $\hat{\mathrm{H}}$, which fulfills $[\hat{\mathrm{H}}, \boldsymbol{PT}]=0$:

$$\hat{\mathrm{H}} = \begin{pmatrix} i\mathrm{A} & \mathrm{B} \\ \mathrm{B} & -i\mathrm{A} \end{pmatrix}, \quad \mathrm{A}, \mathrm{B} \text{ real}$$

- $\boldsymbol{P}$: parity operator $P = \hat{\sigma}_{\mathrm{x}}$, $\boldsymbol{T}$: time-reversal operator $\boldsymbol{T}=\boldsymbol{K}$
- ***exact PT*** symmetry: the eigenvalues of $\hat{\mathrm{H}}$ are real
- For $\mathrm{V}_{\mathrm{ri}}=0$ $\hat{\mathrm{H}}_{\mathrm{DL}}$ can be brought to the form of $\hat{\mathrm{H}}$ with the unitary transformation

$$\hat{\mathrm{U}} = e^{i\varphi\hat{\sigma}_{\mathrm{y}}} e^{i\tau/2\hat{\sigma}_{\mathrm{z}}}$$

- At the EP its eigenvalues change from purely real to purely imaginary

→ ***exact PT*** symmetry is spontaneously broken

→ Talk by Uwe Günther

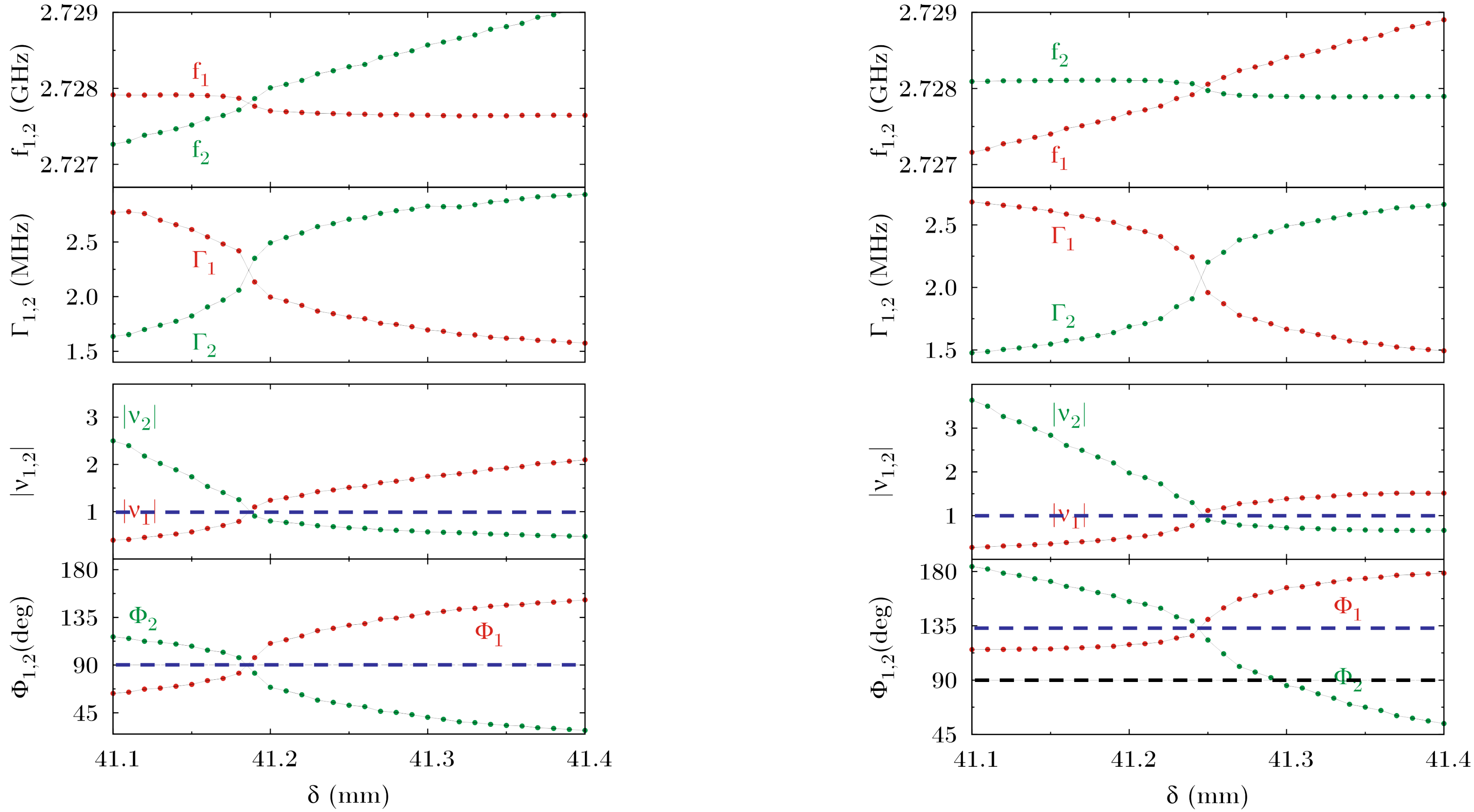
Summary

- High precision experiments were performed in microwave billiards with and without T violation at and in the vicinity of an EP
- The behavior of the complex eigenvalues and ratios of the eigenvector components of the associated two-state Hamiltonian were investigated
- Encircling an EP:
- Eigenvalues: $\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \to \begin{pmatrix} e_2 \\ e_1 \end{pmatrix}$ Eigenvectors: $\begin{pmatrix} |r_1\rangle \\ |r_2\rangle \end{pmatrix} \to \begin{pmatrix} -|r_2\rangle e^{i\gamma_1(t_1)} \\ |r_1\rangle e^{-i\gamma_1(t_1)} \end{pmatrix} \to \begin{pmatrix} -|r_1\rangle e^{i\gamma_1(t_2)} \\ -|r_2\rangle e^{-i\gamma_1(t_2)} \end{pmatrix}, \gamma_1(t) = -\gamma_2(t)$
- T-invariant case: $\gamma_1(t) \equiv 0$
- Violated T invariance: $\gamma_{1,2}(t_1) \neq \gamma_{1,2}(0)$, different loops: $\gamma_{1,2}(t_2) \neq \gamma_{1,2}(0)$
- The size of T violation at the EP is determined from the phase of the ratio of the eigenvector components
- *Exact PT* symmetry is observed along a line in the parameter plane
- *Exact PT* symmetry is spontaneously broken at the EP

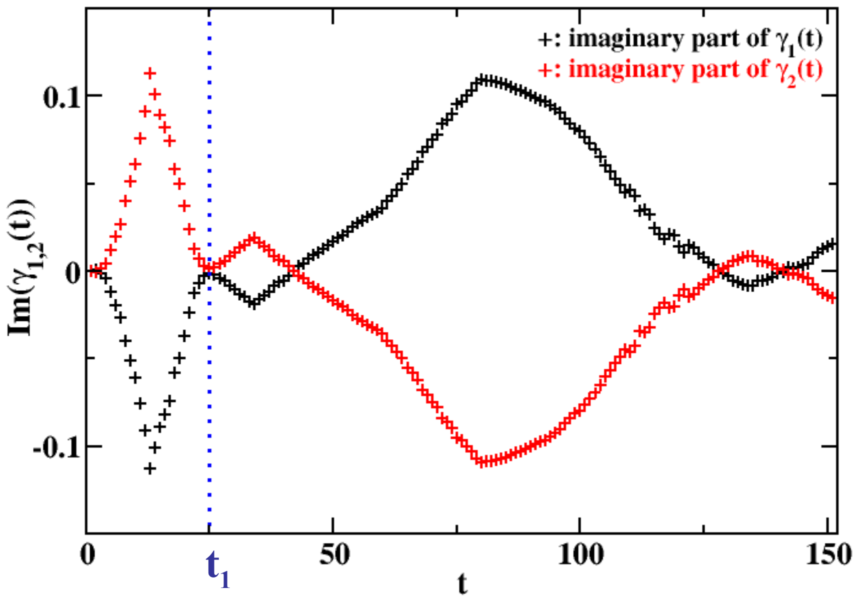
Localization of EP for B=0 and B=53mT

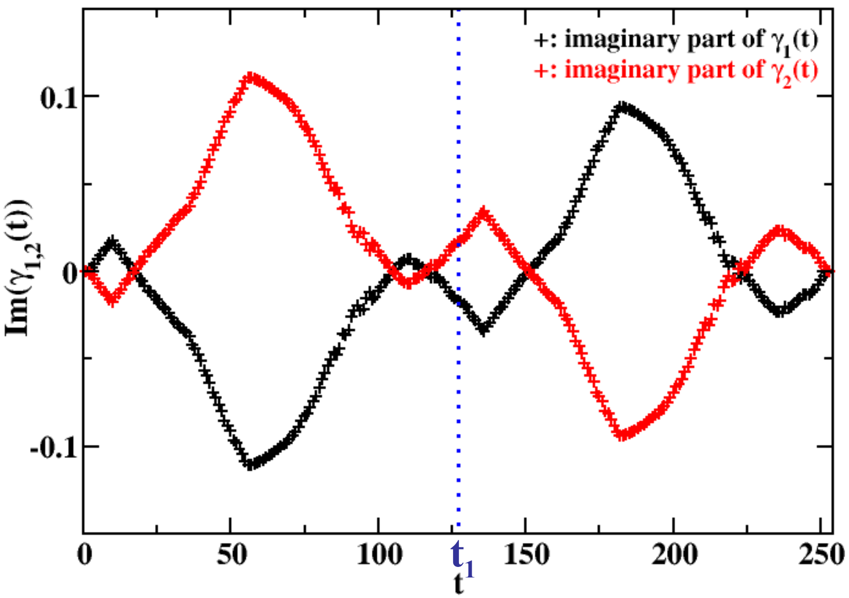
B = 0: s_{EP}=1.68 mm δ_{EP}=41.19 mm

B = 53 mT: s_{EP}=1.66 mm δ_{EP}=41.25 mm



Geometric Amplitude $e^{-\mathrm{Im}(\gamma_1(t))}$ Gathered Along Two Different/Equal Loops



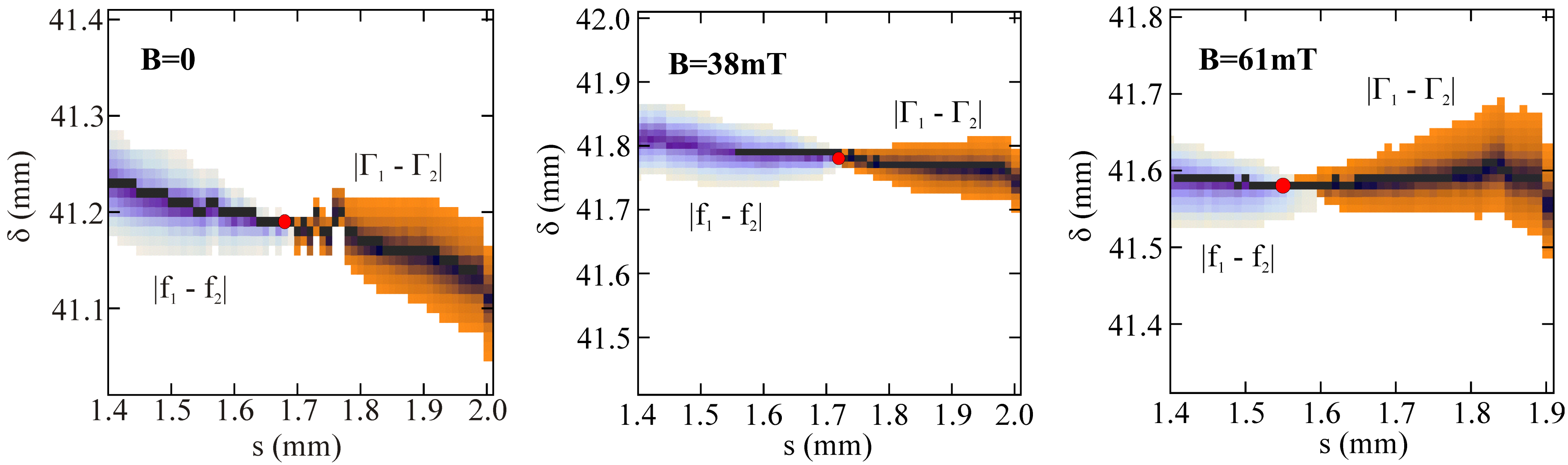


$t = 0: \mathrm{Im}(\gamma_1(0)) = \mathrm{Im}(\gamma_2(0)) = 0$ $t = 0: \mathrm{Im}(\gamma_1(0)) = \mathrm{Im}(\gamma_2(0)) = 0$

$t = t_1: \mathrm{Im}(\gamma_1(t_1)) = -\mathrm{Im}(\gamma_2(t_1)) = -0.00148$ $t = t_1: \mathrm{Im}(\gamma_1(t_1)) = -\mathrm{Im}(\gamma_2(t_1)) = -0.0166$

$t = t_2: \mathrm{Im}(\gamma_1(t_2)) = -\mathrm{Im}(\gamma_2(t_2)) = 0.01522$ $t = t_2: \mathrm{Im}(\gamma_1(t_2)) = -\mathrm{Im}(\gamma_2(t_2)) = -3.5\cdot 10^{-}$

Difference of Eigenvalues in the parameter plane

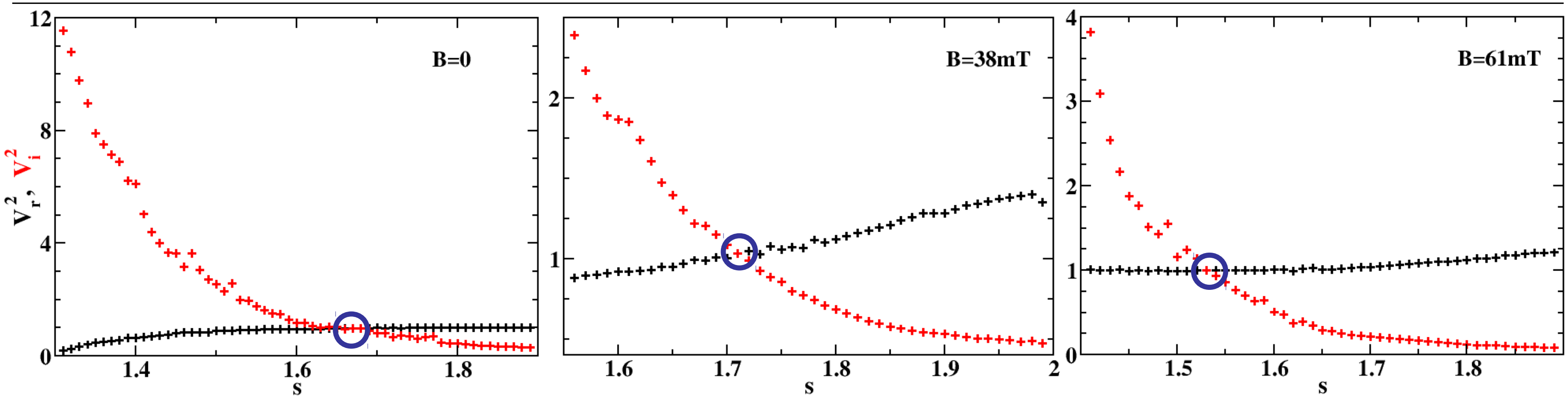


- Dark line: $|f_1-f_2|=0$ for $s<s_{EP}$, $|\Gamma_1-\Gamma_2|=0$ for $s>s_{EP}$

$\rightarrow (e_1-e_2) = (f_1-f_2)+i(\Gamma_1-\Gamma_2)$ is real for $s>s_{EP}$ and purely imaginary for $s<s_{EP}$

- $\pm\ (e_1-e_2)$ are the eigenvalues of $\hat{H}_{PT} = \hat{H}_{eff} - \frac{E_1+E_2}{2}\hat{I} = \begin{pmatrix} \frac{E_1-E_2}{2} & H_{12}^S - iH_{12}^A \\ H_{12}^S + iH_{12}^A & -\frac{E_1-E_2}{2} \end{pmatrix}$

Eigenvalues of H_{PT} along Dark Line



- Eigenvalues of $\hat{H}_{PT}$: $$\varepsilon_\pm = \pm\left|H_{12}^S\right|\sqrt{V_r^2 - V_i^2 + 2iV_{ri}},$$
- Dark Line: $$0 = V_{ri} \propto \left(\mathrm{Re}\,H_{12}^S\,\mathrm{Im}\,H_{12}^S + \mathrm{Re}\,H_{12}^A\,\mathrm{Im}\,H_{12}^A + \mathrm{Re}(E_1 - E_2)\,\mathrm{Im}(E_1 - E_2)/4\right)$$ $$\left|H_{12}^S\right|^2 V_r^2 = \left(\left(\mathrm{Re}\,H_{12}^S\right)^2 + \left(\mathrm{Re}\,H_{12}^A\right)^2 + \left(\mathrm{Re}[E_1 - E_2]\right)^2/4\right)$$
- V_r^2 and V_i^2 cross at the EP

PT symmetry of H_{PT} along Dark Line

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- For $V_{ri}=0$ $\hat{H}_{PT}$ can be transformed into a *PT*-symmetric Hamiltonian

$$\hat{U}\hat{H}_{PT}\hat{U}^{-1} = \frac{1}{\cos\tau}\begin{pmatrix} i\,\frac{\mathrm{Im}\,H_{12}^S}{\sin 2\varphi} & \frac{\mathrm{Re}\,H_{12}^S}{\cos 2\varphi} \\ \frac{\mathrm{Re}\,H_{12}^S}{\cos 2\varphi} & -i\,\frac{\mathrm{Im}\,H_{12}^S}{\sin 2\varphi} \end{pmatrix}$$

- Unitary transformation: $\hat{U} = e^{i\varphi\hat{\sigma}_y} e^{i\tau/2\hat{\sigma}_z}$ with $\tan 2\varphi = \frac{2}{\cos\tau}\frac{\mathrm{Im}H_{12}^S}{\mathrm{Im}(E_1 - E_2)}$

→ Talk by Uwe Günther