Quantum multiple scattering

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Nice, France

Quantum Physics with Non-Hermitian Operators
DRESDEN June 15 - 25, 2011

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<u>Collaborations</u>
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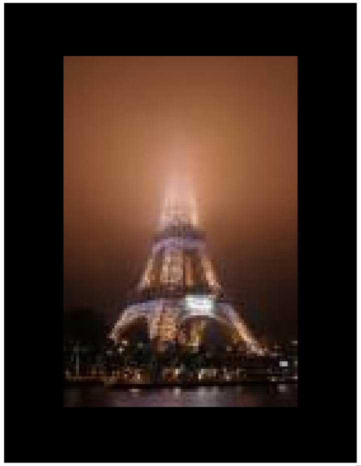
€ : CNRS, UNSA, PACA, CG06, ANR, INTERCAN

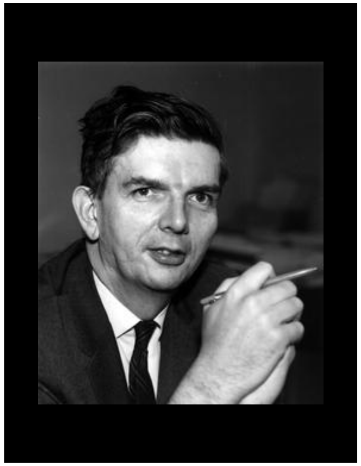
Storage of Light in Atomic samples : Disorder vs cooperative effects



Multiple Scattering

Anderson Localization

Dicke States



Wave propagation in disordered media :

1958 : on average : interferences washed out : random walk / diffusion

Light : radiation trapping in stars

Electrons : metal (Drude model)

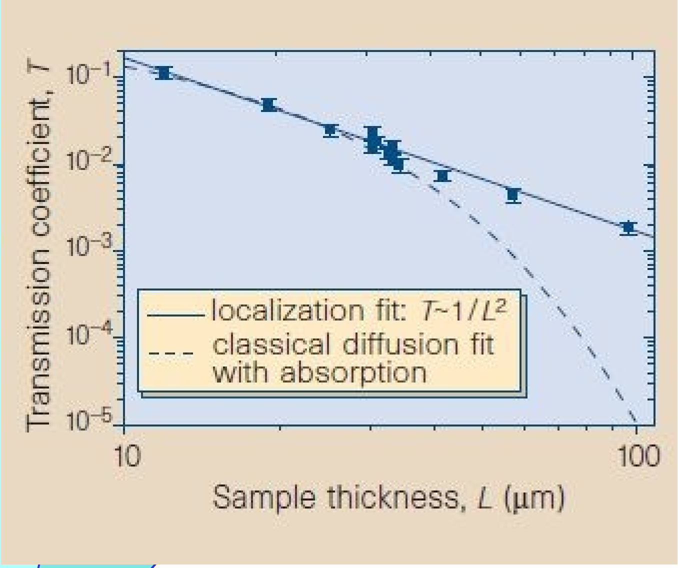
1958 : P.W. Anderson : vanishing diffusion for strong disorder !

- **Solid State Physics :**
  Metal-Insulator Transitions for electrons
- **Light Scattering :**
  Photonic Crystals, Colloidal Systems, White Paint, **Atoms**
- **Matter Waves :**
  BEC in Disordered Potential, Kicked Rotator
- **Accoustics :**
  Metal Rods, Aluminium Beads

Anderson Localization in 3D :

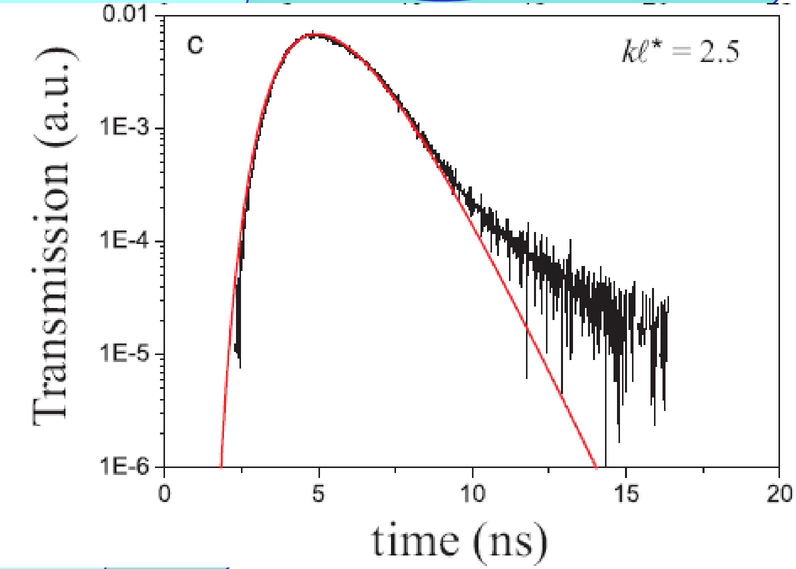
phase transition ⇒ strong scattering required

Semi-conductor powder



D.Wiersma et al., Nature 1997

White Paint



C.Aegerter et al., EPL 2006

Why light localization with atoms ?

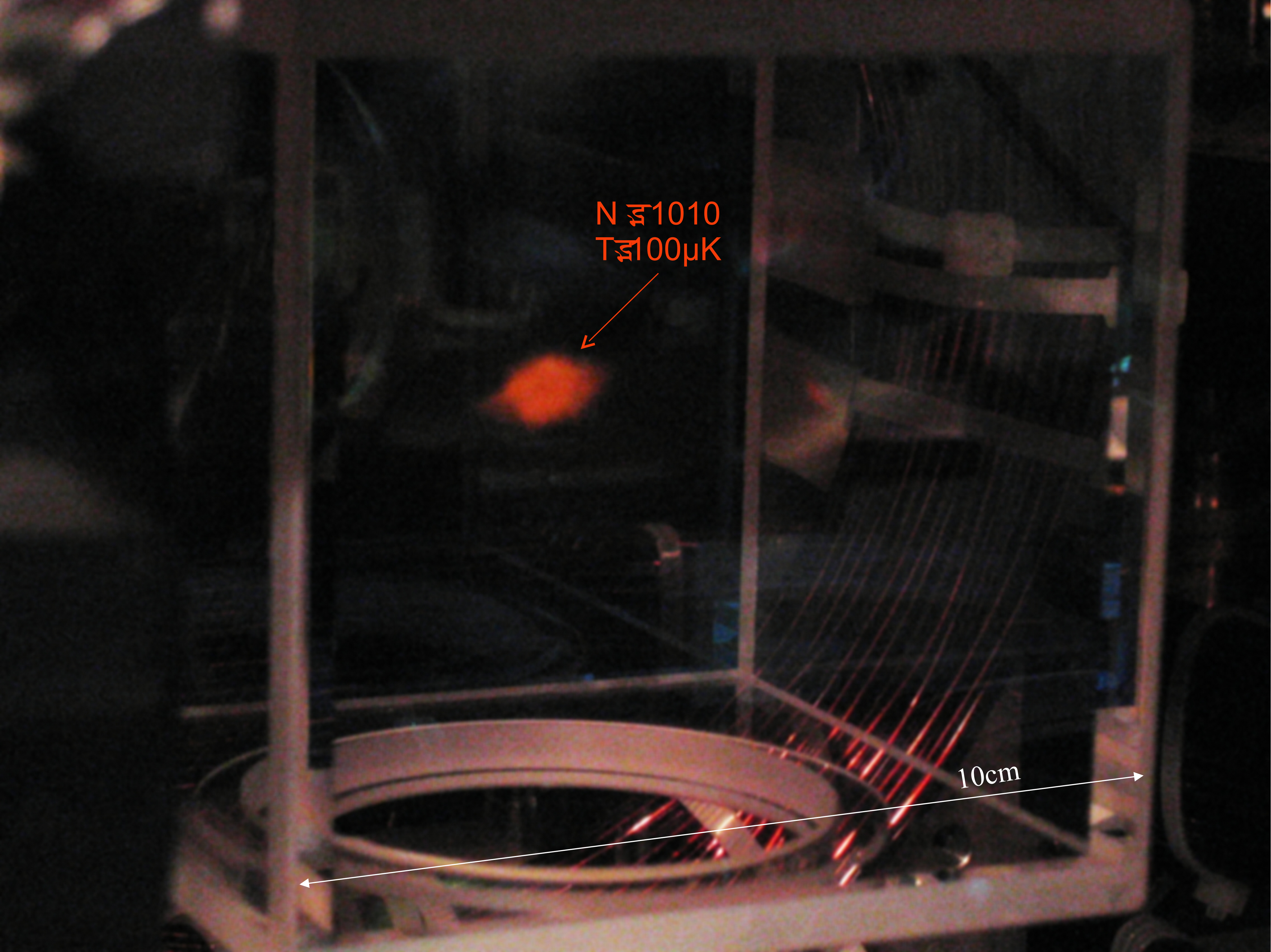
- role of quantum fluctuations (quantum optics)
- role of entanglement of scatterers (non local potential?)
- ab initio calculations

Open system : resonant scattering :

- quality factor ~ 108
- 'monodisperse' sample : cold atoms

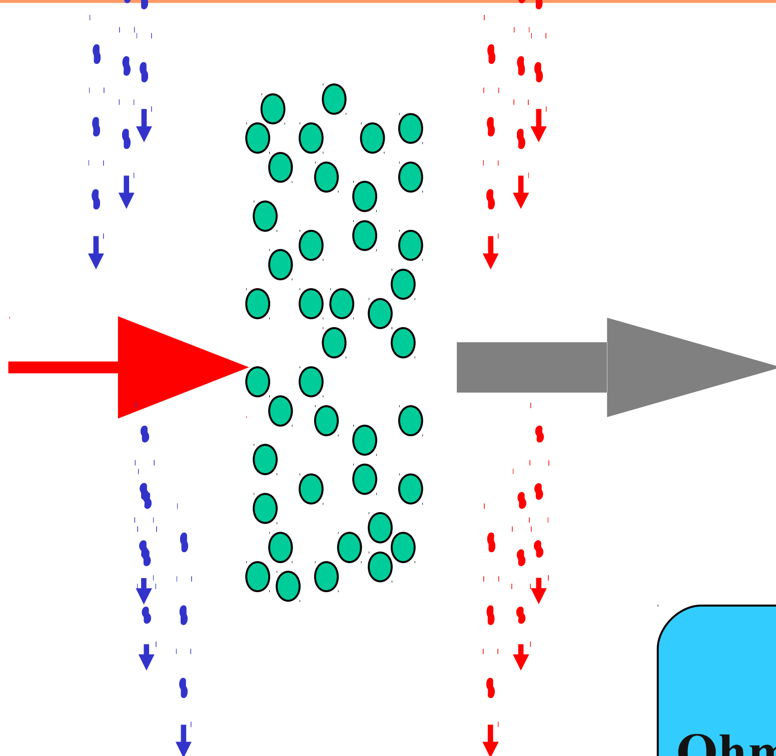
⇒'delay time' at resonance : τd ~ 50ns

⇒on resonance nat ड1014 at/cm3 **(Ioffe-Regel)**



N 1010
T 100µK
10cm

Scattering Experiments



$$\ell scat = \frac{1}{n\sigma}$$

Single scattering

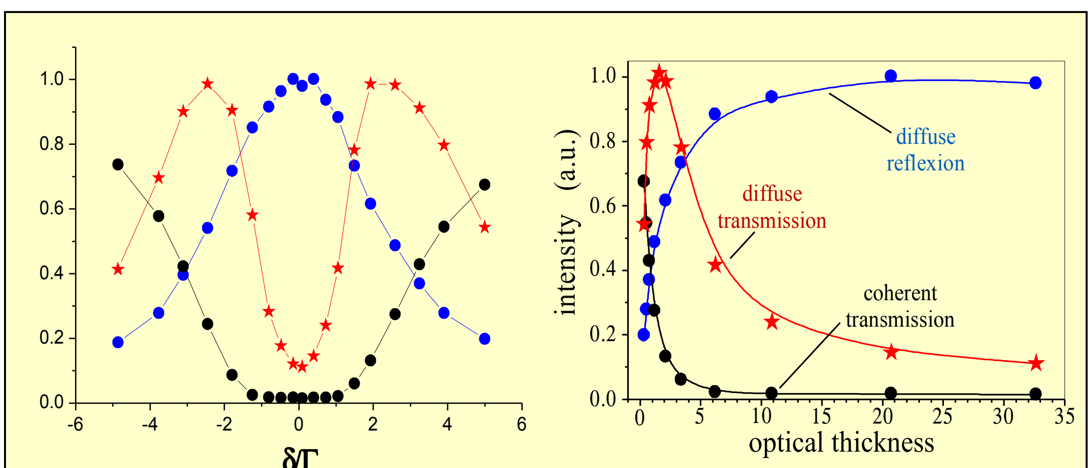
Beer-Lambert :

Tcoh =exp(-L/ ℓscat)

Multiple scattering

Ohm's law : Tdiff = ℓscat / L

Localized : TD∝exp(-L/Ll)		Critical : T∝1/L2



MOT parameters :

number of atoms : $N \approx$ few 10^{10}

MOT diameter : $\varnothing_{FWHM} \approx 1$ cm

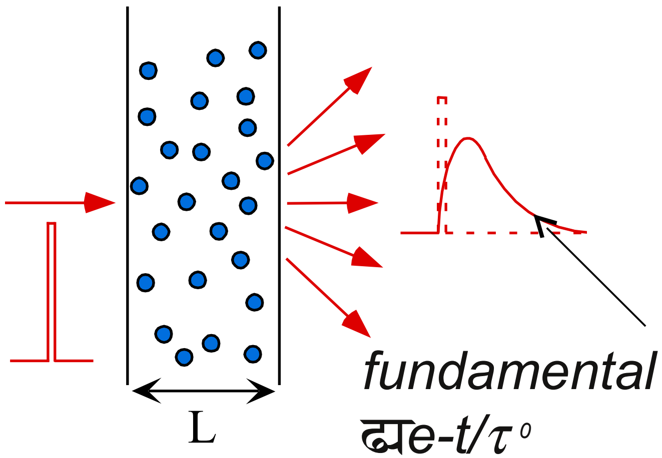
optical thickness : $b \approx 300$

velocity distribution : $v_{rms} \approx 10$ cm/s

Université Nice Sophia Antipolis

institut non linéaire de Nice SOPHIA ANTIPOLIS

Time Resolved Experiments : Radiation Trapping



fundamental (Holstein) mode
∝ e^{-t/τ_0}

diffusion theory

$$D = \frac{1}{3}\,\frac{l_{tr}^2}{\tau_{tr}}$$

l_{tr}: *transport mean-free path*

τ_{tr}: *transport time*

$$= \frac{1}{3}\, l_{tr} v_{tr}$$

v_{tr}: *transport velocity*

$$\tau_0 \approx \frac{L^2}{\pi^2 \Delta} \qquad \frac{3}{\pi^2} = \beta^2 \tau_{tr}$$

$b = \frac{L}{l}$ optical thickness

Time Resolved Experiments

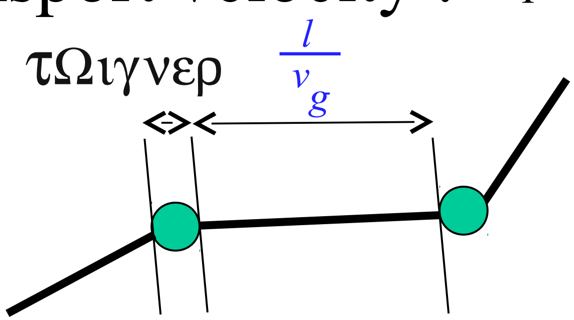
- Phase velocity : $c = \frac{c0}{n}$ propagation of phase for a monochromatic wave

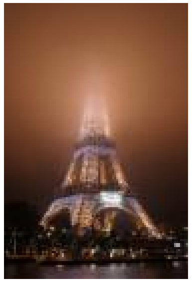
  $c \lessgtr c0$

- Group velocity : vg= त्ज / तk propagation of transmitted gaussian pulse with slowly varying envelope

  cold atoms on resonance : vg < 0 |vg| << 0

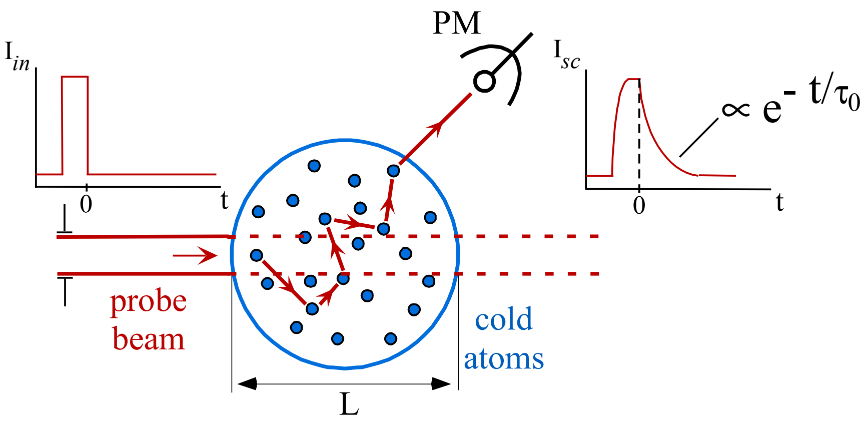
- Transport velocity : propagation of scattered wave energy 0 < vtr < c0

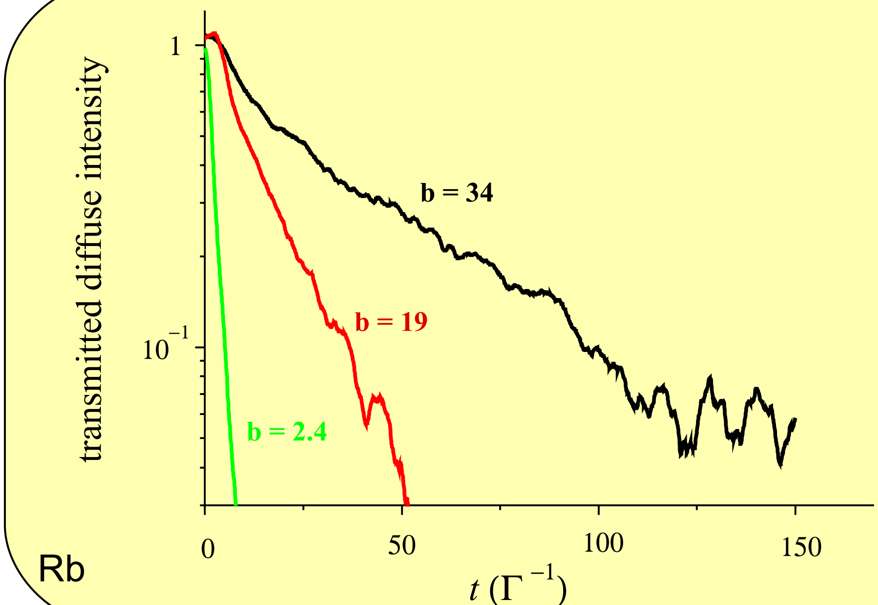




Phys. Rep. **270**, 143 (1996).

Slow Diffusion of Light





vtr / c = ıtr / cττρ झ3 ·10-5

D : smaller than in Ti02 with kl झ1 (Dझ4m2/s)

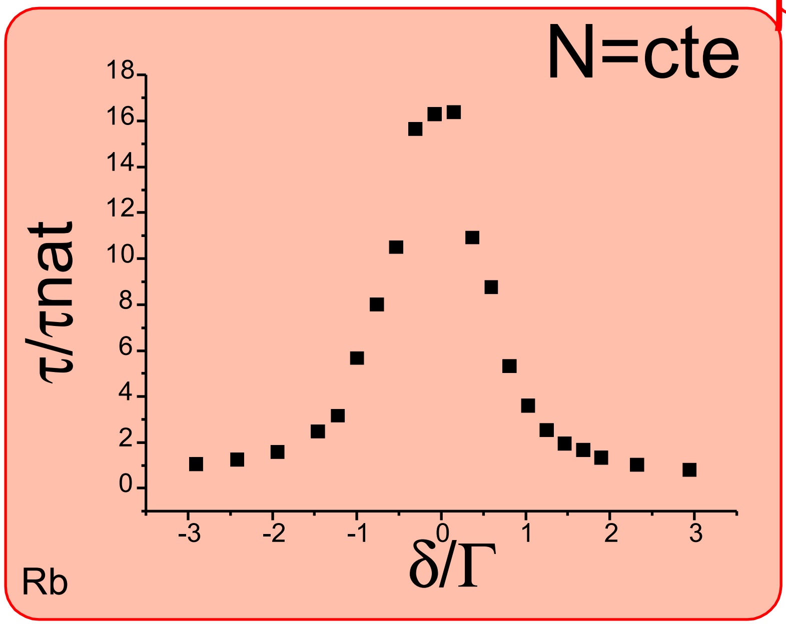
τ0 झ L2 / π2D   Dझ0.66m2/s

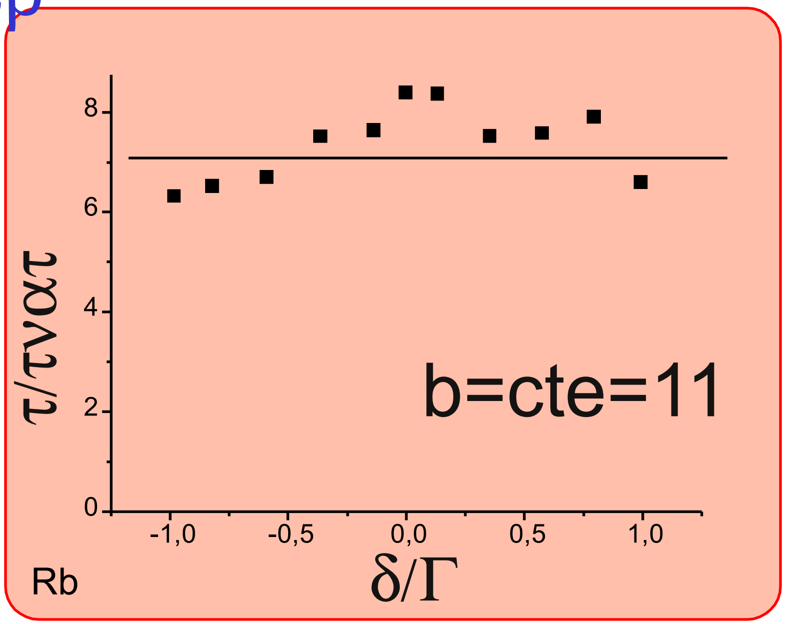
NO interference effect !
ड्डLocalization

for b=34 : τ0 झ52 τnat L=4mm

Transport time for light in cold atoms

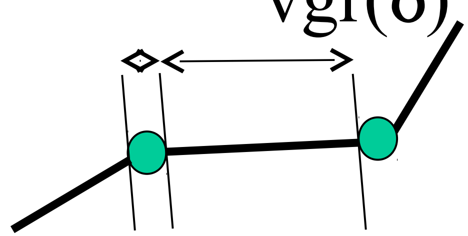
τ0 ≈ L2/π2Δ   3/π2 =
β2 ττρ

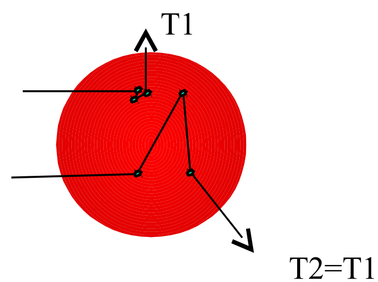




Transport Time :
~Independent of δ

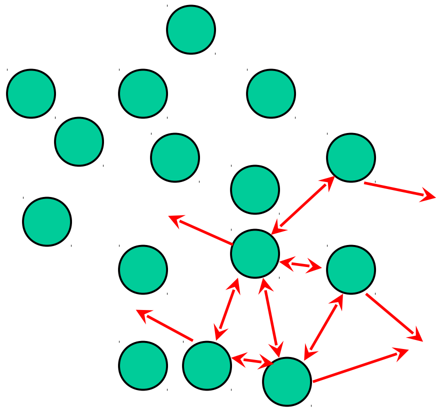
τtr ≈ τWigner (δ) + l(δ)/vgr(δ)

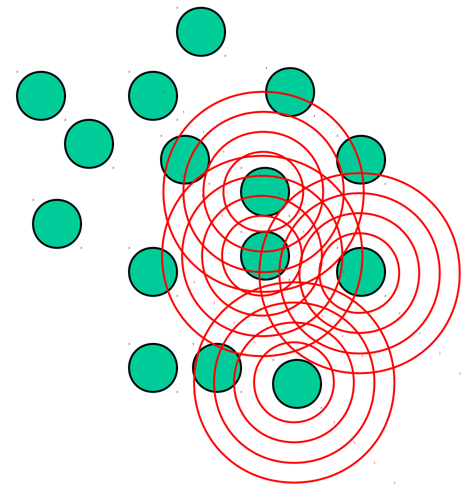




Phys. Rev. Lett. **91**, 223904 (2003)

Photons ... ... are waves





Random walk :

Diffusion coefficient

D0 ड्ड **l** 2 / **τ**

l = 1/n σ

Interference correction to

Diffusion coefficient

D ड्ड D0 [1- 1/(k **l**)2]

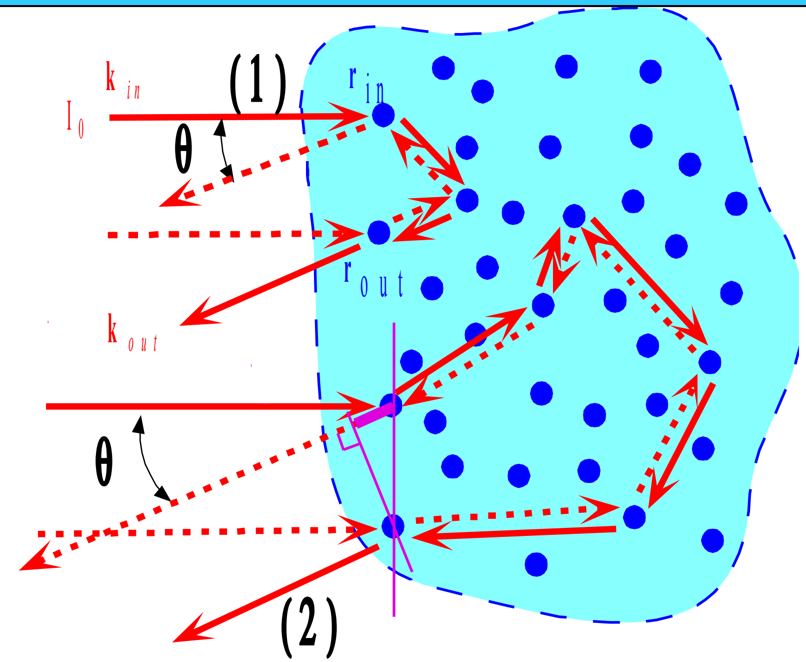
Strong Localization (D=0) :

Ioffe-Regel criterium : k **l** ड्ड 1

(near field scattering **l** ड्ड l)

Weak Localization => Coherent Backscattering

- **uncorrelated** paths add incoherently
- **correlated** (i.e. reciprocal) paths add coherently



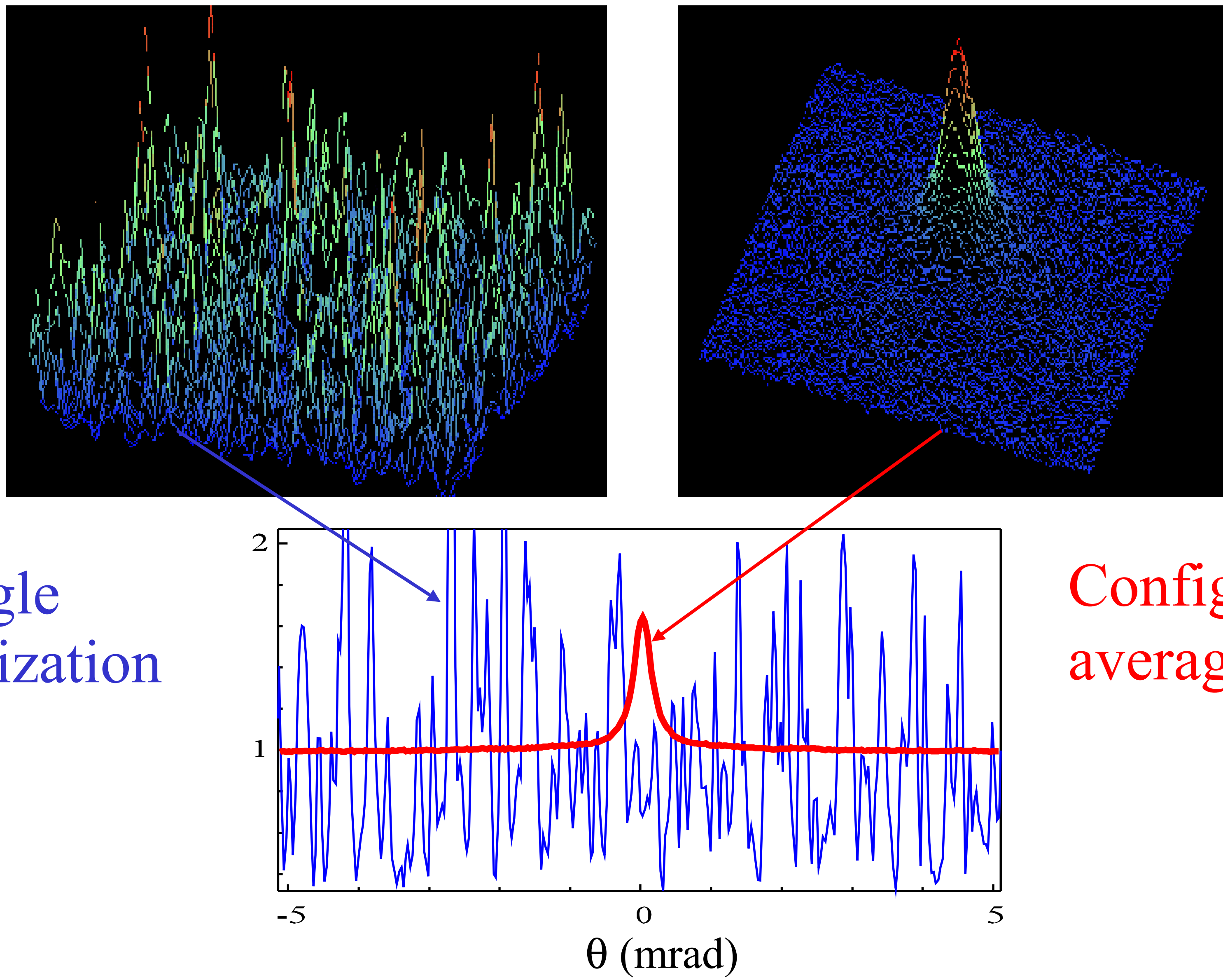
Δφ= (κιν+ κουτ).(ριν−ρουτ) ⇒ Δφ = 0 φορ ανψ πατη

Coherent Backscattering $\frac{<I(0)>}{<I(\theta)>} = 2$

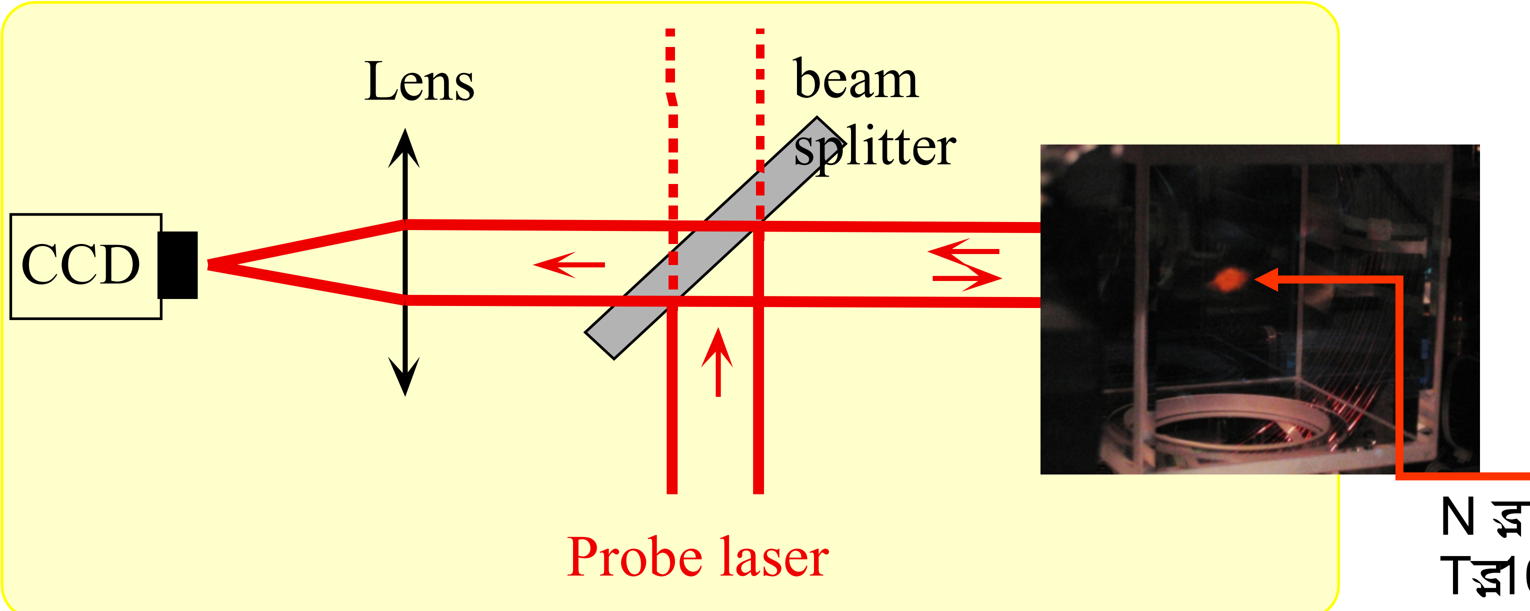
multiple self-aligned Sagnac interferometer

Configuration Average

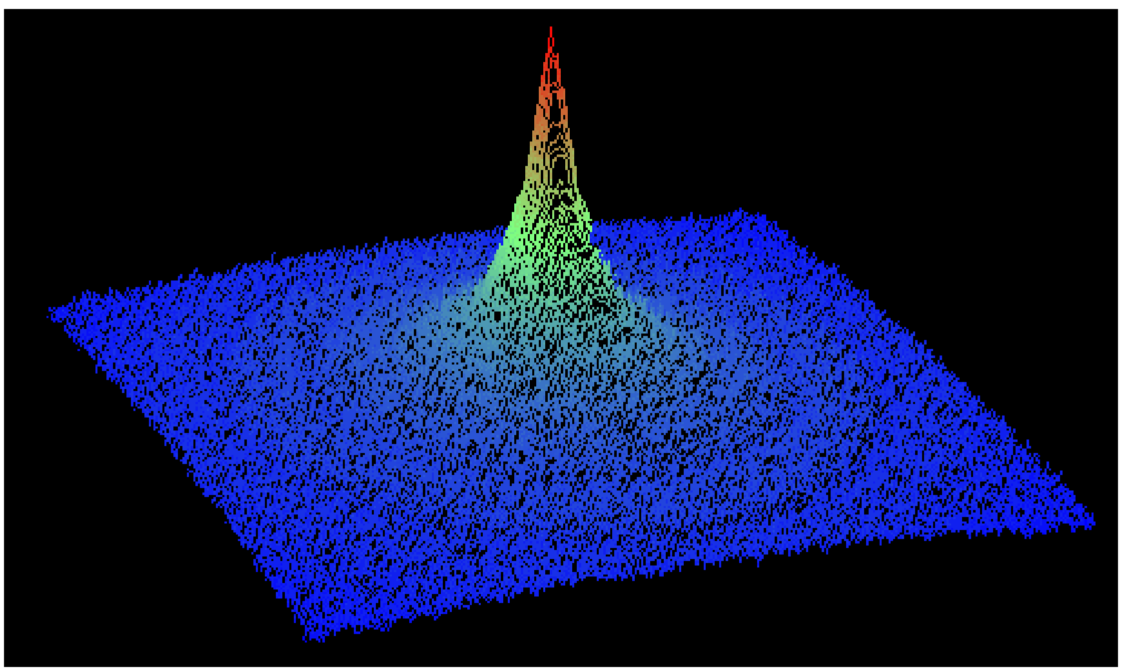
Single realization



Configuration average



N ≈ 1010
T ≈ 100µK
kl ≈ 1000



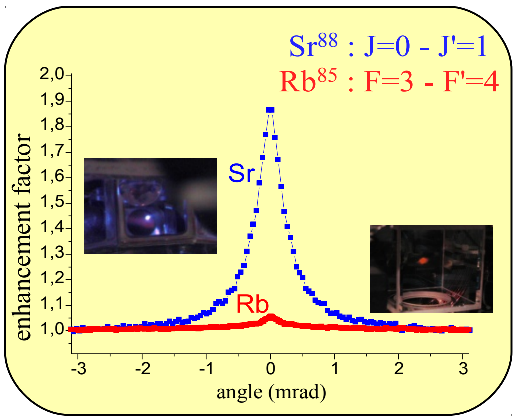
Phys. Rev. Lett., **83**, 5266 (1999)

Coherence after resonant scattering with atoms !

also : M. Havey et al.

COHERENT BACKSCATTERING = coherent probe

Internal structure :
Rb = quantum magnets
Sr = classical dipole



PRL, **85**, 4269 (2000)

PRL, **88**, 203902 (2002)

PRL, **89**, 163901 (2002)

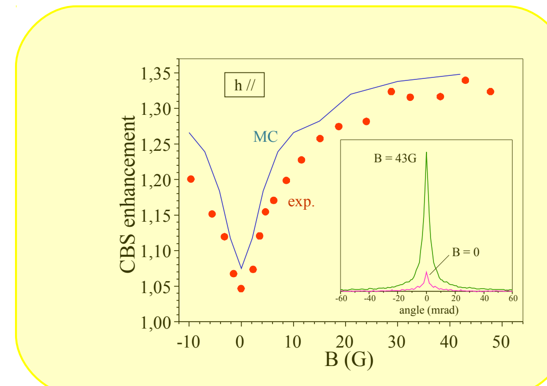
PRL, **93**, 143906 (2004)

Quantum fluctuations :
inelastic scattering
non linear response

Sr88

PRE, **70**, 036602 (2004)

Restoring two level atoms:



PRL, **93**, 143906 (2004)

Temperature :
'fast' atomic dynamics
'slow' transport of light

PRL, **97**, 013004 (2006)

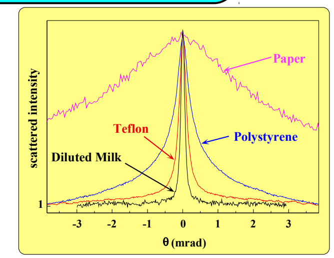
Rb85

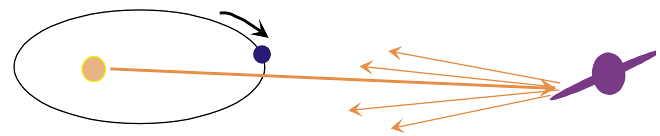
T = 80µK ↑

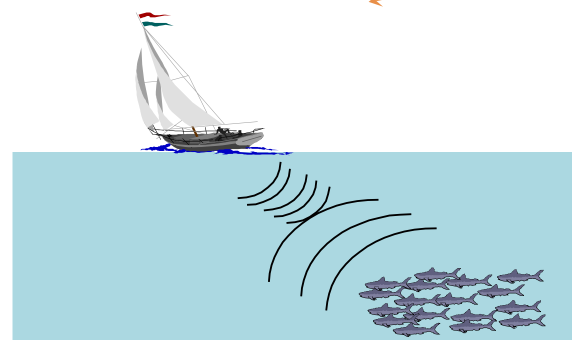
T = 50mK ↓

Coherent Backscattering

Light waves : white paint (TiO_2)
teflon, milk, paper
tissue
rings of Saturn



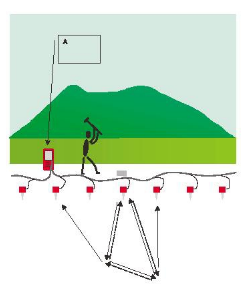




Acoustic waves : metal rods
fish (?)

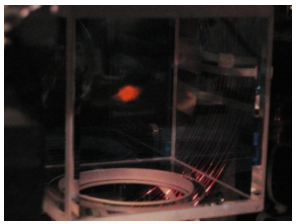
Matter waves : electrons : negative magneto-resistance
not (yet) with ultra-cold atoms

Seismic waves :

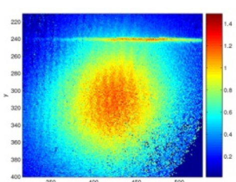


Perspectives :Towards strong localization of light

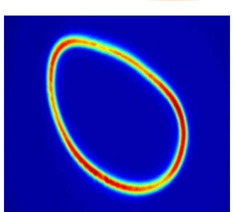
Ioffe-Regel : k l झ1

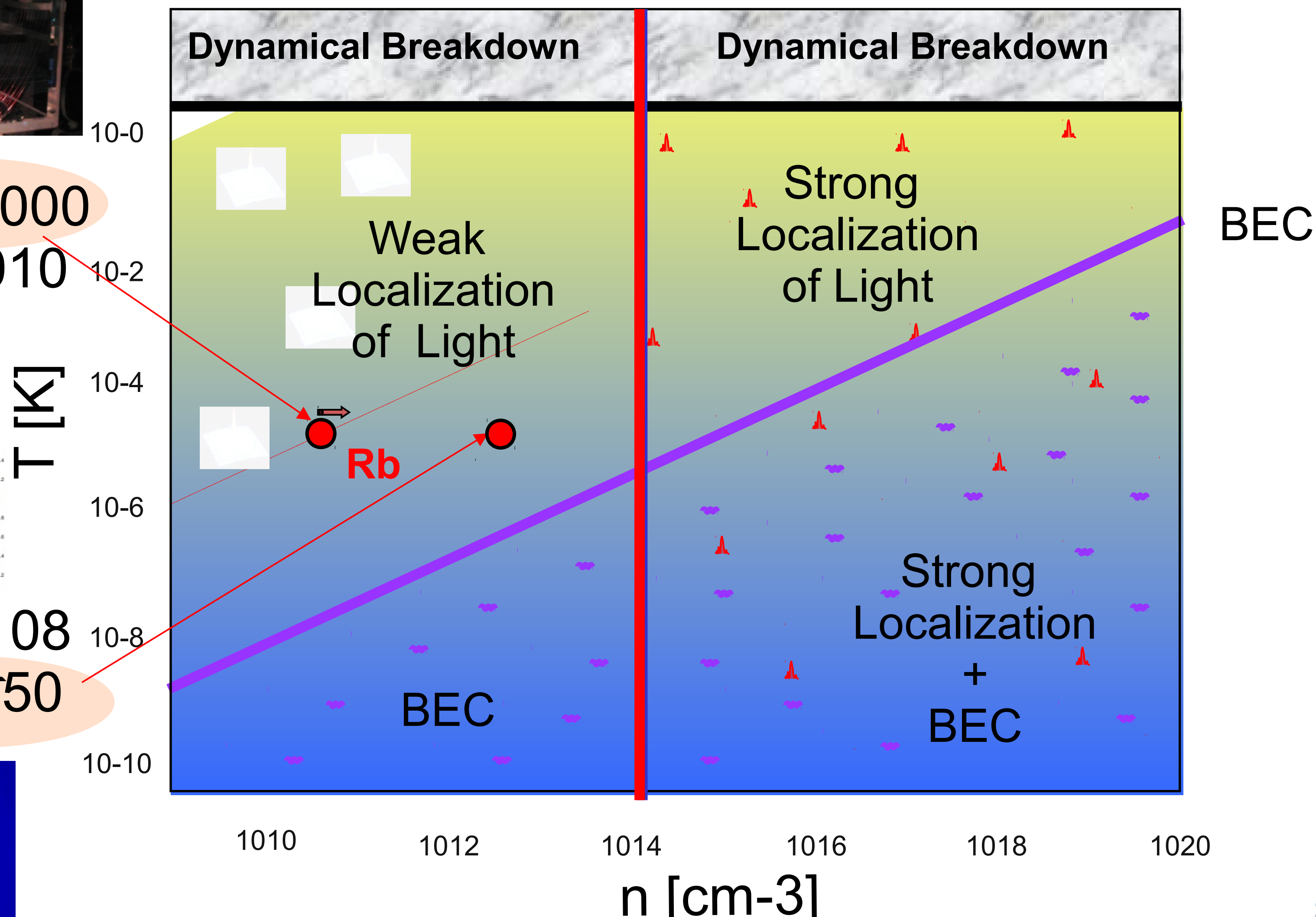


k l झ1000
N= 1010

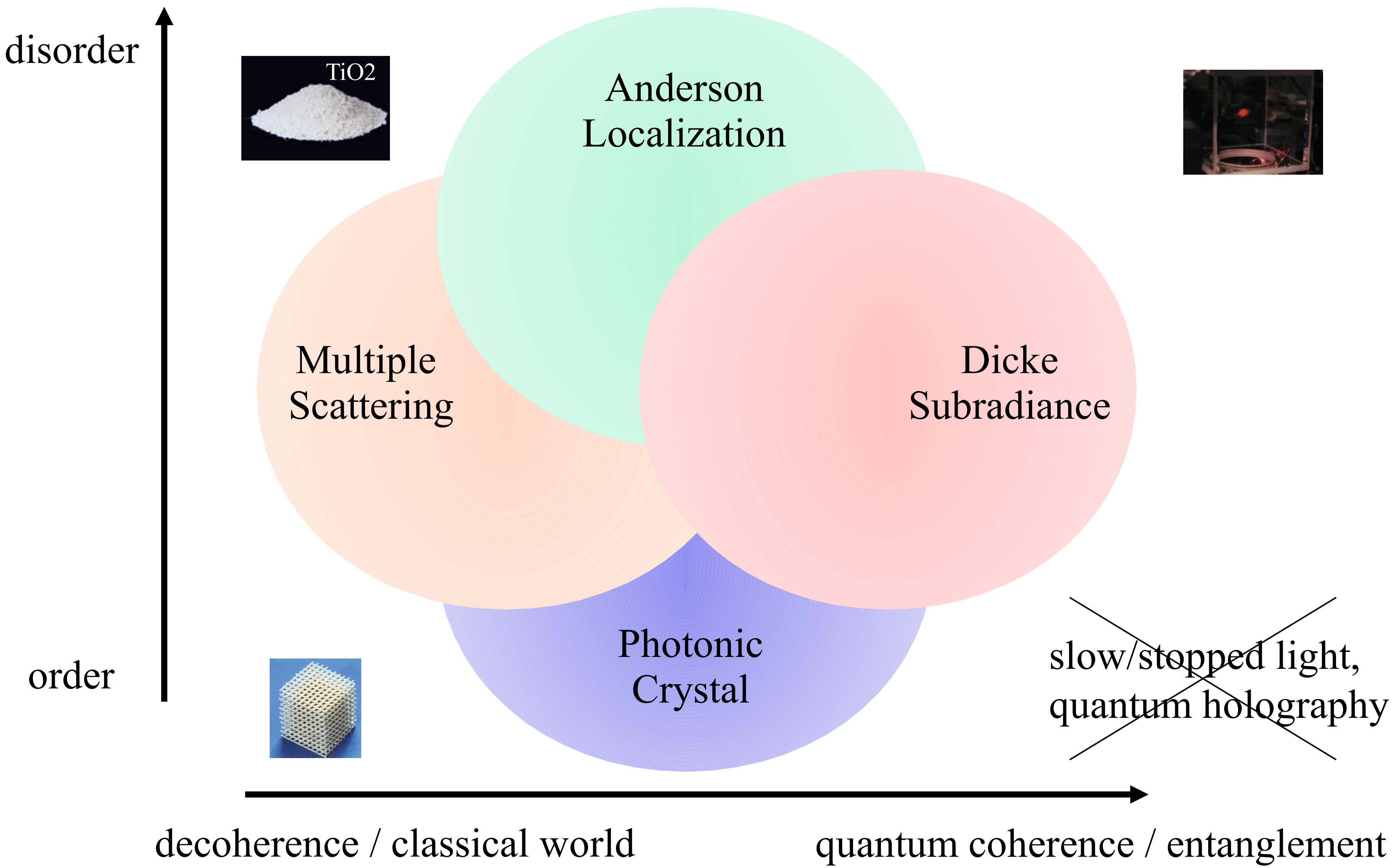


N= 108
k l झ50



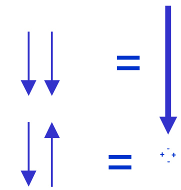


How to trap a 'photon' with N atoms?



1954 : Dicke super- and subradiant states

Superradiance = symetric state (easy to observe)
Subradiance = antisymetric states (‘fragile’ : difficult to observed)



Correlated dipoles

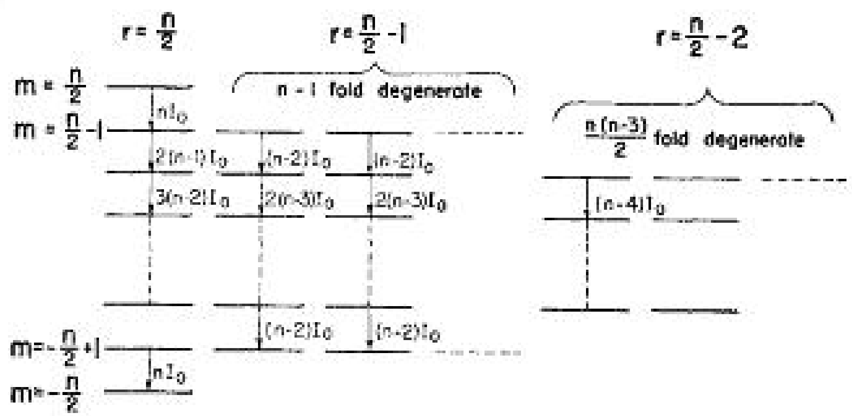
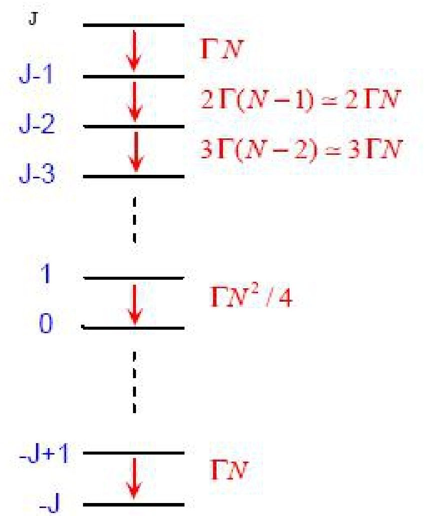
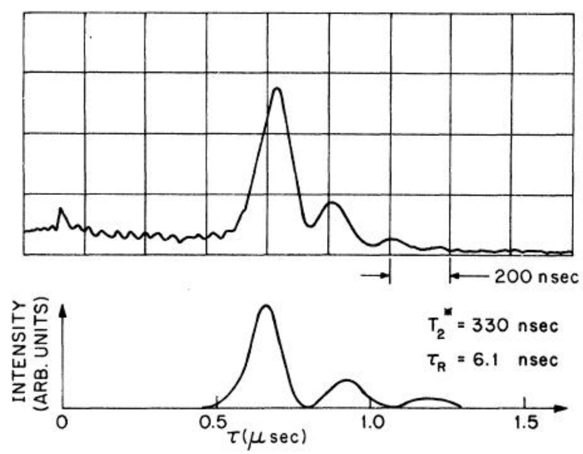


FIG. 1. Energy level diagram of an n-molecule gas, each molecule having 2 nondegenerate energy levels. Spontaneous radiation rates are indicated. $E_m = mE$.



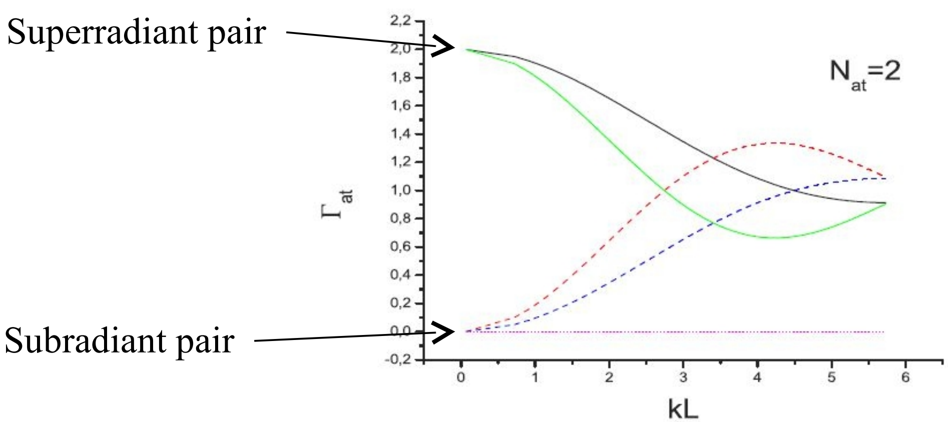
R. Dicke 1954

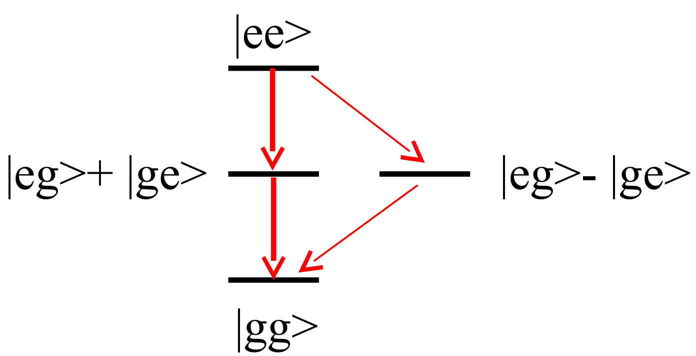


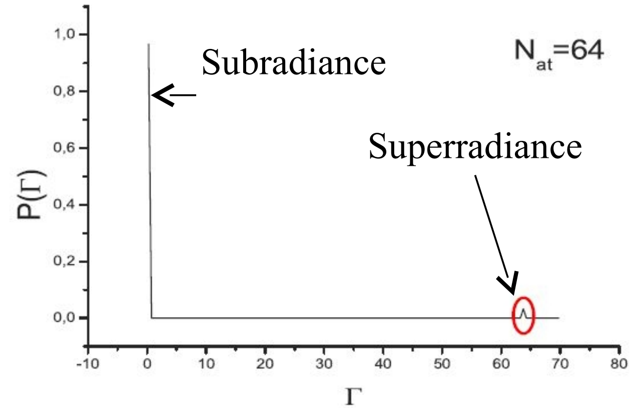
First experimental observation

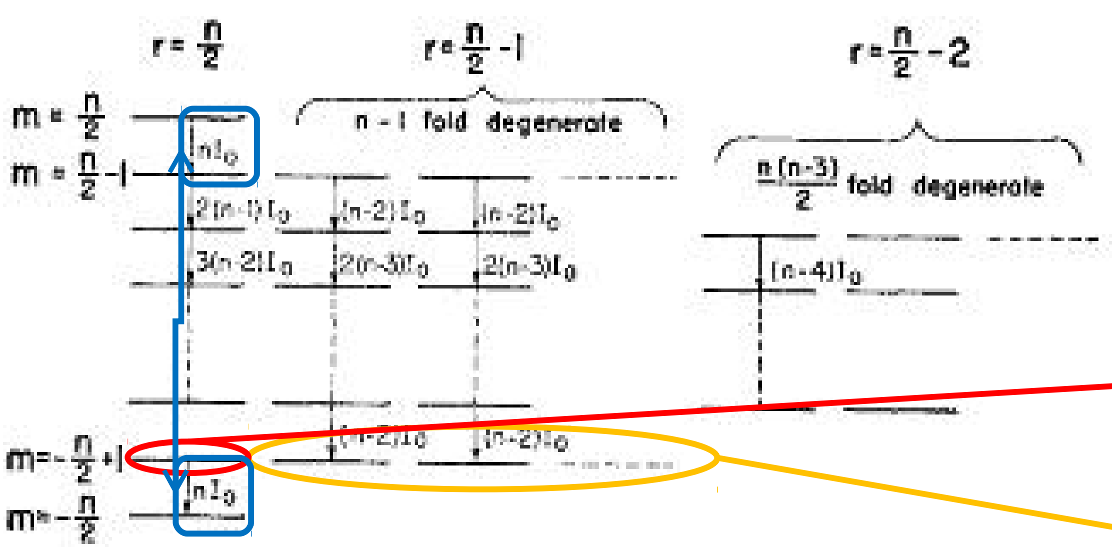
Feld et al. 1973

Small volume limit









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$\Gamma max \sim N\, \Gamma 0$

Subradiant
(metastable)
states

1) Effective Hamiltonian

- Open System
- Properties and eigenvalues of Heff

2) Driven System

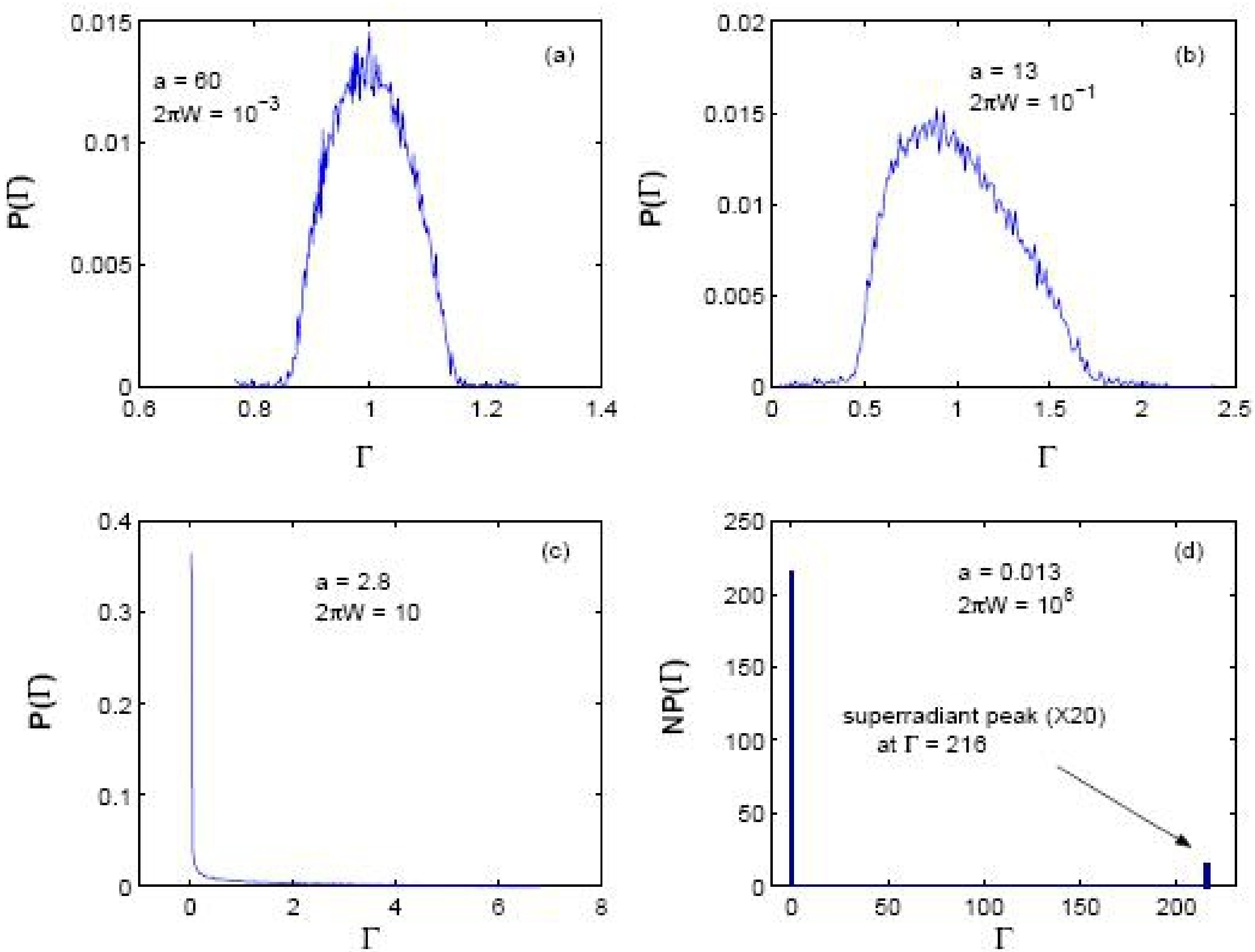
- Initial State preparation
- Scattering

1) Effective Hamiltonian

p, q : angle and polarisation dependant

- Open System
- Quantum Extension of Anderson Hamiltonian
- Heisenberg model with global coupling

Photon Escape Rate = Spectrum {Im (Heff) }

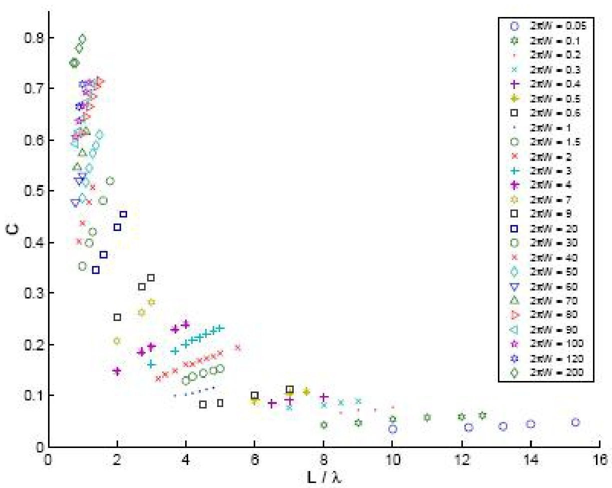


size : a = L/l
disorder parameter W=1/kl

See also F. Pinheiro et al. 2004

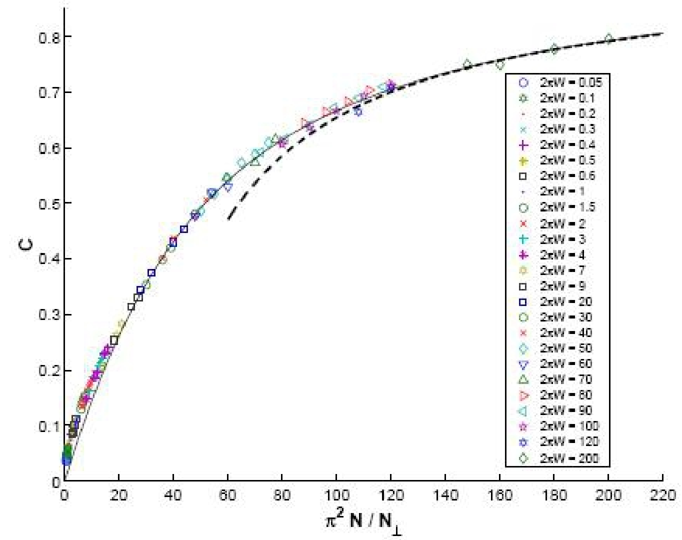
Photon Escape Rates

Measure of long lived photons



Single parameter scaling

N/N⊥



cooperative effects dominate over disorder !
no phase transition observed with P(Γ)

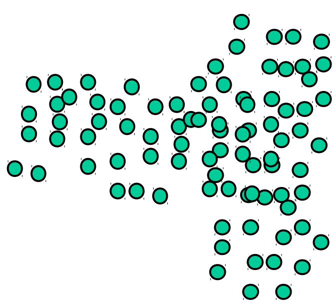
Dicke > Anderson

E. Akkermans, A. Gero, RK, PRL, **101**, 103602 (2008)

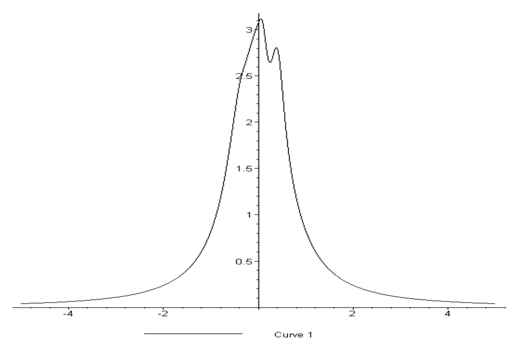
Eigenvalues

Beyond Photon escape times :

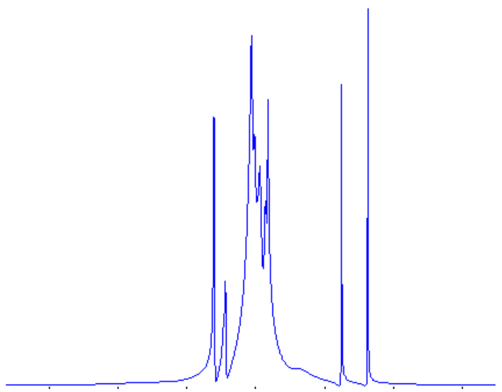
Cloud of Atoms = Large Molecule (with 1010 atoms)



'dilute' molecule



'dense' molecule



molecular spectrum ?

SHENG LI AND ERIC J. HELLER   PHYSICAL REVIEW A 67, 032712 (2003)

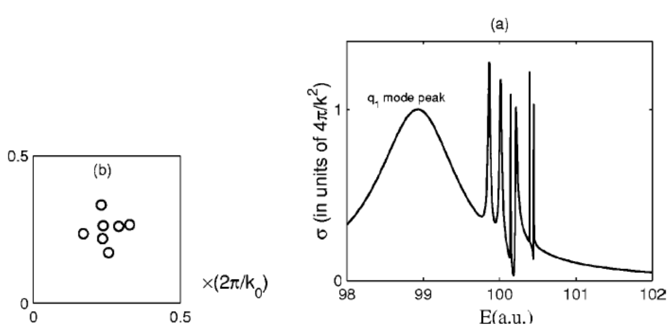
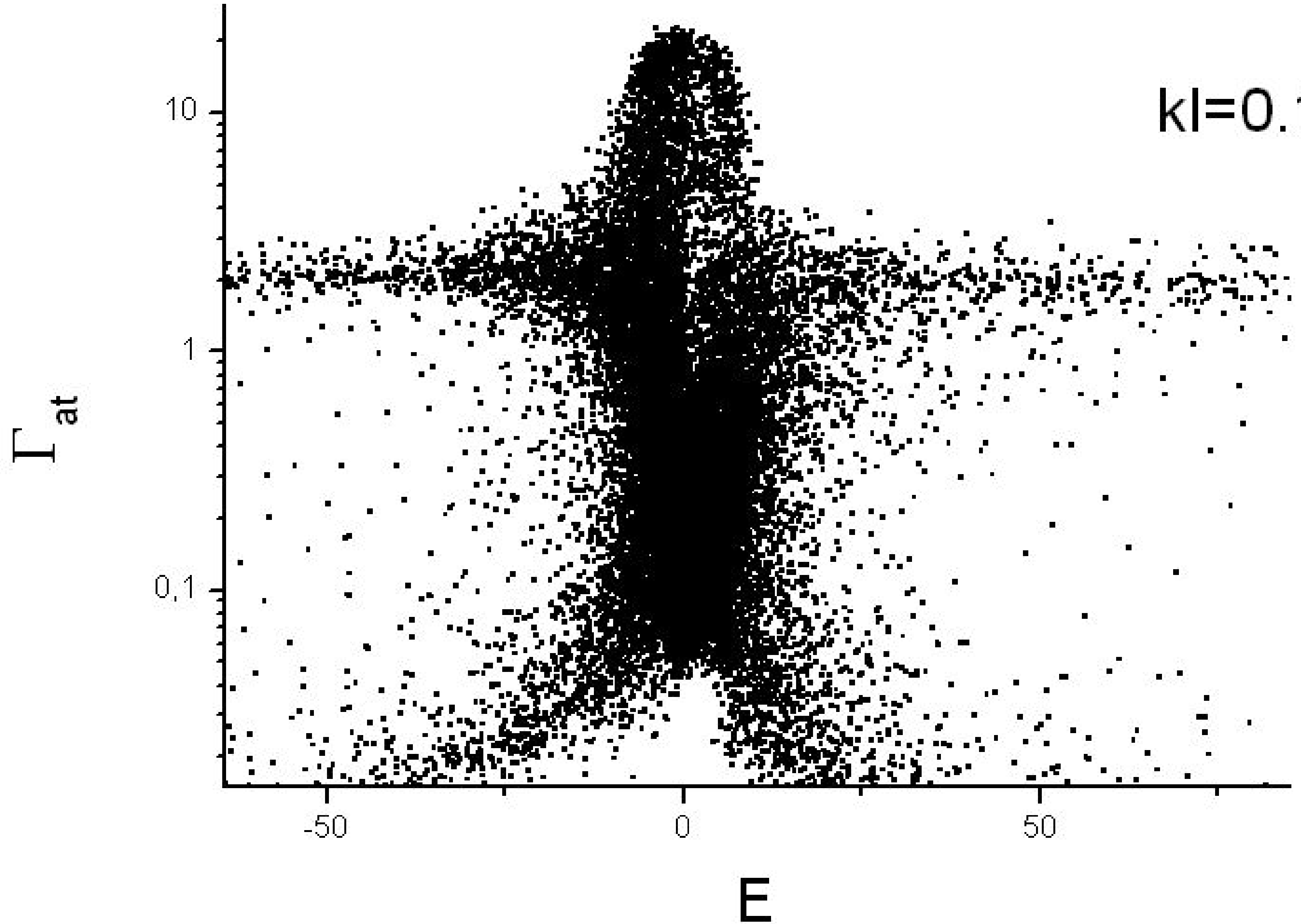


FIG. 4. (a) Total cross section as a function of energy for a system of seven identical scatterers randomly placed on a plane. Each scatterer is the same as used in Fig. 1. The positions of the scatterers are shown in (b).

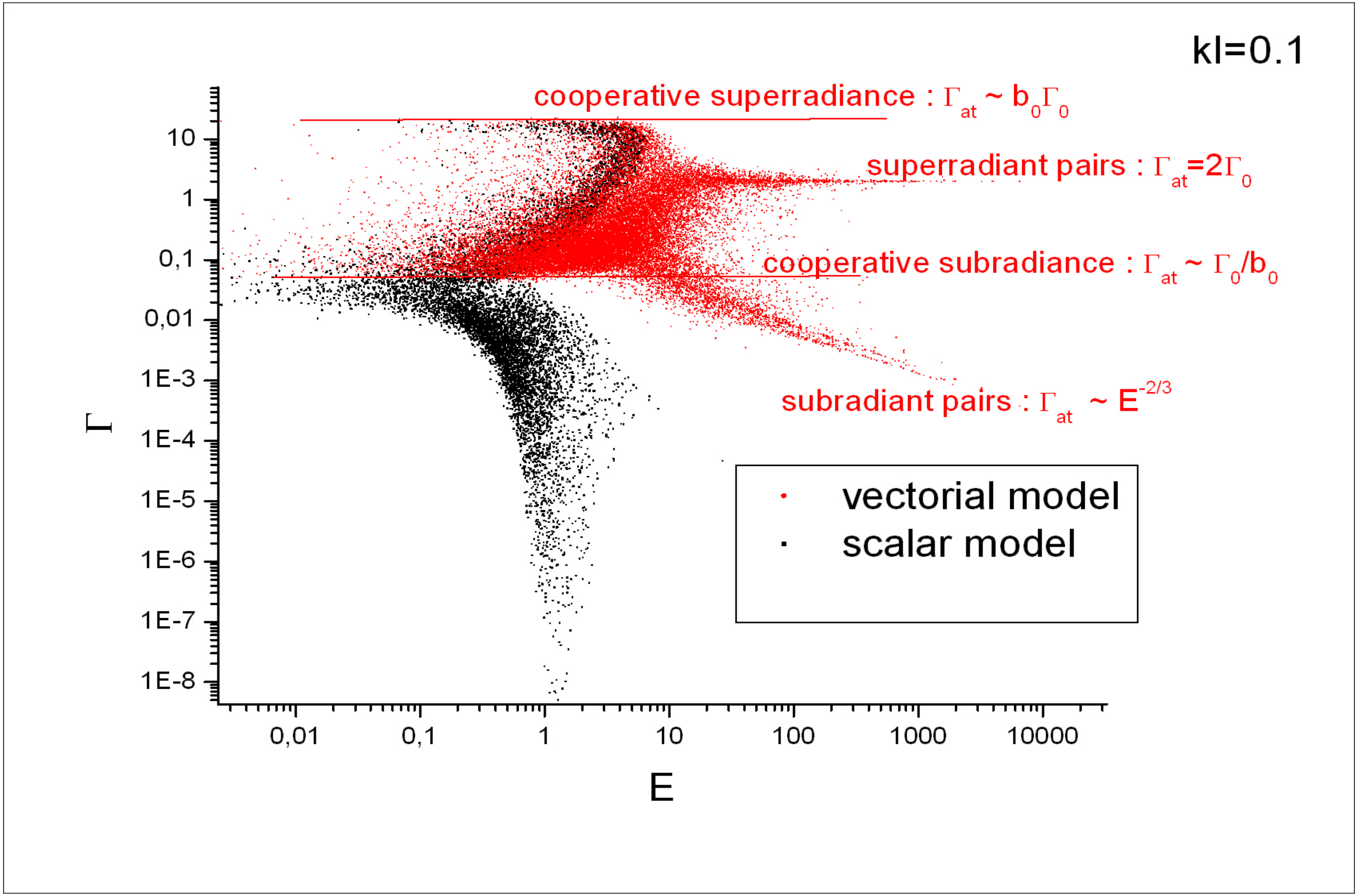
proximity resonances

Institut non linéaire de Nice SOPHIA ANTIPOLIS

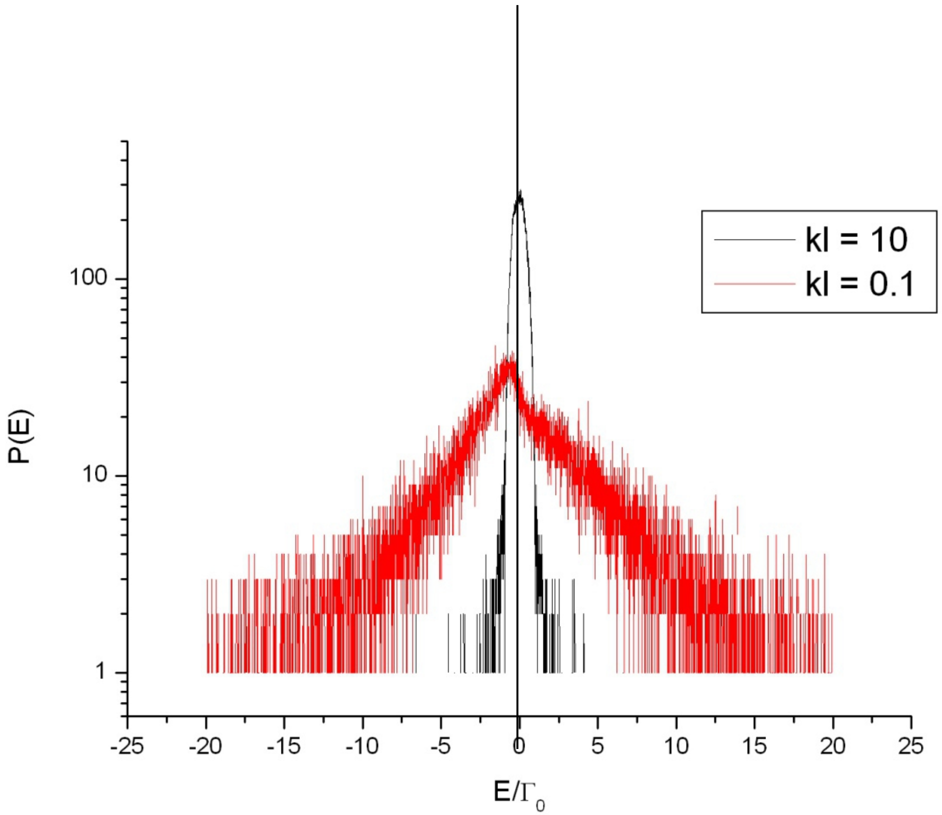
Eigenvalues of Heff



Eigenvalues



Eigenvalues



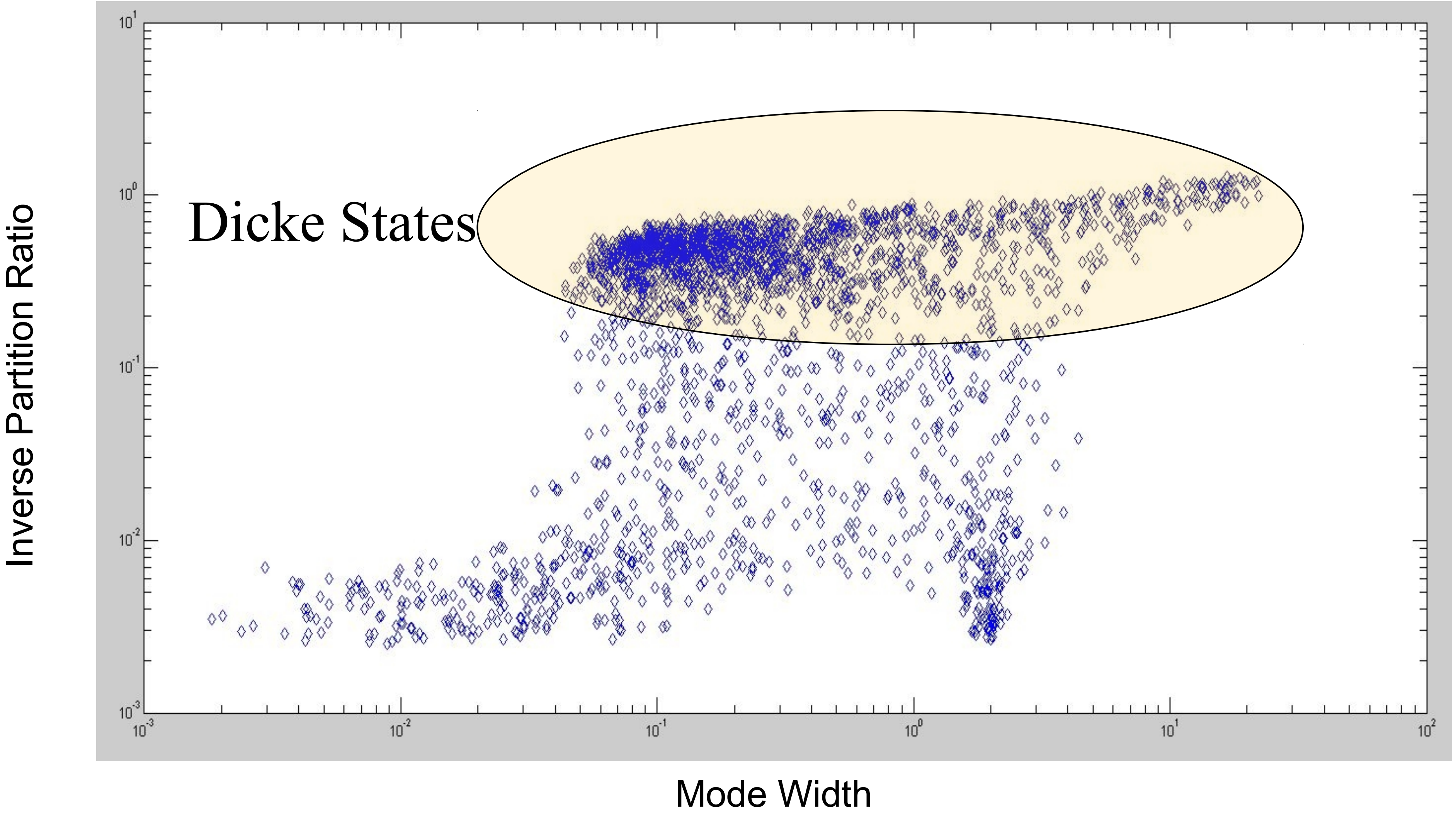
Width distribtution ≠ Escape Rates

Density of States
(Lorentz-Lorenz / cooperative Lamb shift)

Random Matrix Theory not valid : Euclidean Random Matrices

Eigenvectors

kl=0.1
N=800
Full Vectorial



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- Open System
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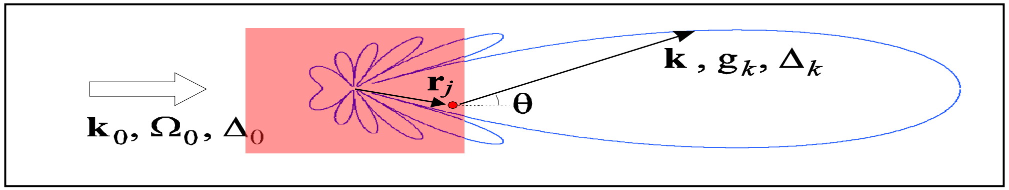


Driven Timed Dicke State :

Hamiltonian (RWA)

$$\hat{H} = \hbar \sum_{j=1}^{N} \left[\frac{\Omega_0}{2} \hat{\sigma}_j e^{i\Delta_0 t - i\mathbf{k}_0\cdot\mathbf{r}_j} + \text{h.c.} \right] + \hbar \sum_{j=1}^{N} \sum_{\mathbf{k}} \left[g_k \hat{\sigma}_j \hat{a}_{\mathbf{k}}^{\dagger} e^{i\Delta_k t - i\mathbf{k}\cdot\mathbf{r}_j} + \text{h.c.} \right]$$

wave function

$$|\Psi(t)\rangle = \alpha(t)|0\rangle_a|0\rangle_{\mathbf{k}} + e^{-i\Delta_0 t} \sum_{j=1}^{N} \tilde{\beta}_j(t) e^{i\mathbf{k}_0\cdot\mathbf{r}_j} |j\rangle_a|0\rangle_{\mathbf{k}} + \sum_{\mathbf{k}} \gamma_{\mathbf{k}}(t)|0\rangle_a|1\rangle_{\mathbf{k}}$$

time evolution

$$\dot{\alpha} = -\tfrac{i}{2}\Omega_0 \sum_{j=1}^{N} \tilde{\beta}_j ,$$

$$\dot{\tilde{\beta}}_j = i\Delta_0\tilde{\beta}_j - \tfrac{i}{2}\Omega_0\alpha - i\sum_{\mathbf{k}} g_k\gamma_{\mathbf{k}} e^{i(\Delta_0-\Delta_k)t + i(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{r}_j},$$

$$\dot{\gamma}_{\mathbf{k}} = -ig_k e^{-i(\Delta_0-\Delta_k)t} \sum_{j=1}^{N} \tilde{\beta}_j e^{-i(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{r}_j}.$$

Trace over environment

Markov approx. $\tau_N \gg$

Low saturation $\alpha \approx 1$

$$\dot{\tilde{\beta}}_j = \underbrace{i\Delta_0\tilde{\beta}_j - \tfrac{i}{2}\Omega_0}_{\text{coherent drive}} \underbrace{- \tfrac{1}{2}\Gamma \sum_{m=1}^{N} \gamma_{jm}\tilde{\beta}_m}_{\text{Heff}}$$

coherent drive

Heff

Average force on center of mass (easy to measure)

Ansatz : βj=βTD Average Force

$$\mathbf{F}_a = \langle \hat{\mathbf{F}}_{aj} \rangle = -\frac{\hbar \mathbf{k}_0 \Omega_0}{\sqrt{N}} \mathrm{Im}\left[\beta(t)\right]$$

$$\mathbf{F}_e = -\hbar \mathbf{k}_0 \Gamma \left|\beta(t)\right|^2 f_N$$

$$\frac{F_{c,N}}{F_1} = \frac{4\Delta_0^2 + \Gamma^2}{4\Delta_0^2 + \left(1 + \frac{b_0}{12}\right)^2 \Gamma^2} \left[1 + \frac{b_0}{24(k\sigma_R)^2}\right]$$

Superradiance

Disorder

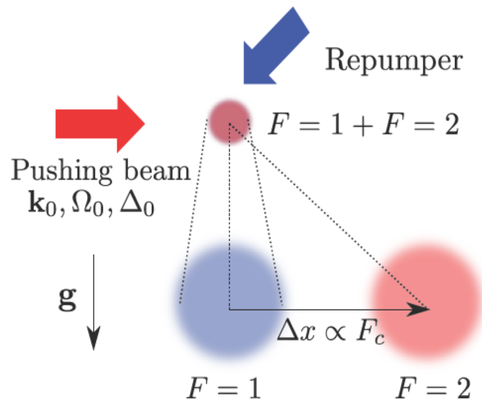
Σ ढ्ढै

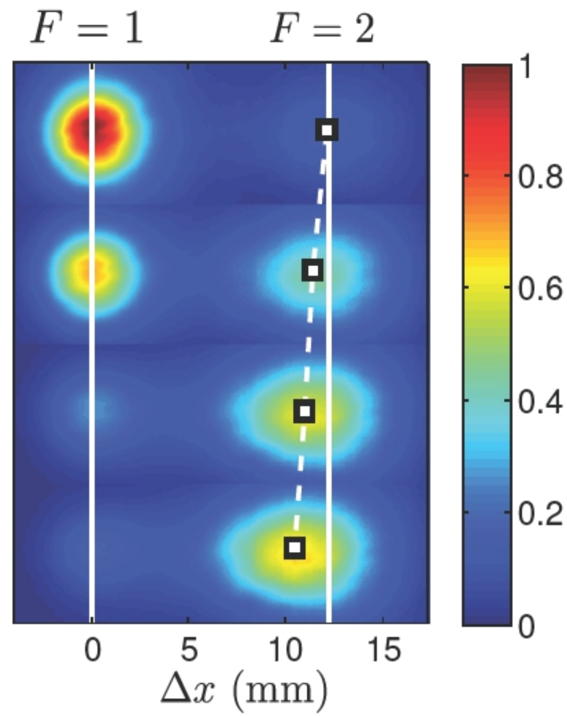
Emission Diagram (Mie)

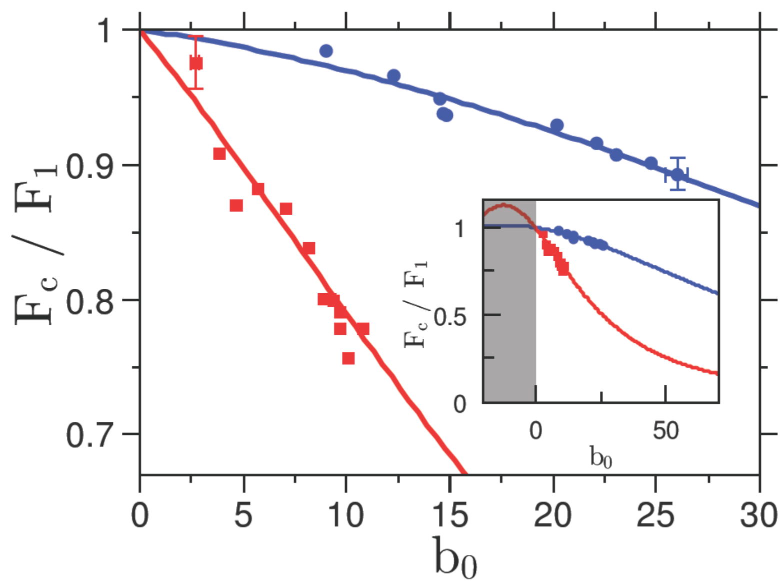
Nat/Nmodes द्वाNat / (L/λ)2
द्वाb0

Experimental sequence:

- MOT loading (2 s)
- Dark MOT (50 ms)
- Optical pumping
- Pushing beam (0.8 ms)
- Time of flight (13 ms)
- Fluorescence imaging !





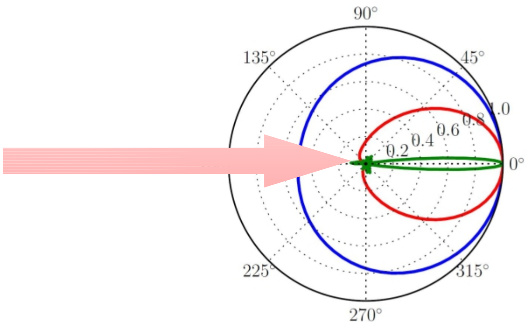


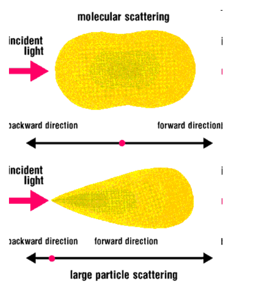
radiation pressure force reduced by **cooperative scattering** (absorption+emission)

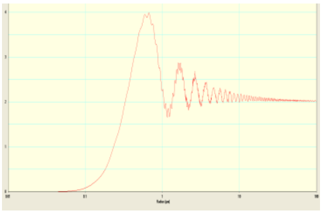
PRL, **104**, 183602 (2010)

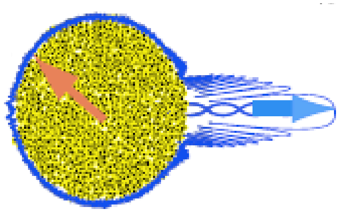
Large volume limit

Modified Angular emission diagram : Mie scattering









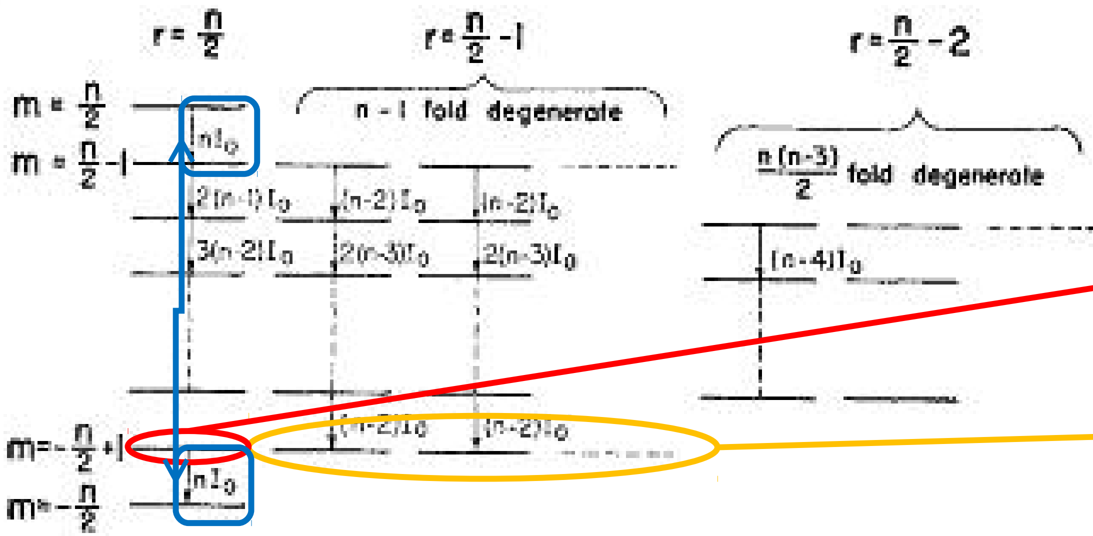


FIG. 1. Energy level diagram of an n-molecule gas, each molecule having 2 nondegenerate energy levels. Spontaneous radiation rates are indicated. $E_m = mE$.

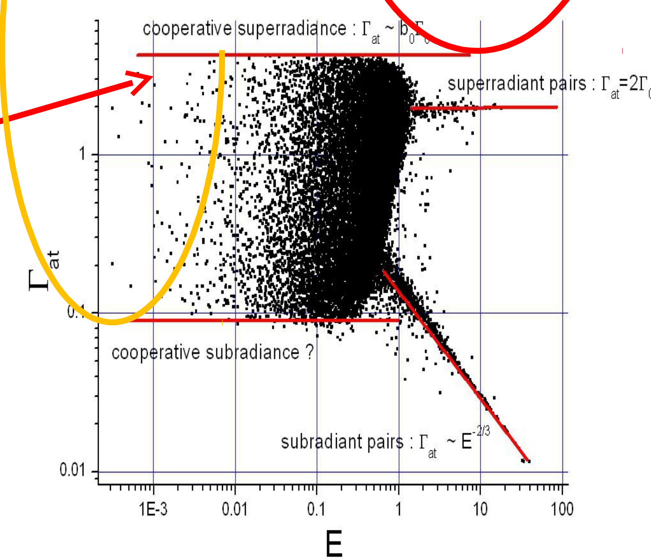
Timed Dicke State

Small volume :

$\Gamma max \sim N\ \Gamma 0$

Subradiant
(metastable)
states

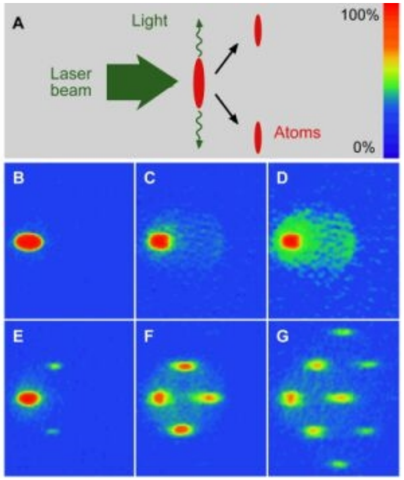
Large Volume :
$\Gamma max \sim b0\ \Gamma 0$



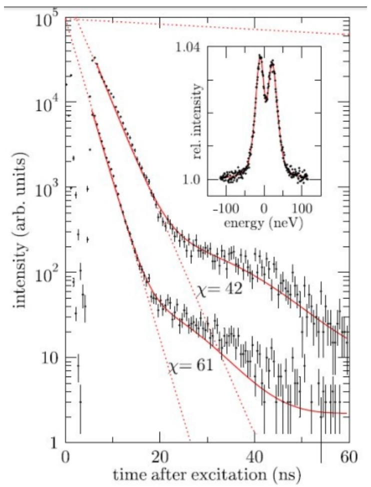
Superradiance in :

AMO

Ketterle et al.,
Science **285**, 571 (1999)



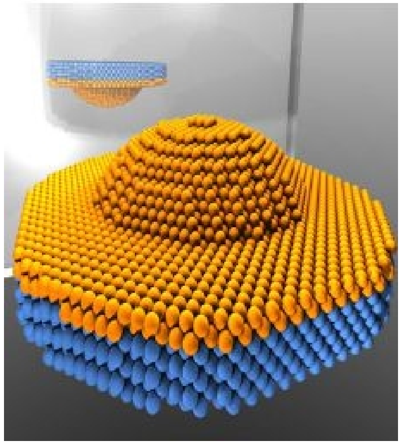
Nuclear physics



Röhlsberger et al.,
Science **328**, 1239 (2010)

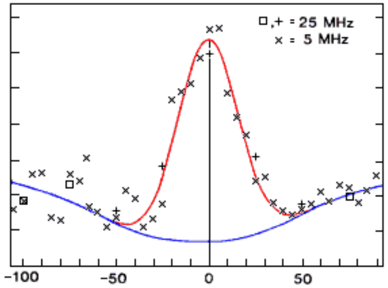
Quantum Dots :
Giant oscillator strength

Lodahl et al.,
Nat. Phys., 19th Dec 2010



Astrophysics

Mars
CO2



Non thermal emission lines

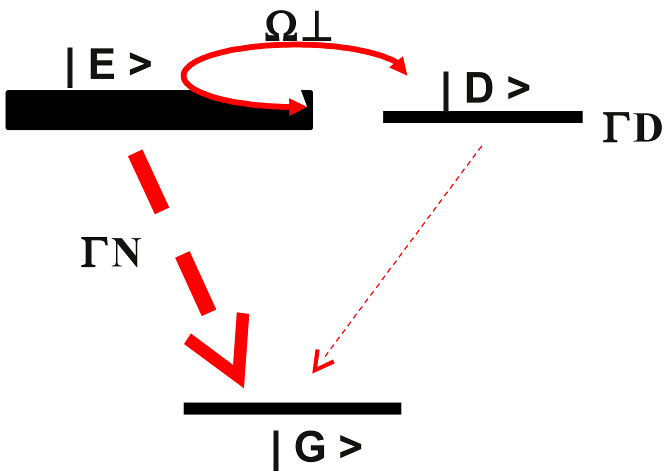
Letokhov et al.,
New Astronomy Reviews **51**, 443(2007)

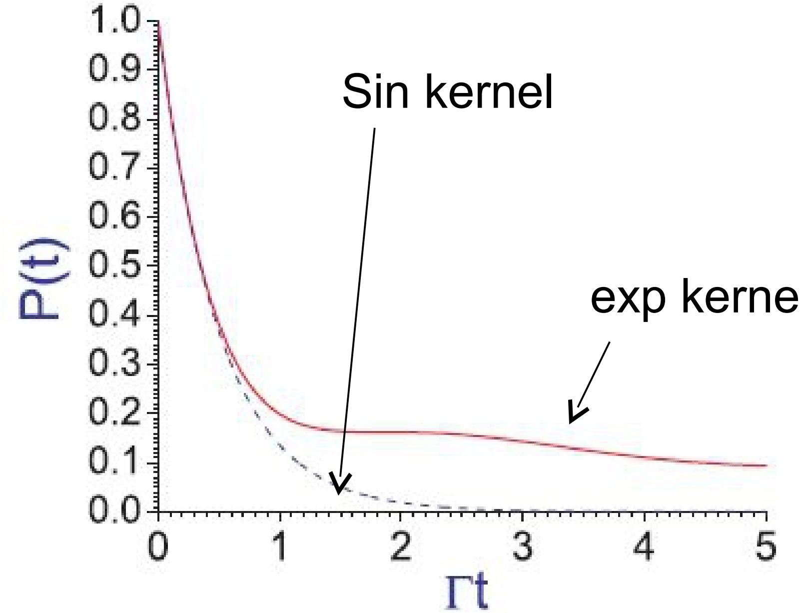
Looking for subradiance

Dicke subradiance for N two level systems (in free space, N>>1) has **not yet been observed**

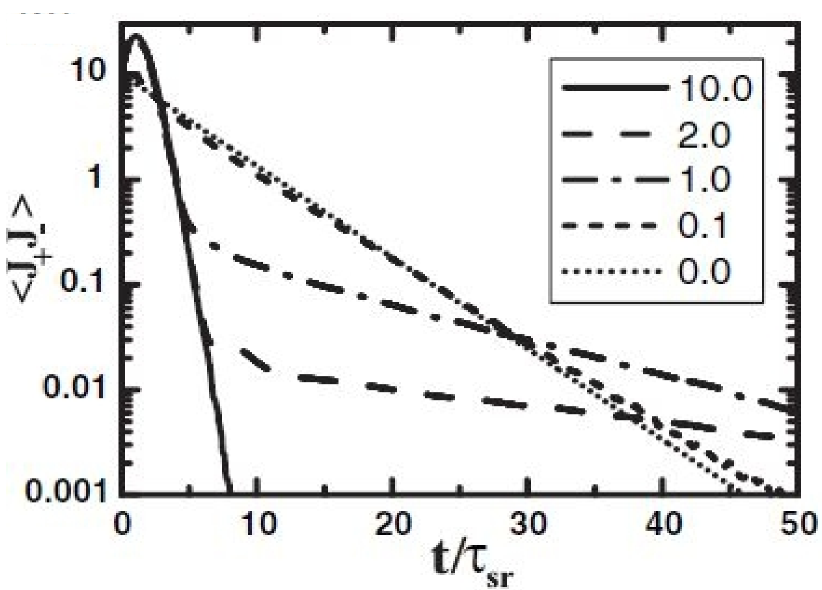
- Does not require large spatial densities
- Requires large optical densities
- Requires careful coupling : fragile state

Fano Coupling and controlled subradiance





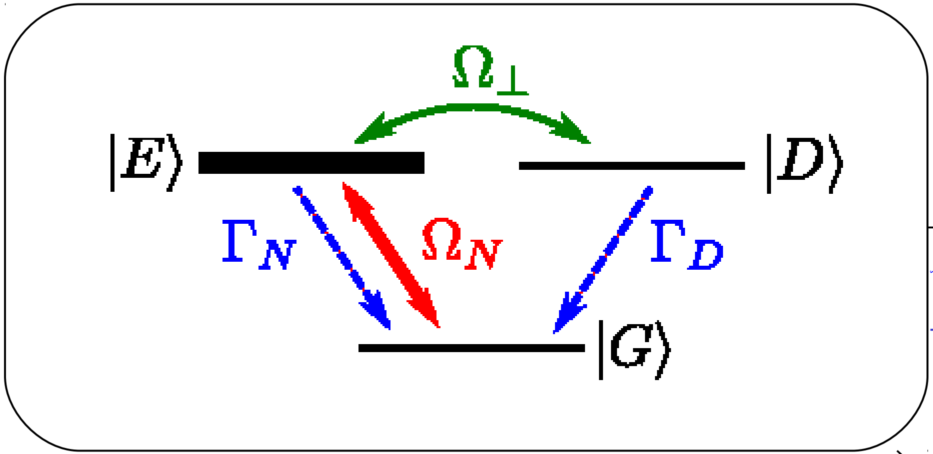
Svidzinsky et al. PRA 81, 053821 (2010)
|व(0)>=|TD>



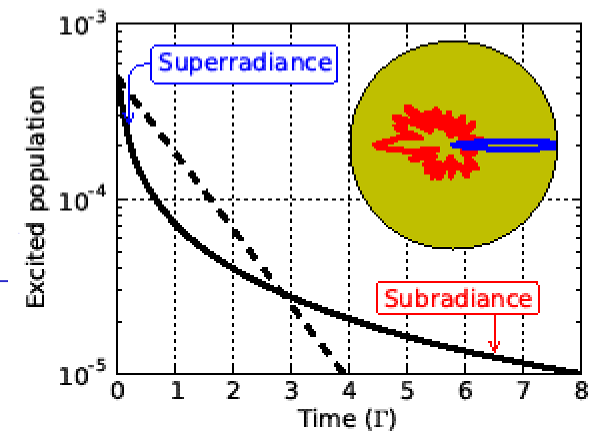
Temnov, Woggon,
PRL 95, 243602 (2005)

Inhomogeneous broadening in |ei>
=> coupling in |TD>

Fano Coupling and controlled subradiance



Fano

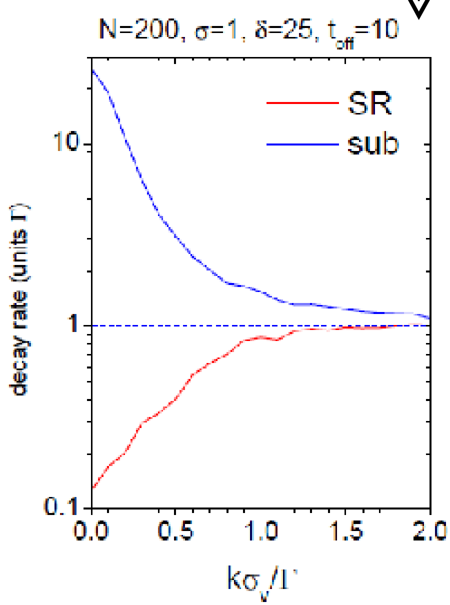


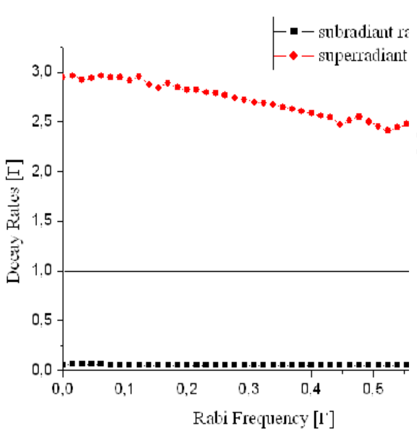
$$\dot{\beta}_i = \left[-\frac{\Gamma}{2} + i(\Delta_0 - \mathbf{k}_0 \cdot \mathbf{v}_i)\right]\beta_i - \frac{i}{2}\Omega_0 + \frac{i\Gamma}{2}\sum_{j\neq i} V_{ij}\beta_j$$

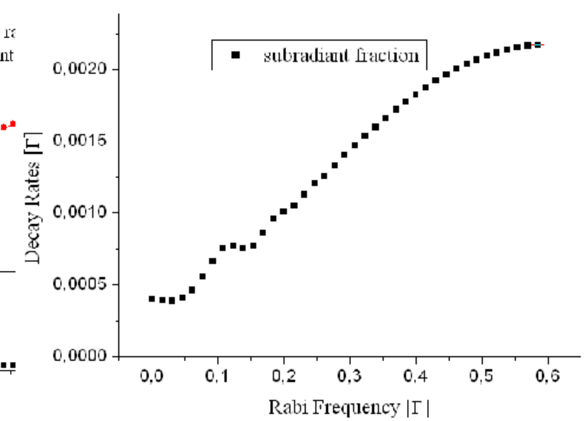
$$V_{ij}(t) = \frac{e^{ik_0|\mathbf{r}_i - \mathbf{r}_j + (\mathbf{v}_i - \mathbf{v}_j)t|}}{k_0|\mathbf{r}_i - \mathbf{r}_j + (\mathbf{v}_i - \mathbf{v}_j)t|} e^{-i\mathbf{k}_0\cdot[\mathbf{r}_i - \mathbf{r}_j + (\mathbf{v}_i - \mathbf{v}_j)t]}$$

Doppler

Random Light shift







What's next :

- Subradiance experiments
- Look for Anderson (help with 'decoherence')
- Add order (coupled spins on lattice)
- Multiple scattering of quantum fluctuations
- Entanglement


Thank you for your attention
And enjoy …