Resonance of Open Quantum Systems and Spontaneous Breaking of Time-Reversal Symmetry

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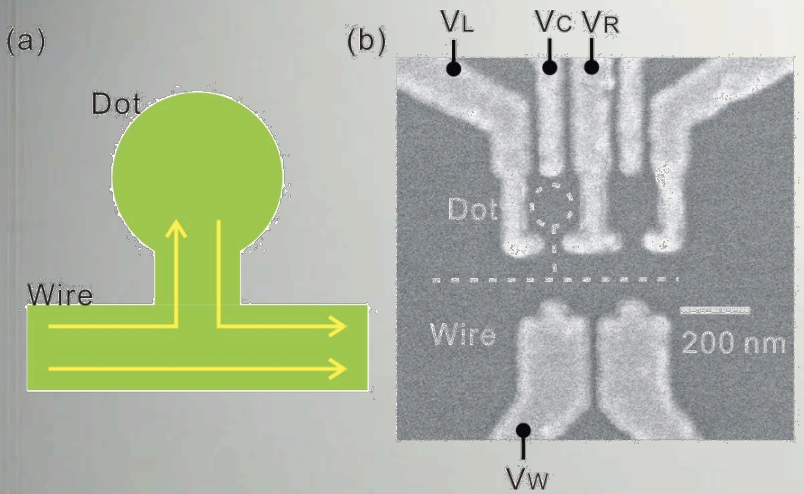
Open quantum systems

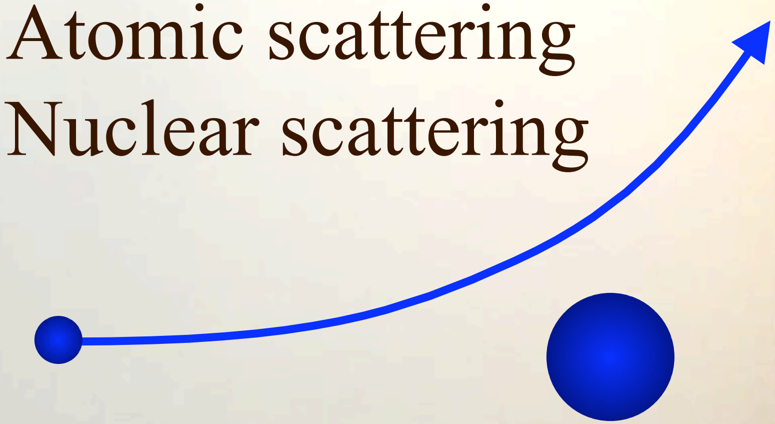
Electronic conduction
in mesoscopic systems

$$G = \frac{2e^2}{h} T(E_F)$$



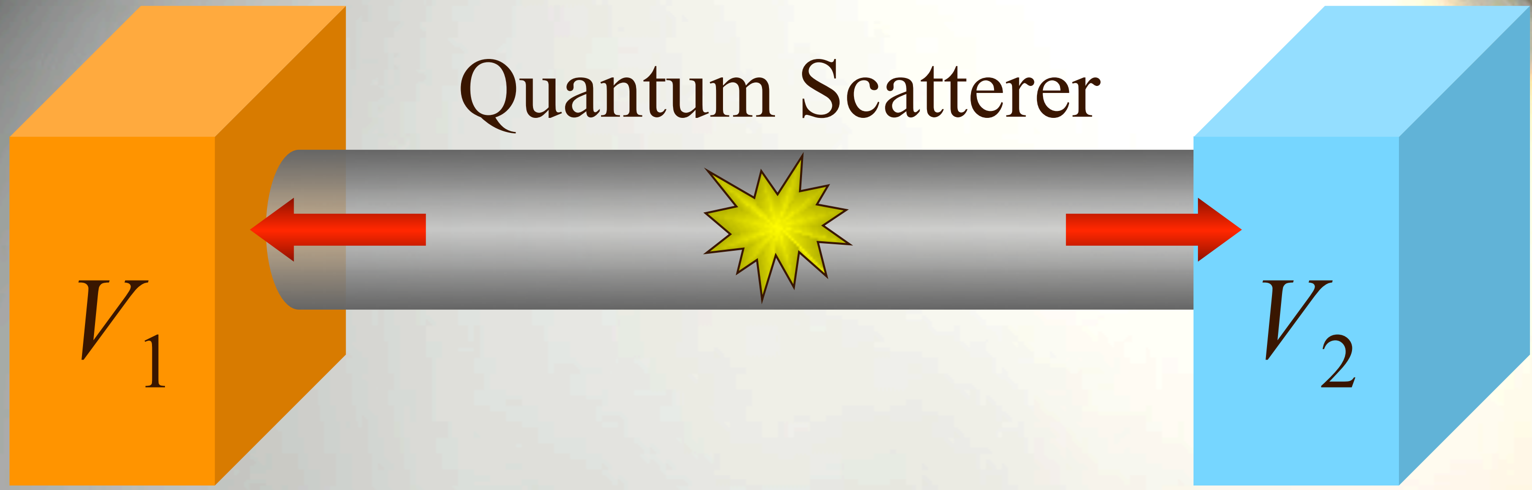
S. Katsumoto (ISSP, U. Tokyo)

Atomic scattering
Nuclear scattering



Once the particles go out of the central region,
they never come back.

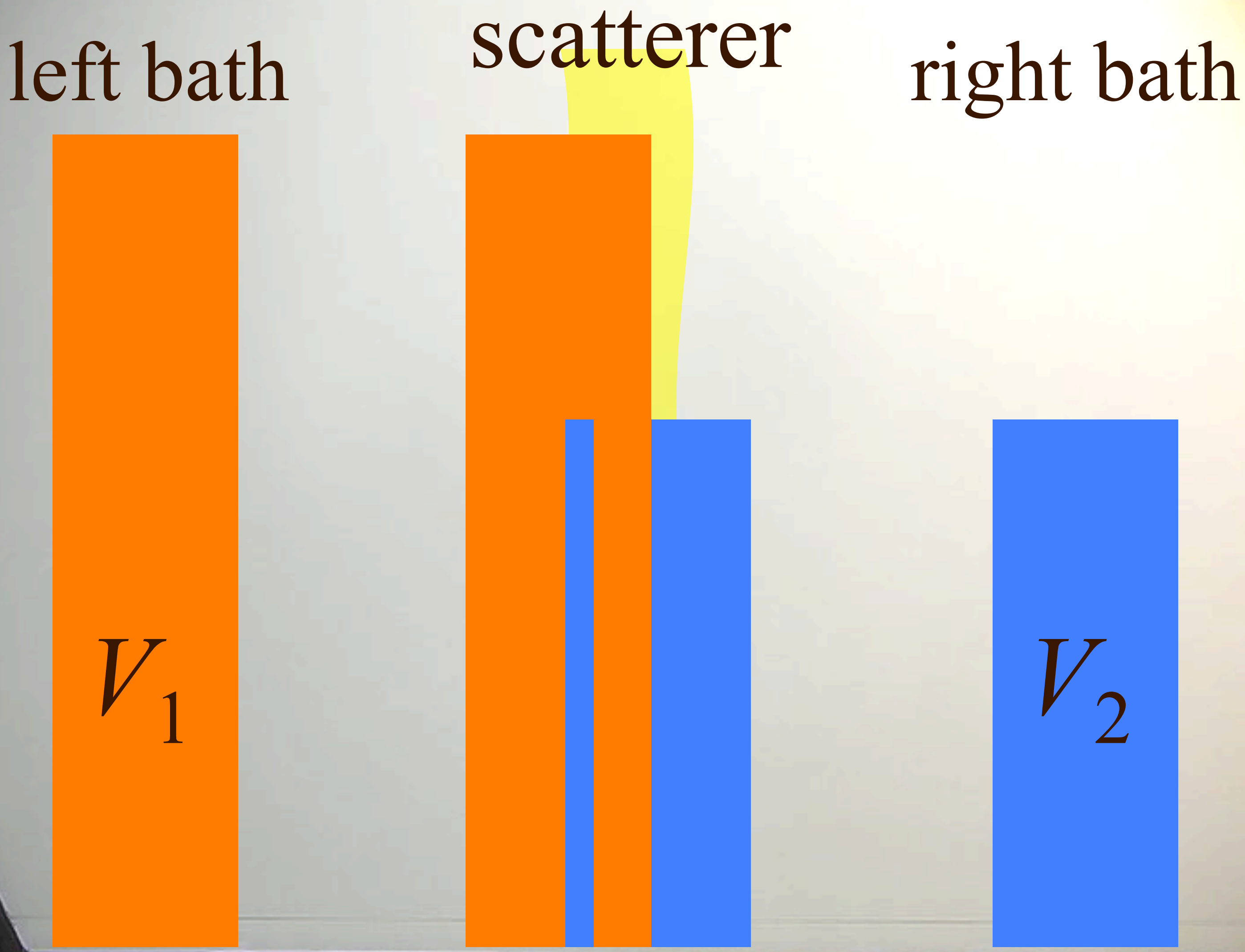
Landauer Formula



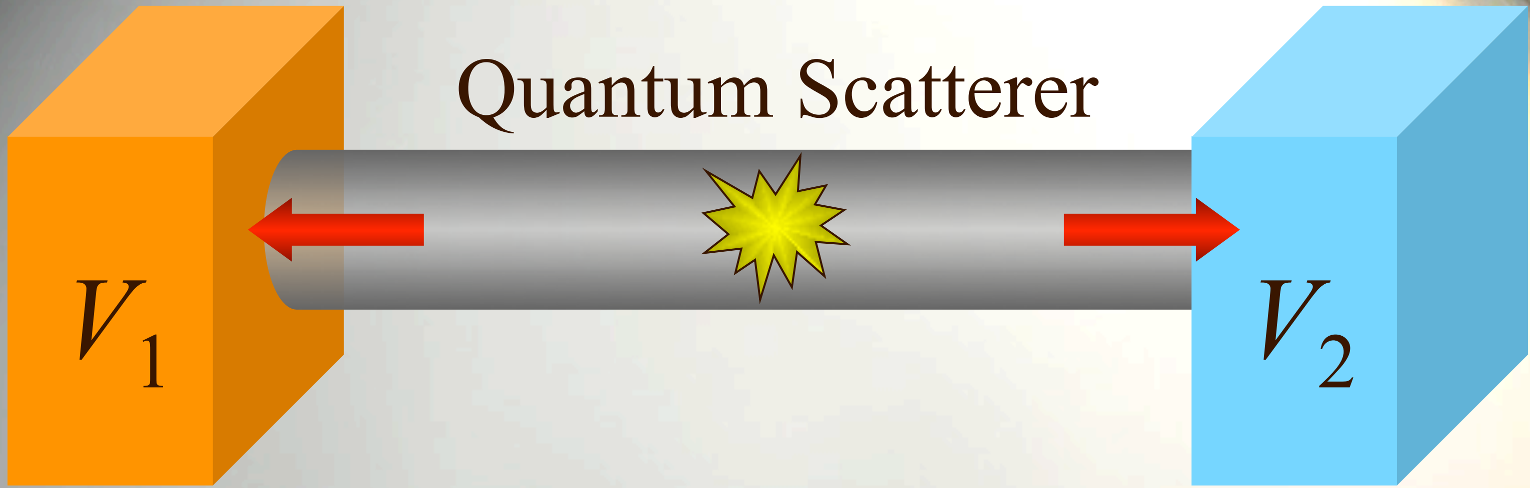
Conductance (Inverse Resistance): $G = \frac{2e^2}{h} T(E_F)$

Transmission probability: $T(E)$

Where's dissipation?



Landauer Formula

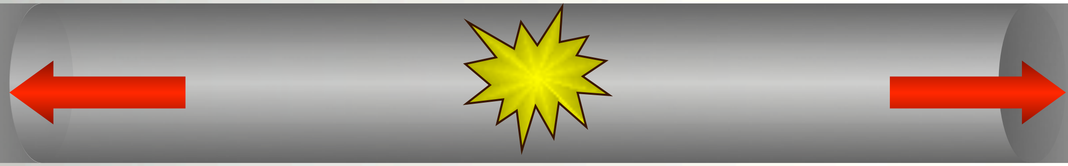


Conductance (Inverse Resistance): $G = \frac{2e^2}{h} T(E_{\mathrm{F}})$

Transmission probability: $T(E)$

Where's dissipation?

resonance



spontaneous breaking of
time-reversal symmetry

anti-resonance

Dissipation in quantum systems

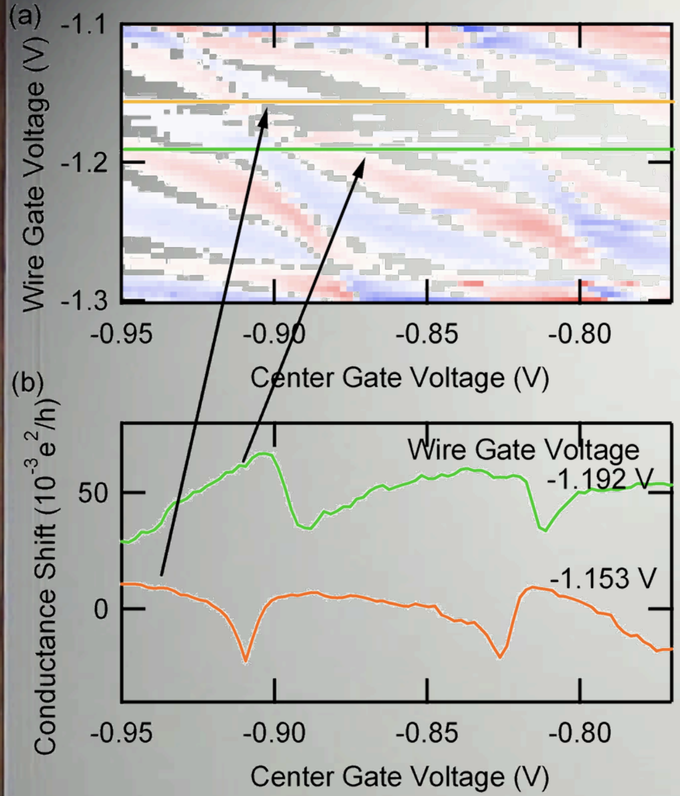
•<u>Finite System + Dissipation added by hand</u>

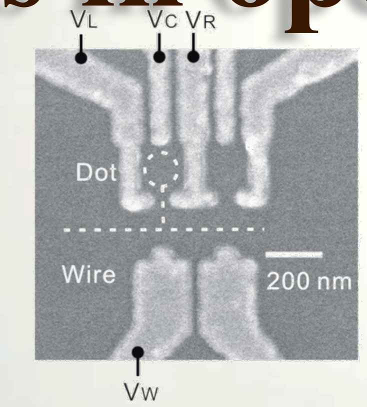
•<u>Finite System + Infinite Heat Bath</u>
Traces out the heat bath + Markovian approximation.
Time-reversal symmetry is broken somewhere.

•<u>Finite System + Finite System</u>
Everything microscopically.
"We do not live long enough to see the recurrence."

•<u>Finite System (quantum dot) + Infinite System (lead)</u>
Everything microscopically.
Spontaneous breaking of time-reversal symmetry.

Resonances in open systems





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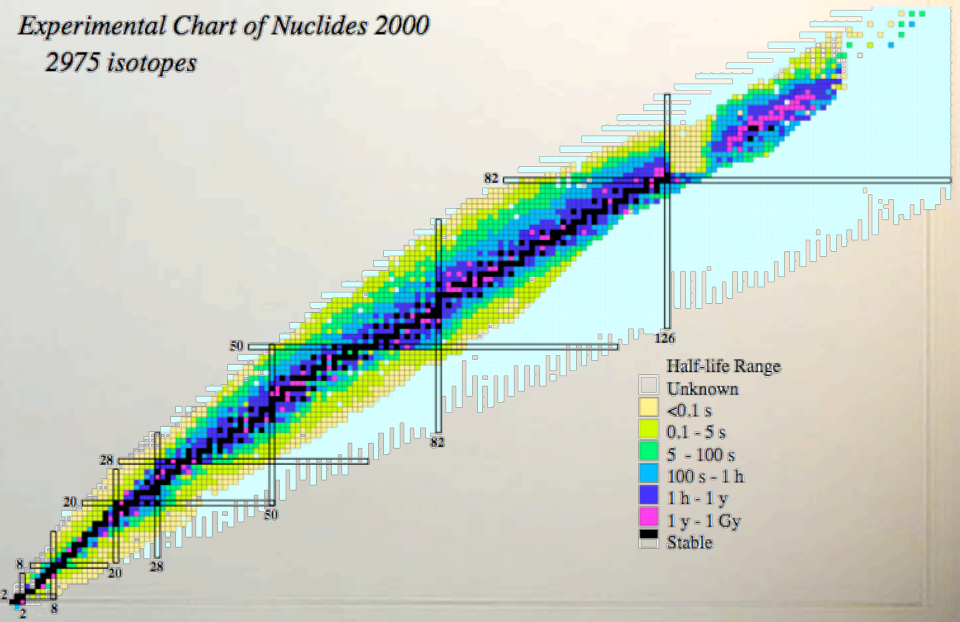
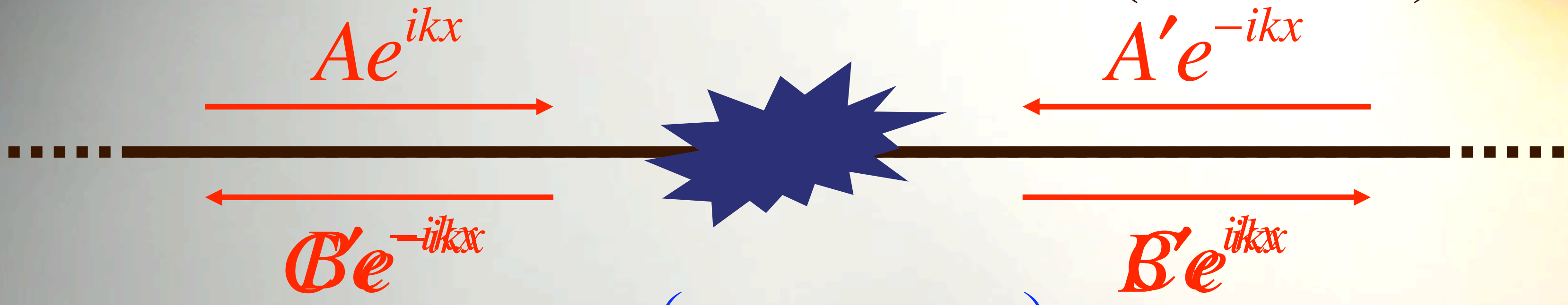
Search of Unstable Nuclei



Definition of resonance

Resonance: Pole of Trans. Prob. (S-Matrix)



$$S(E) \equiv \begin{pmatrix} r(E) & t'(E) \\ t(E) & r'(E) \end{pmatrix} \equiv \begin{pmatrix} \dfrac{B(E)}{A(E)} & \dfrac{C'(E)}{A'(E)} \\ \dfrac{C(E)}{A(E)} & \dfrac{B'(E)}{A'(E)} \end{pmatrix}$$

where $E = E(k)$

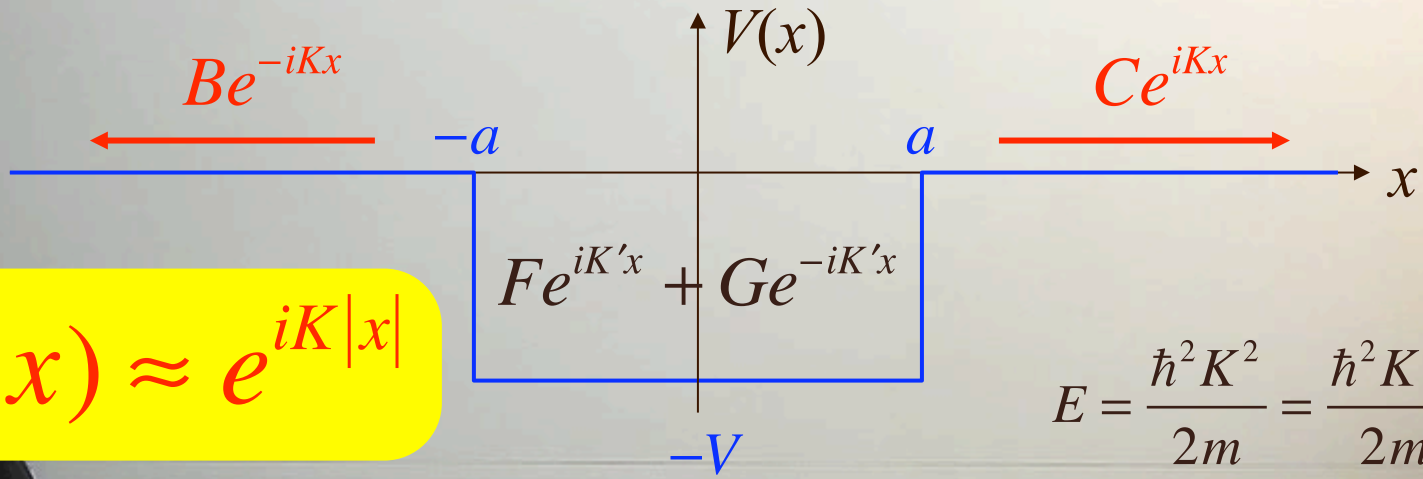
$$G(E) = \frac{2e^2}{h}\left|t(E)\right|^2 = \frac{2e^2}{h}\left|\frac{C(E)}{A(E)}\right|^2$$

Pole $\rightarrow$ $A(E) = 0$ or $A'(E) = 0$, where $E \in \mathbb{C}$

Definition of resonance

Siegert condition (1939)
Resonance: Eigenstate with outgoing waves only.

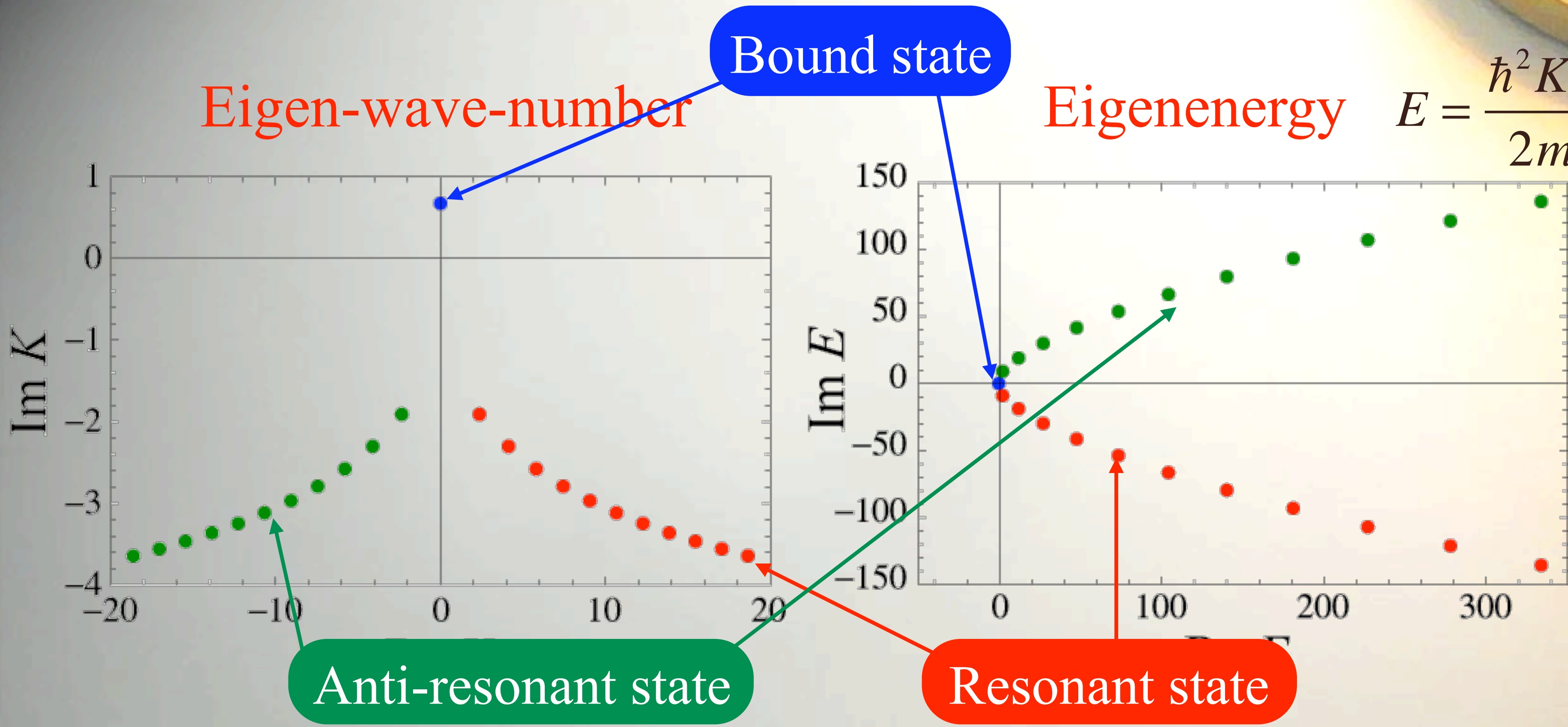
$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)\right)\psi(x)=E\psi(x)$$



$$\psi(x)\approx e^{iK|x|}$$

$$E=\frac{\hbar^2K^2}{2m}=\frac{\hbar^2K'^2}{2m}-V$$

Definition of resonance



$$E = \frac{\hbar^2 K^2}{2m}$$

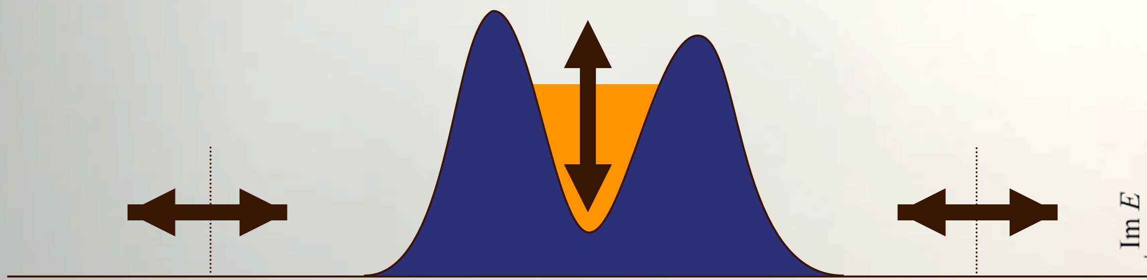
$$\psi(x) \approx e^{iK|x|}$$

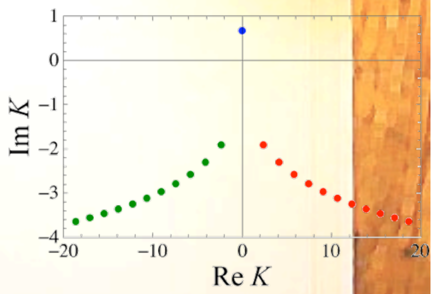
$$\begin{cases} \mathrm{Re}\, K_n \gtrless 0 \Leftrightarrow \mathrm{Im}\, E_n \lessgtr 0 \\ \mathrm{Im}\, K_n < 0 \end{cases}$$

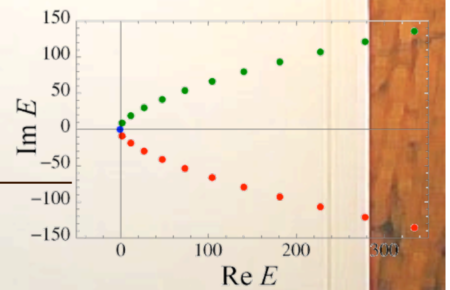
Resonant state as a stationary eigenstate

N. Hatano, K. Sasada, H. Nakamura and T. Petrosky, Prog. Theor. Phys. **119** (2008) 187

$$\langle x | \Psi_n(t) \rangle \approx e^{iK_n |x| - iE_n t}$$



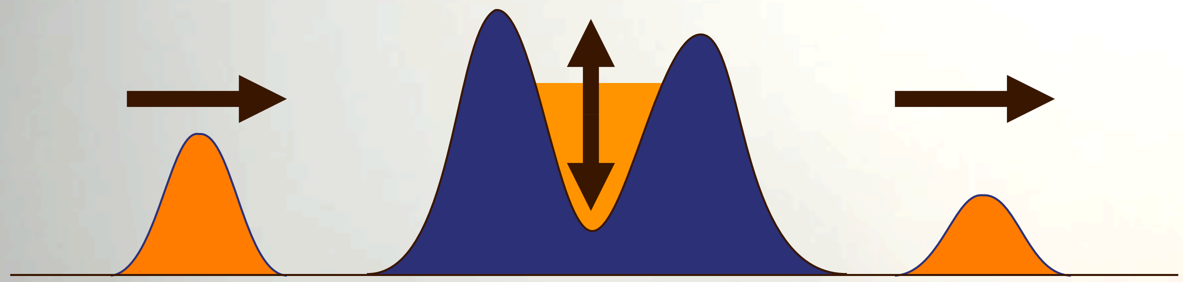




$$\mathrm{Im}\, E_n \lessgtr 0 \Leftrightarrow \mathrm{Re}\, K_n \lessgtr 0$$

"Resonant state" as an eigenstate

Dynamical View of Resonance

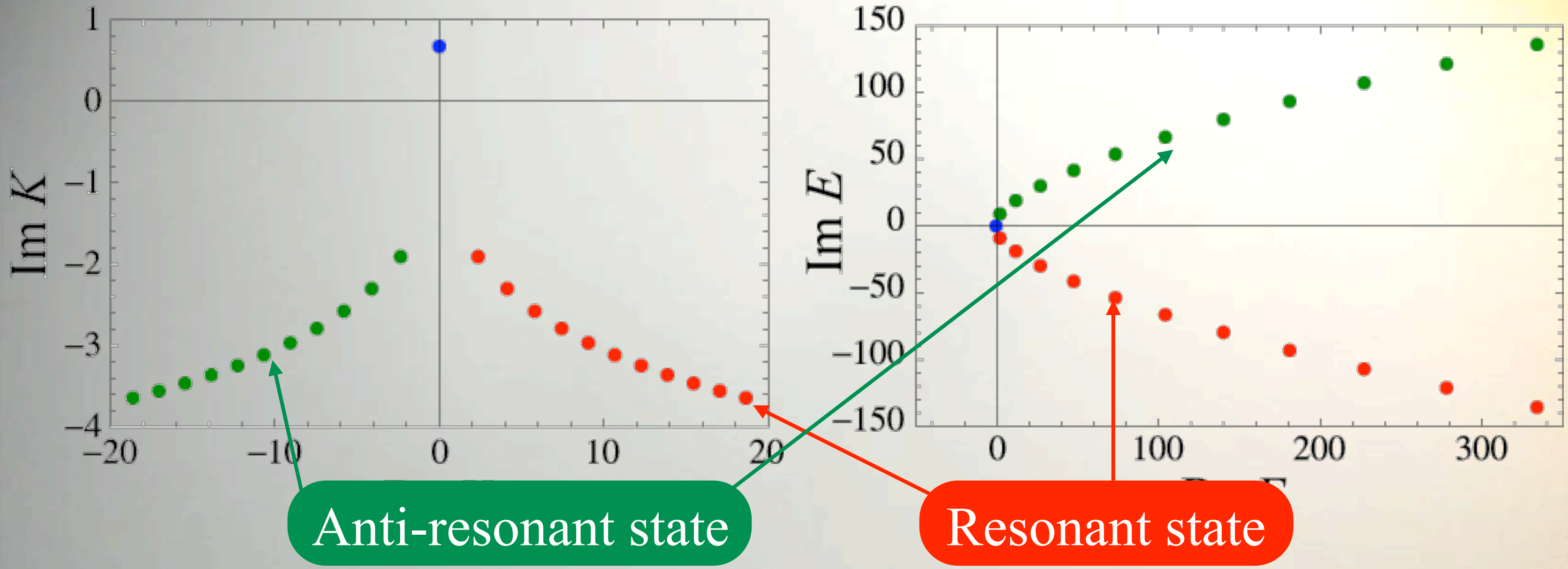


Break down the process to
two "eigenstates"
(a resonant state and an anti-resonant state)

Time-Reversal Symmetry

N. Hatano, K. Sasada, H. Nakamura and T. Petrosky, Prog. Theor. Phys. **119** (2008) 187

Eigen-wave-number Eigenenergy

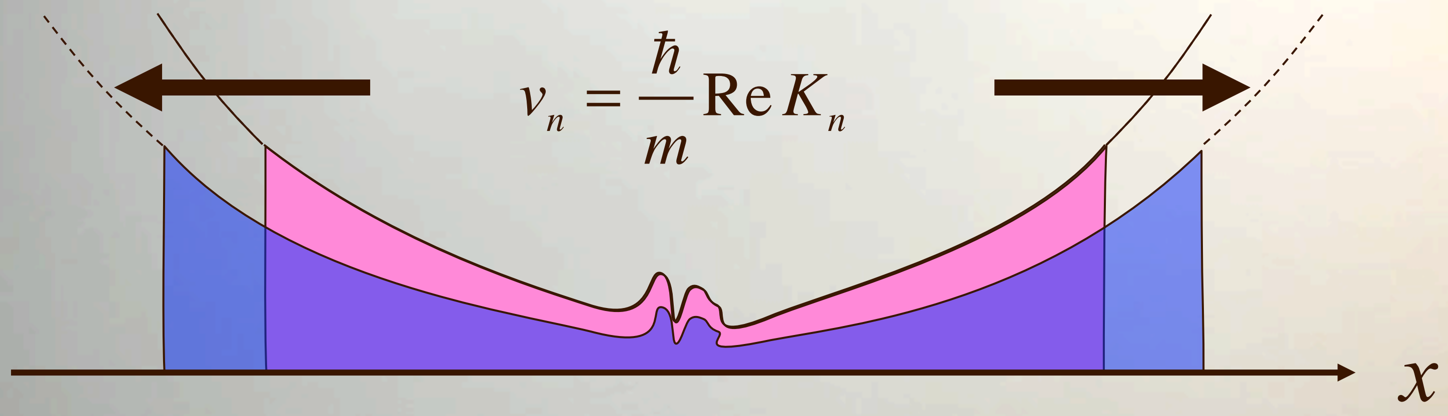


Each resonant or anti-resonant state breaks the time-reversal symmetry spontaneously!
(Dissipation into Infinities)

Particle-Number Conservation

N. Hatano, K. Sasada, H. Nakamura and T. Petrosky, Prog. Theor. Phys. **119** (2008) 187

$$\langle x|\Psi_n(t)\rangle \approx e^{iK_n|x|-iE_nt} \Rightarrow \left|\langle x|\Psi_n(t)\rangle\right|^2 \approx e^{2|\mathrm{Im}\,K_n||x|-2|\mathrm{Im}\,E_n|t}$$



Probabilistic Interpretation

Non-Hermiticity of open systems

- Seemingly Hermitian Hamiltonian can be **non-Hermitian outside the Hilbert space** (and it is in general), having complex eigenvalues.
- The eigenfunction of a resonant state is **indeed outside the Hilbert space**, being divergent in space.
- Nonetheless, the eigenfunction **supports the probabilistic interpretation**.
- And yet, it breaks the time-reversal symmetry spontaneously!
- The breaking is due to dissipation into infinities.

Approach to Equilibrium

$$\rho(t) \to \rho_{\text{eq}} \equiv \frac{1}{Z}\sum_n e^{-\beta E_n} |n\rangle\langle n|$$

Mixed State

Liouville-von Neumann equation

$$i\hbar \frac{d\rho}{dt} = [H, \rho] =: L\rho$$

$$L\rho_n = z_n \rho_n \qquad z_n \in \mathbb{C}$$

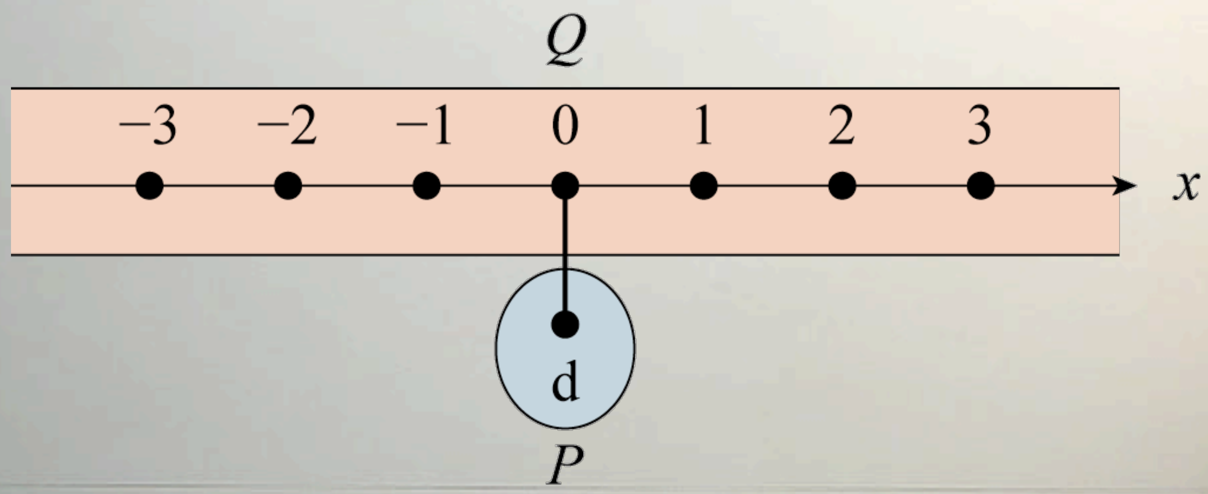
$$\rho(t) \simeq \rho_0 + e^{-iz_1 t}\rho_1 \simeq \rho_0 + e^{-(\operatorname{Im} z_1)t}\rho_1$$

Liouville von Neumann Eq.

R. Nakano, T. Mori, N. Hatano, T. Petrosky

$$i\hbar \frac{d\rho}{dt} = [H, \rho] =: L\rho$$

$$L\rho_n = z_n \rho_n \qquad z_n \in \mathbb{C}$$



Liouville von Neumann Eq.

T. Petrosky, I. Prigogine

$$i\hbar \frac{d\rho}{dt} = [H, \rho] =: L\rho$$

$$H|n\rangle = E_n |n\rangle$$
$$\langle m|n\rangle = \delta_{mn}$$
$$\sum_n |n\rangle\langle n| = 1$$

→

$$L|m,n\rangle\rangle = \lambda_{mn} |m,n\rangle\rangle$$
$$\langle\langle m,n|k,l\rangle\rangle = \delta_{mk}\delta_{ln}$$
$$\sum_{m,n} |m,n\rangle\rangle\langle\langle m,n| = 1$$

$$|m,n\rangle\rangle \coloneqq |m\rangle\langle n|$$
$$\lambda_{mn} \coloneqq E_m - E_n$$
$$\langle\langle A|B\rangle\rangle \coloneqq \mathrm{Tr}\, A^\dagger B$$

Liouville von Neumann Eq.

T. Petrosky, I. Prigogine

$$L|m,n\rangle\rangle = \lambda_{mn}|m,n\rangle\rangle$$

$$\langle\langle m,n|k,l\rangle\rangle = \delta_{mk}\delta_{ln}$$

$$\sum_{m,n}|m,n\rangle\rangle\langle\langle m,n| = 1$$

$$|m,n\rangle\rangle := |m\rangle\langle n|$$

$$\lambda_{mn} := E_m - E_n$$

$$\langle\langle A|B\rangle\rangle := \mathrm{Tr}\, A^\dagger B$$

$$\left[H,|m\rangle\langle n|\right] = H|m\rangle\langle n| - |m\rangle\langle n|H = \left(E_m - E_n\right)|m\rangle\langle n|$$

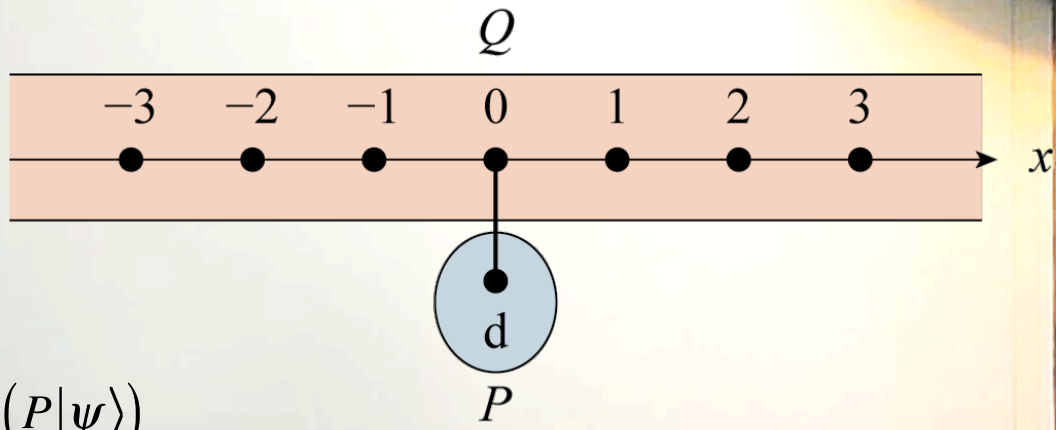
$$\mathrm{Tr}\left[\left(|m\rangle\langle n|\right)^\dagger |k\rangle\langle l|\right] = \langle m|k\rangle\langle l|n\rangle = \delta_{mk}\delta_{ln}$$

$$\sum_{m,n}\langle\langle A|m,n\rangle\rangle\langle\langle m,n|B\rangle\rangle = \sum_{m,n}\mathrm{Tr}\left[A^\dagger|m\rangle\langle n|\right]\mathrm{Tr}\left[\left(|m\rangle\langle n|\right)^\dagger B\right]$$

$$= \sum_{m,n}\langle n|A^\dagger|m\rangle\langle m|B|n\rangle = \mathrm{Tr}\left[A^\dagger B\right] = \langle\langle A|B\rangle\rangle$$

Feshbach Formalism

I. Rotter et al.



$$H|\psi\rangle = E|\psi\rangle$$

$$P + Q = 1$$

$$H(P+Q)|\psi\rangle = E|\psi\rangle$$

$$\begin{cases} (PHP + PHQ)|\psi\rangle = EP|\psi\rangle \\ (QHP + QHQ)|\psi\rangle = EQ|\psi\rangle \end{cases}$$

$$\begin{cases} PHP\left(P|\psi\rangle\right) + PHQ\left(Q|\psi\rangle\right) = E\left(P|\psi\rangle\right) \\ QHP\left(P|\psi\rangle\right) + QHQ\left(Q|\psi\rangle\right) = E\left(Q|\psi\rangle\right) \end{cases}$$

$$Q|\psi\rangle = \frac{1}{E - QHQ} QHP\left(P|\psi\rangle\right)$$

$$\left(PHP + PHQ \frac{1}{E - QHQ} QHP\right)\left(P|\psi\rangle\right) = E\left(P|\psi\rangle\right)$$

$$H_{\mathrm{eff}}(E)\left(P|\psi\rangle\right) = E\left(P|\psi\rangle\right)$$

$$\frac{1}{E - QHQ}$$

Retarded ⇔ Resonance

Advanced ⇔ Anti-Resonance

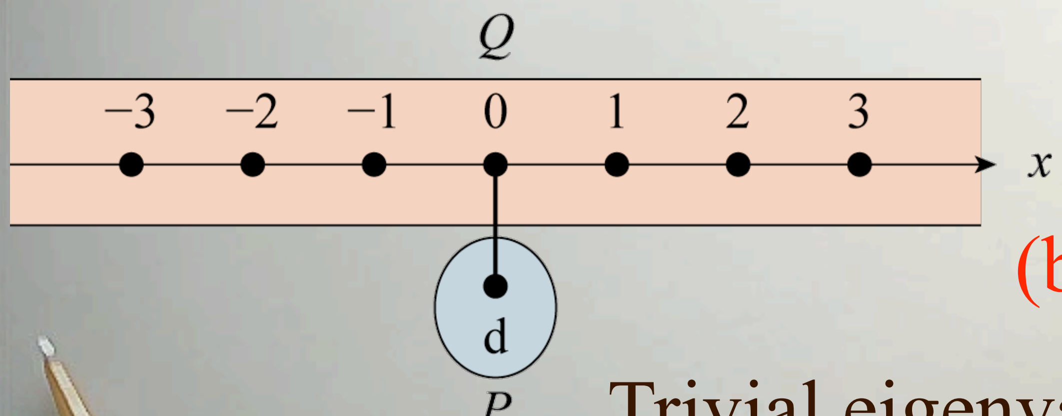
Feshbach Formalism for the Liouvillian

R. Nakano, T. Mori, N. Hatano, T. Petrosky

$$L|\rho\rangle\rangle = z|\rho\rangle\rangle$$

$$\left(P_L L P_L + P_L L Q_L \frac{1}{z - Q_L L Q_L} Q_L L P_L \right) \left(P_L |\rho\rangle\rangle \right) = z \left(P_L |\rho\rangle\rangle \right)$$

$$L_{\mathrm{eff}}(z) \left(P_L |\rho\rangle\rangle \right) = z \left(P_L |\rho\rangle\rangle \right)$$



Interacting two-particle problem (bra and ket particles)

Trivial eigenvalues ($\lambda_{mn} = E_m - E_n$)

Nontrivial eigenvalues

Summary

- Dissipation and resonance
- Definition and physics of resonant states
- Time-reversal symmetry breaking
- Particle-number conservation
- Resonances of the Liouvillian