PT symmetry (breaking) in non-uniform lattice models

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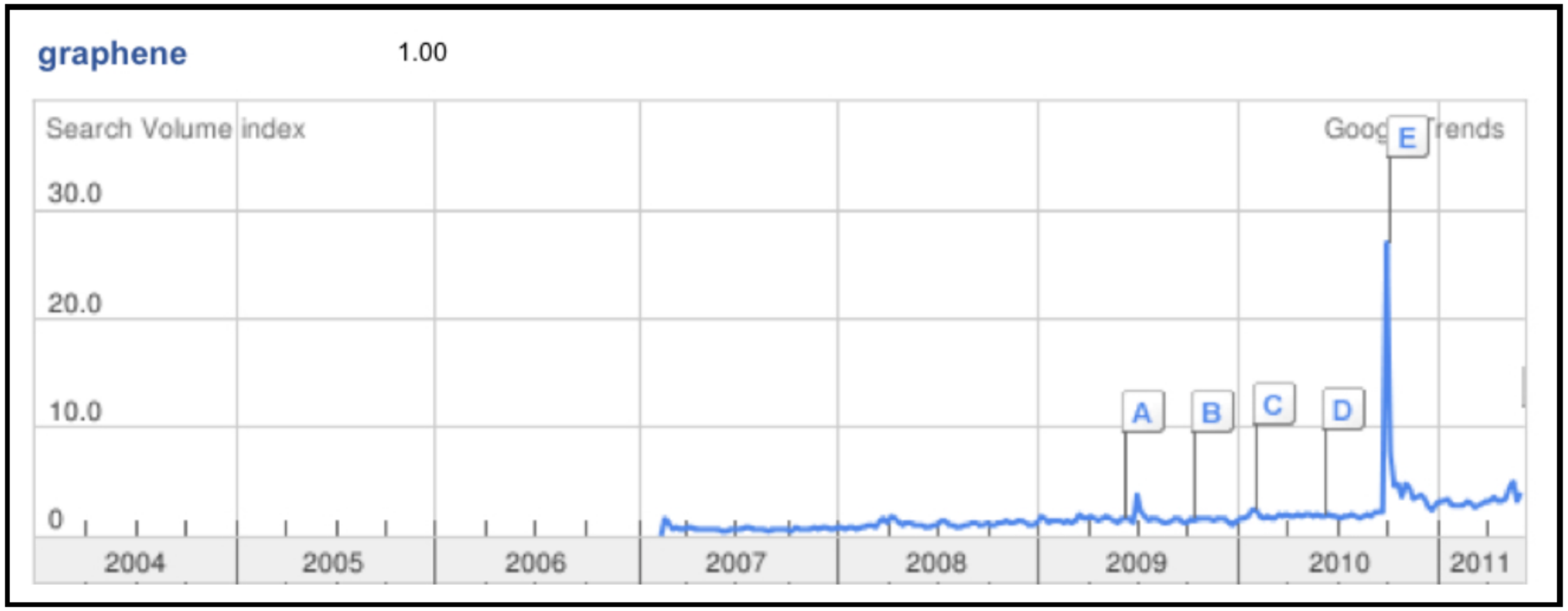
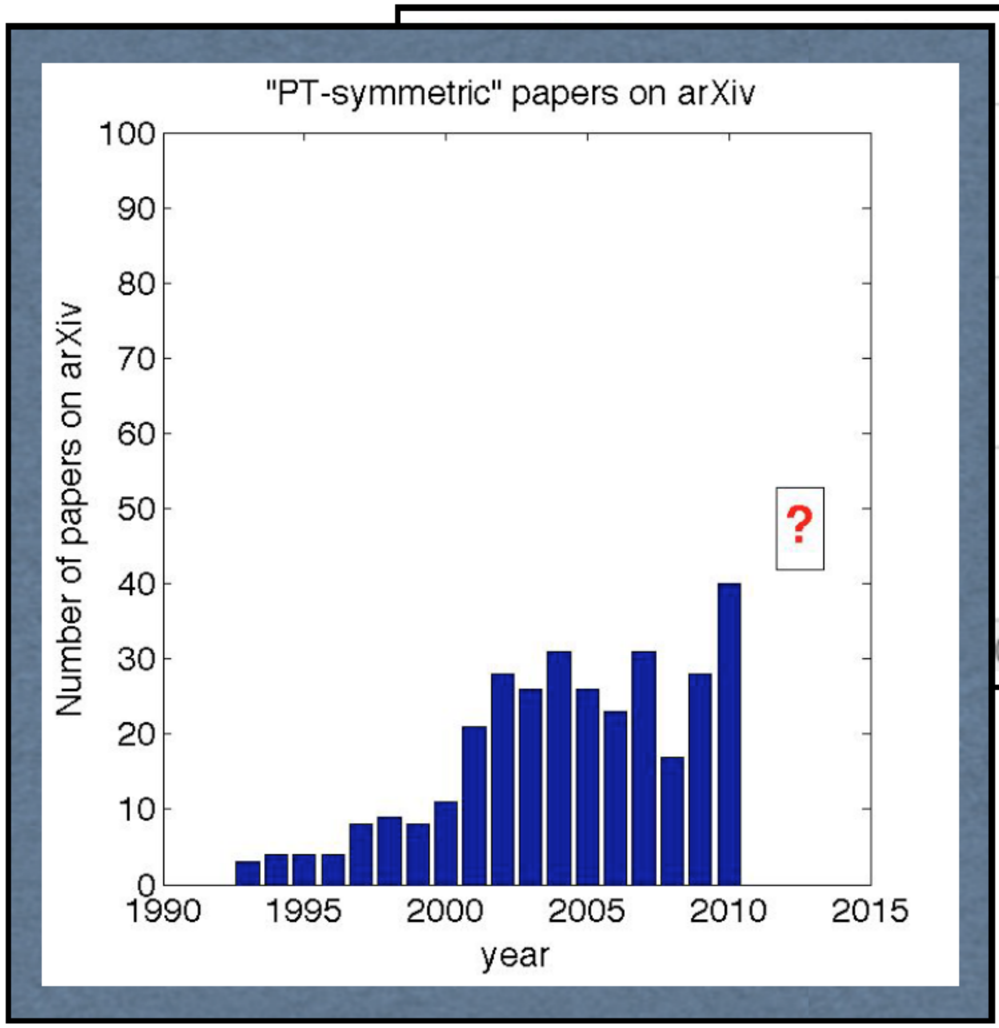
MPI, Dresden 2011

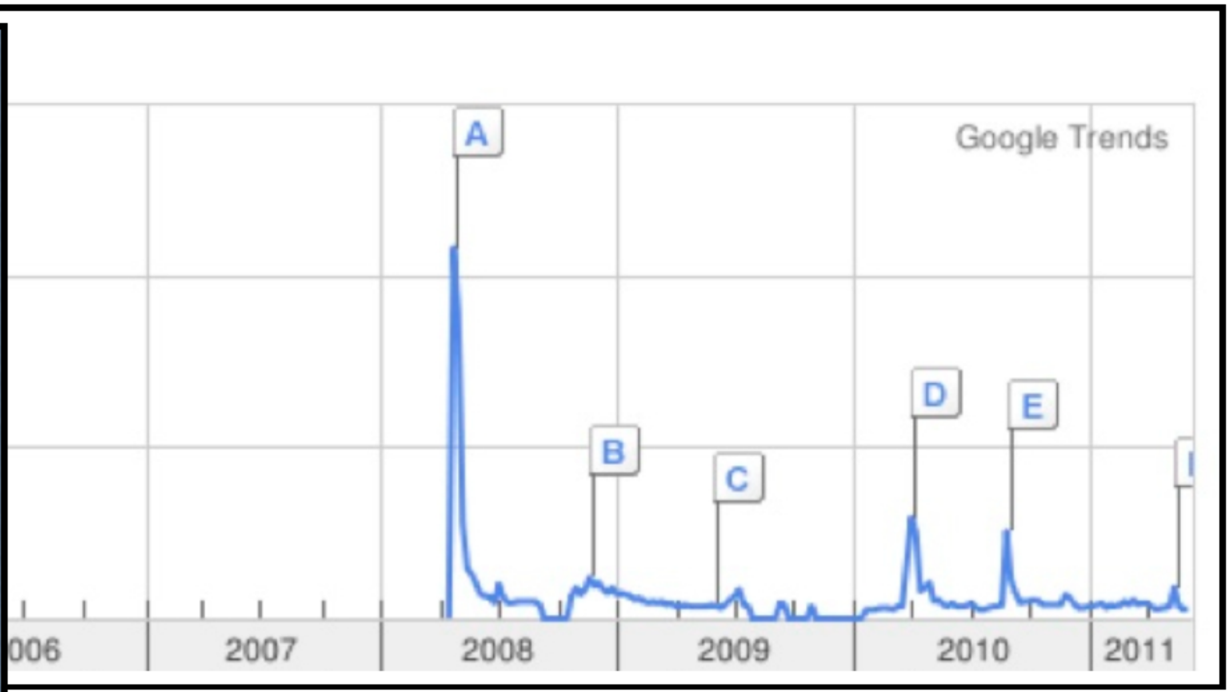
Theory vs. Experiments: popularity!



Graphene
Theory: 1940s
Experiment: 2005



PT
Theory
1990s
Experiment
2010



Memristor: new circuit element
Theory: 1970s
Experiment: 2008

Outline

Introduction: Continuum vs. lattice models

Lattice: non-Hermitian hopping, imaginary potentials

Results: energy spectra, PT-symmetric phase diagram

Results: degrees and signatures of broken PT symmetry

Conclusions

Continuum vs. Lattice Models

Continuum models:
- Hermitian kinetic energy term
- Non-Hermitian, PT-symmetric potential term

Carl Bender,...

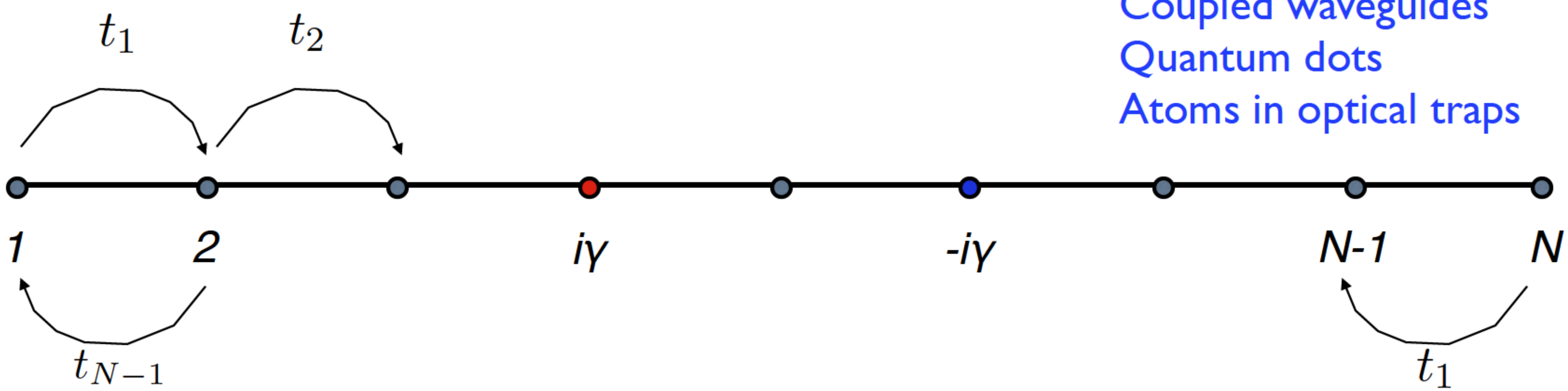
Finite, tight-binding lattice:
- Natural UV and IR cutoffs
- Particle-hole symmetric energy band
- Bound states in repulsive potentials

Miloslav Znojil,...

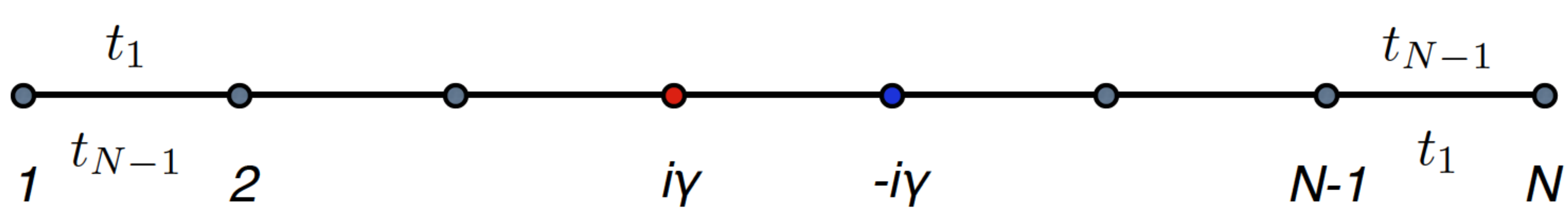
What are PT-symmetric lattice models?

PT-symmetric Lattice Model

Coupled waveguides
Quantum dots
Atoms in optical traps



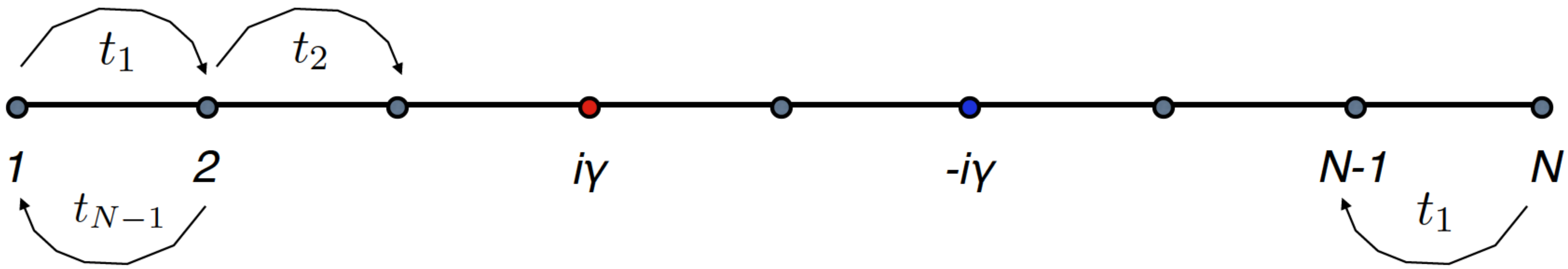
$$\hat{H} = -\sum_{i=1}^{N}\left(t_i a^\dagger_{i+1} a_i + t_{N-i} a^\dagger_i a_{i+1}\right) + i\gamma\left(a^\dagger_{m_0} a_{m_0} - a^\dagger_{N+1-m_0} a_{N+1-m_0}\right)$$



Thermodynamic limit: $N \to \infty$
Continuum limit: $N \to \infty$, $a \to 0$, $Na \to L$

} *These two are different*

PT-symmetric Lattice Model



$$\hat{H} = -\sum_{i=1}^{N}\left(t_i a^\dagger_{i+1} a_i + t_{N-i} a^\dagger_i a_{i+1}\right) + i\gamma\left(a^\dagger_{m_0} a_{m_0} - a^\dagger_{N+1-m_0} a_{N+1-m_0}\right)$$

Non-uniform, non-Hermitian hopping

Non-Hermitian, PT-symmetric gain and loss impurities

When does a clean lattice have real spectrum?

What are the energy levels of the clean lattice?

How robust is the PT phase in the presence of impurities?

Hatano and Nelson. PRL 1996; Jin and Song, PRA 2010
Kottos *et al.* PRL 2009; Makris *et al.* Nat. Phys. 2010

Phys. Rev. A **83**, 050101(R) (2011)

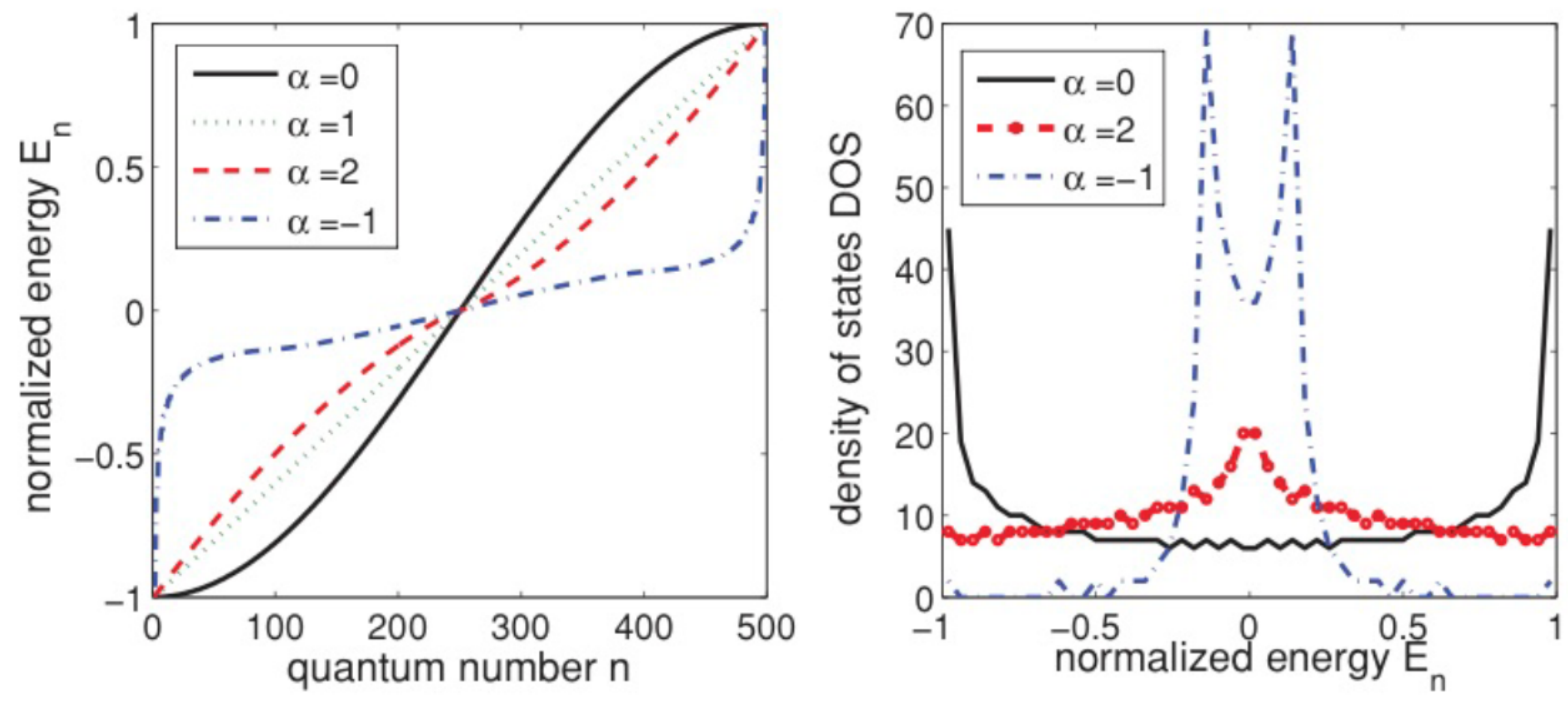
Robust PT Symmetric Lattice

Real spectrum when *hopping elements* have the same sign.

Non-Hermitian Hamiltonian is *similar* to a Hermitian one.

Tunable energy spectrum and density of states.

$$t_\alpha(k) = t_0[k(N-k)]^{\alpha/2}$$



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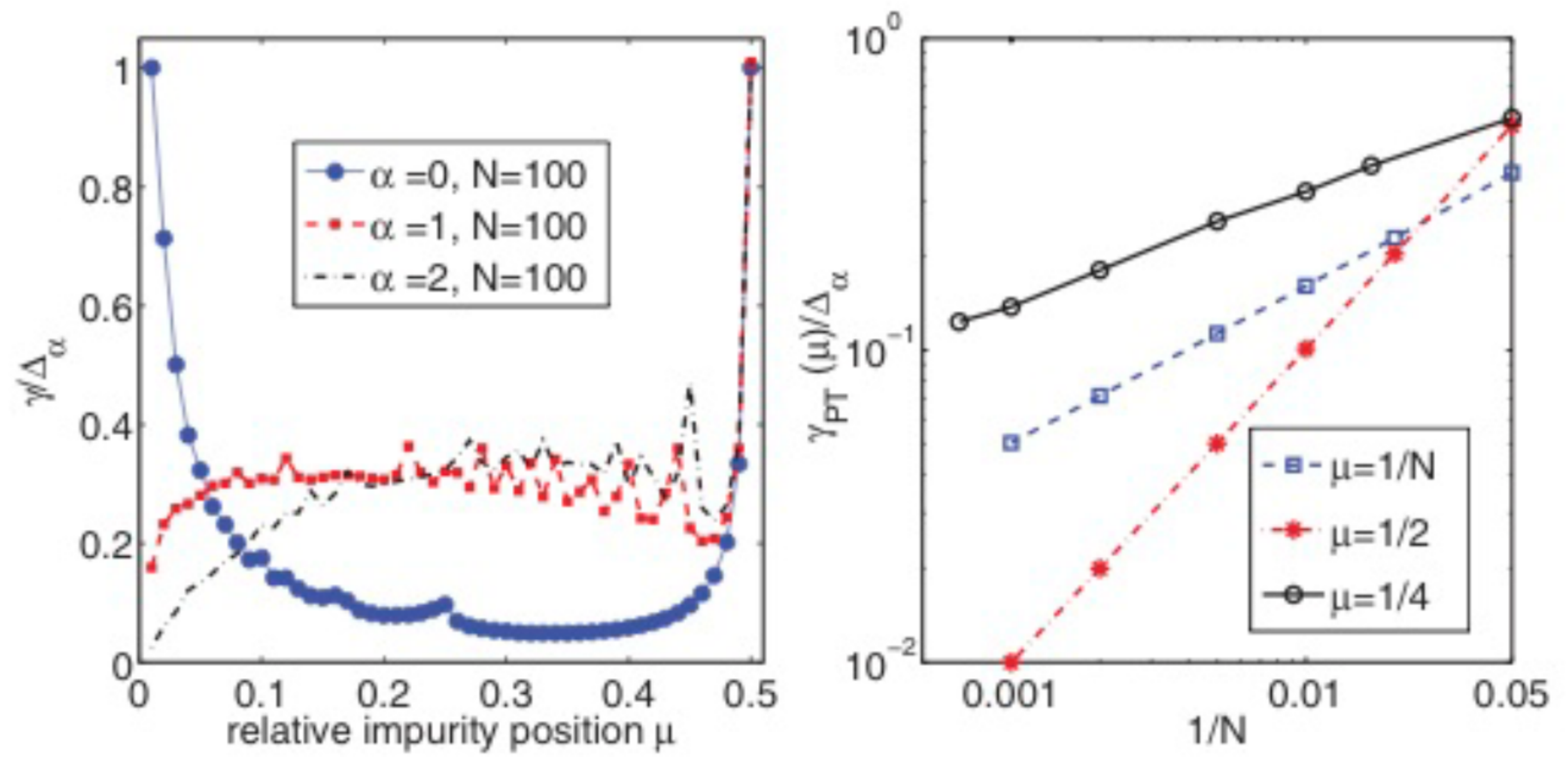
Effects of PT Symmetric Impurities

α=0: PT phase fragile except for closest or farthest impurities



PT phase is relatively strong when α>0.
PT phase is robust when the impurities are closest.

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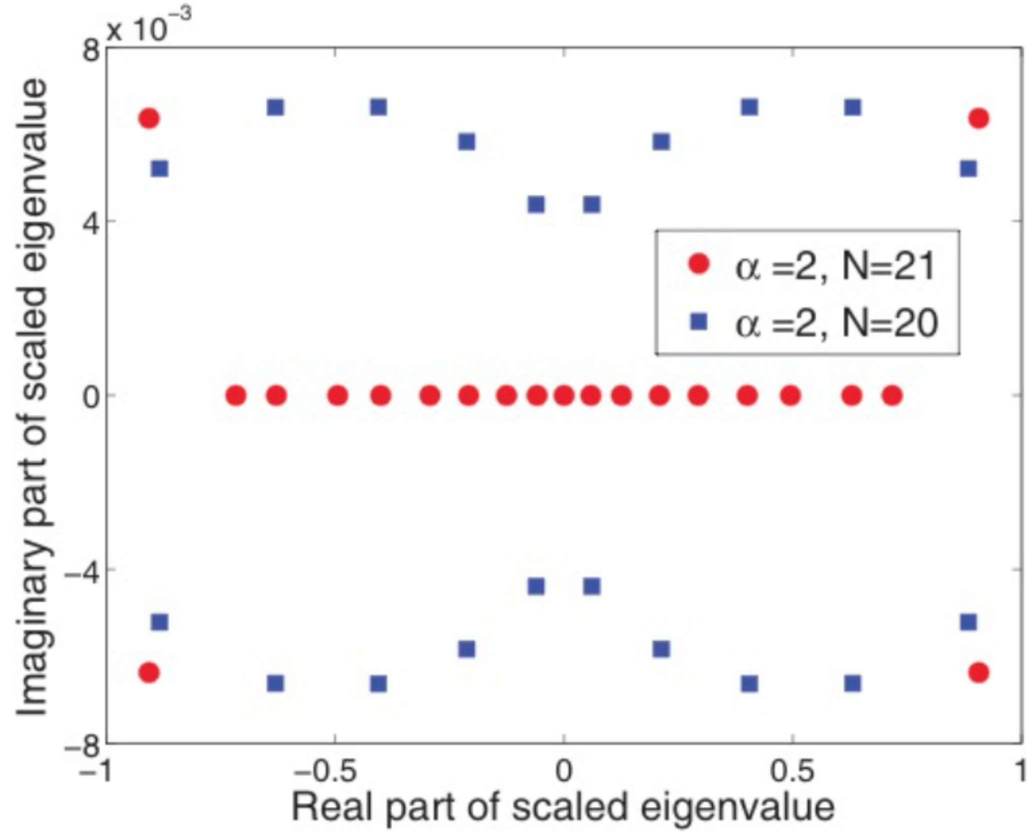
Degrees of PT-symmetry Breaking

For closest impurities, how many eigenvalues become complex?

Even N :
All *N* eigenvalues become complex.

Odd N :
Four eigenvalues at band edges become complex.



What are the signatures of this even-odd effect?

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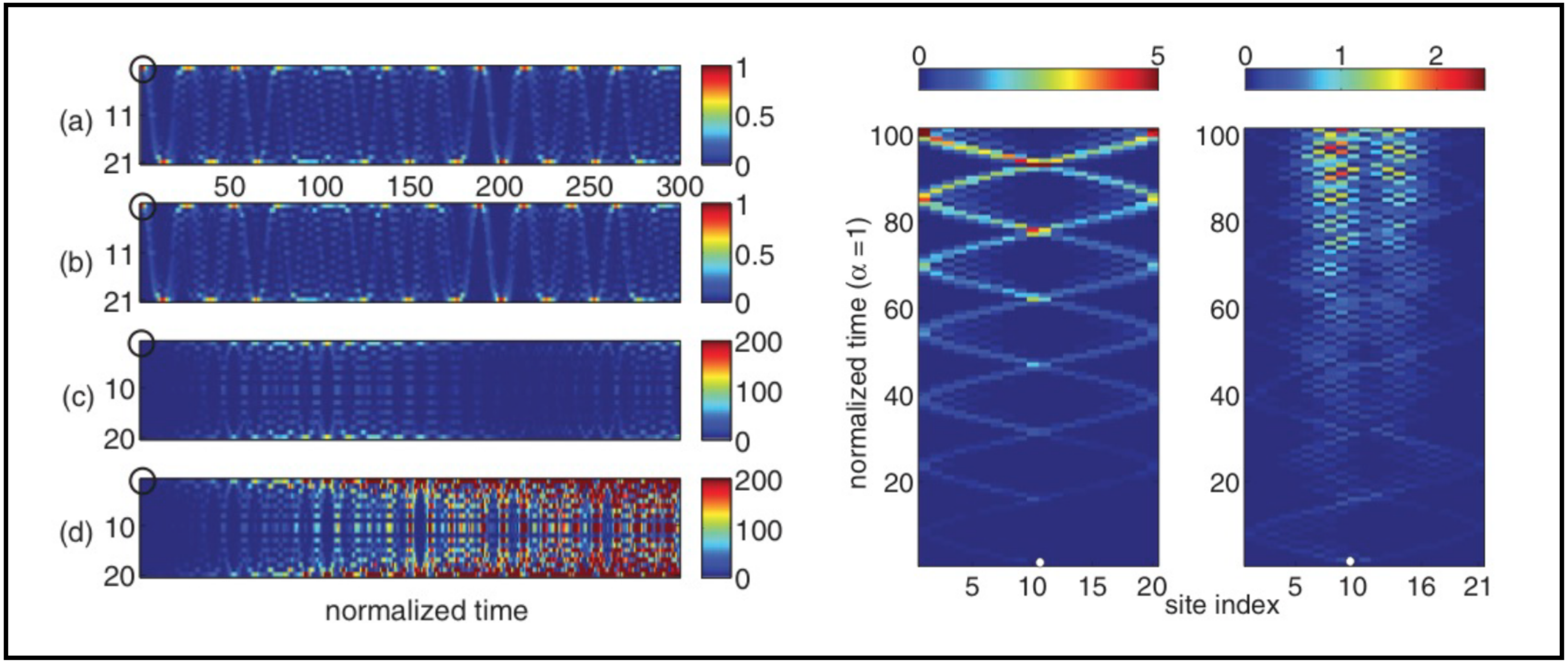
Signatures in Optical Waveguides

Predictions for light intensity in an array of optical waveguides



Before PT breaking : time evolution similar for even/odd N

After PT breaking : time evolution is qualitatively different

Conclusions

- PT lattices have a robust phase when the impurities are closest.
- They can be realized in coupled optical waveguides with tunable hopping, and PT-symmetric loss and gain impurities.
- We predict that PT-symmetry breaking is *qualitatively different* in lattices with even or odd number of sites *N*.
- We predict that even-odd effect manifests in light-intensity time evolution.
- Tunable waveguides: a variable mass particle in a PT-symmetric potential

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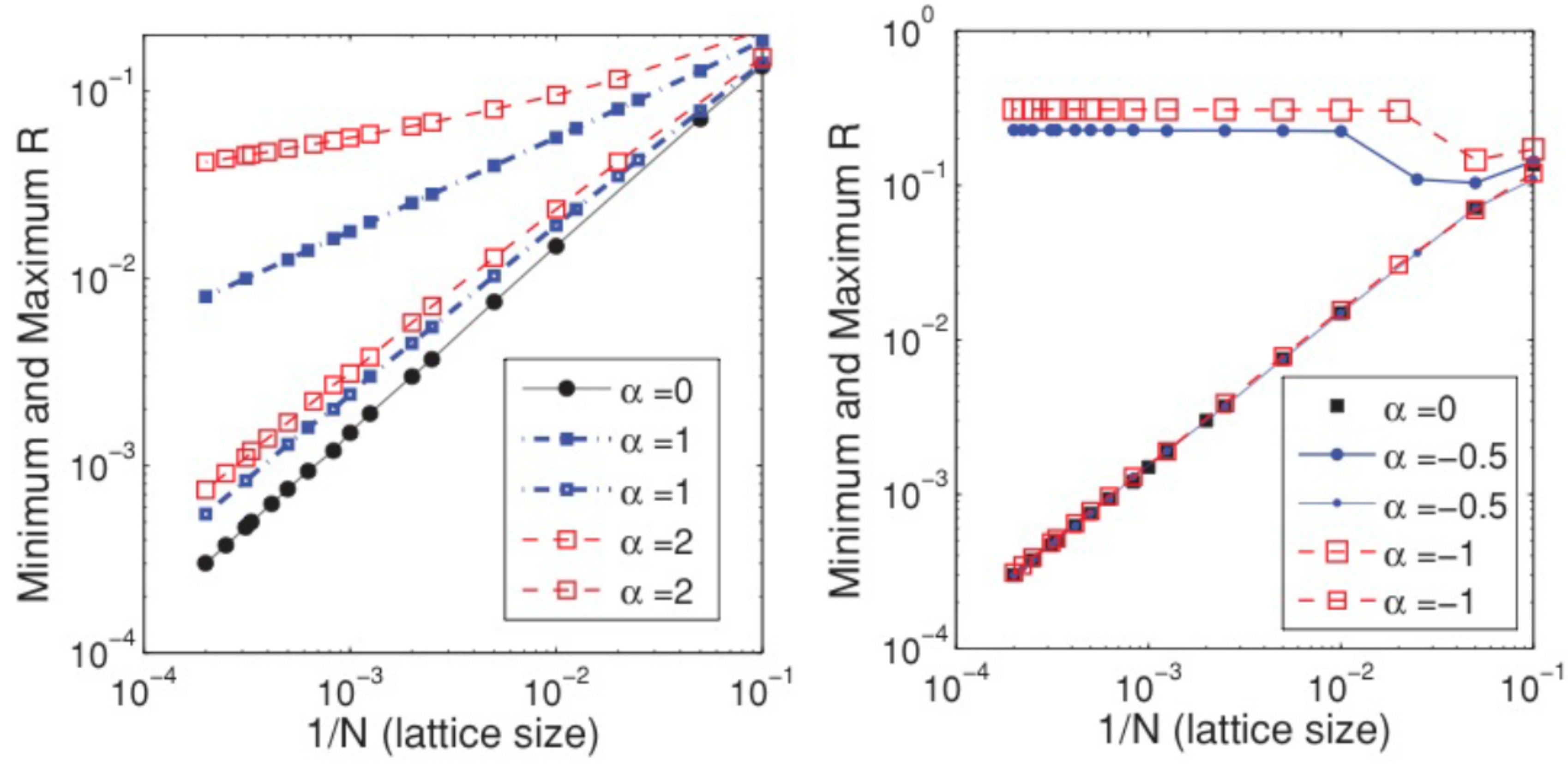
Robust PT Symmetric Lattice

$$t_\alpha(k) = t_0[k(N-k)]^{\alpha/2}$$



All states are extended when $\alpha \geq 0$.

States at the band-edge are localized when $\alpha < 0$.