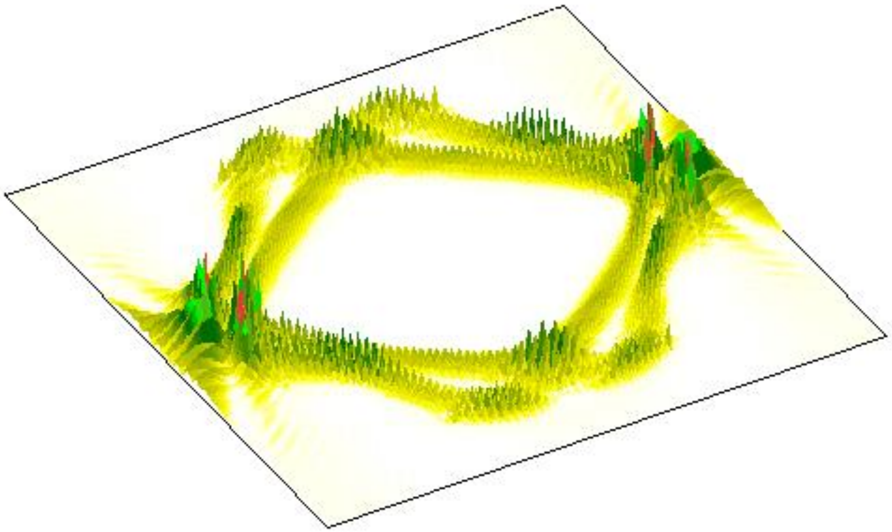
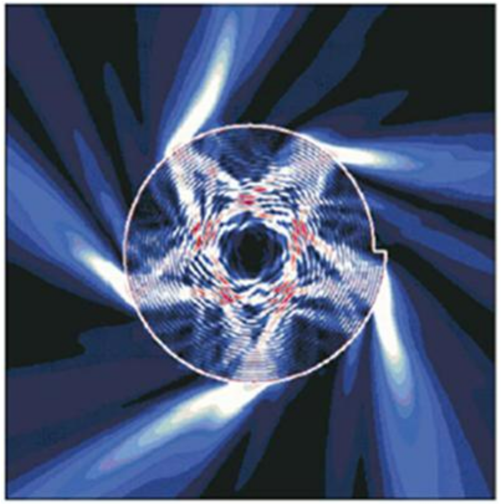
Quasiscarred modes and the role of exceptional point in a deformed microcavity

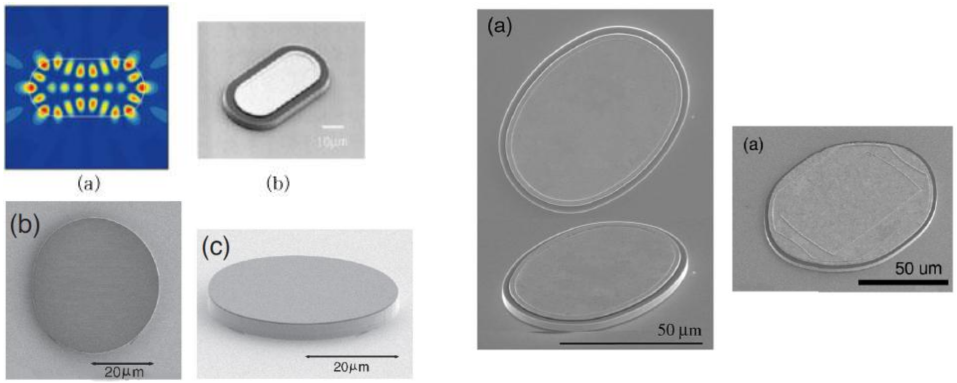




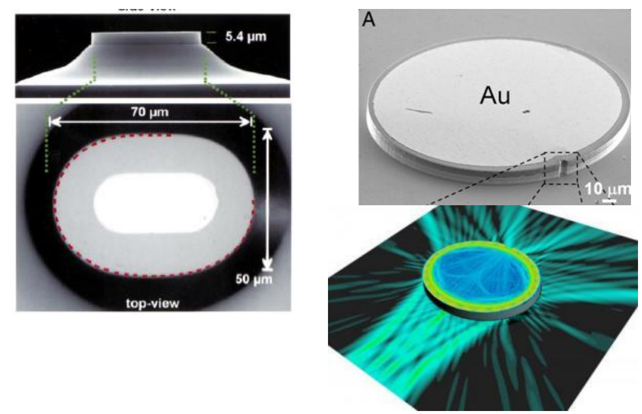
Soo-Young Lee
Pusan National University
15 Jun 2011 PHHQPX11, MPI

Contents

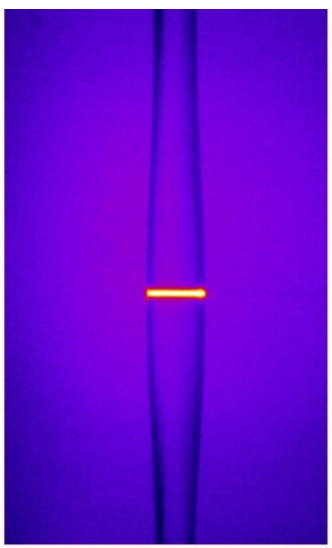
Deformed microcavities



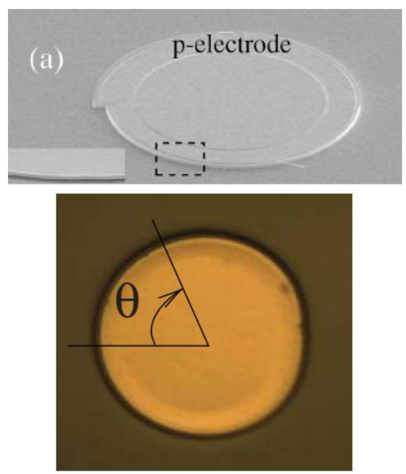
Harayama (Toyo Univ.)



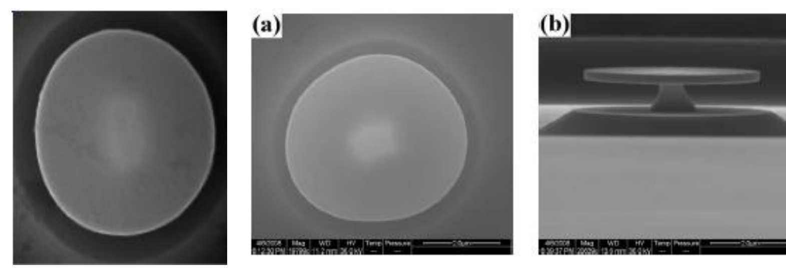
Capasso (Harvard Univ.)



K. An (Seoul Nat. Univ.)

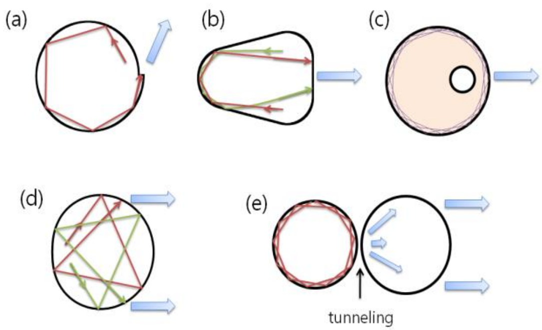


C.-M. Kim (Sogang Univ.)



Cao (Yale Univ.)

Why deformed microcavity?

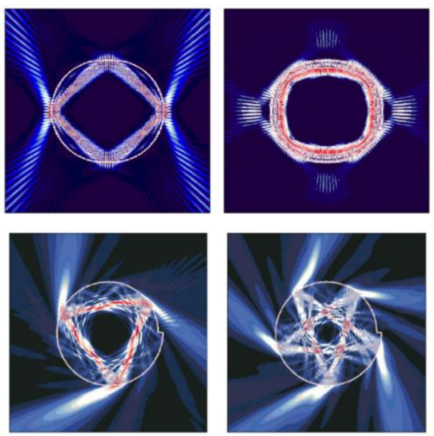


Small optical resonators with directional emission

Good directionality

Unidirectional emission

Chern et al., APL 83, 1710 (2003)
Kurdoglyan et al., OL 29, 2758 (2004)
Wiersig & Hentschel, PRA 73, 013802(R) (2006)
Wiersig & Hentschel, PRL 100, 033901 (2008)
Ryu et al., PRA 79, 053858 (2009)

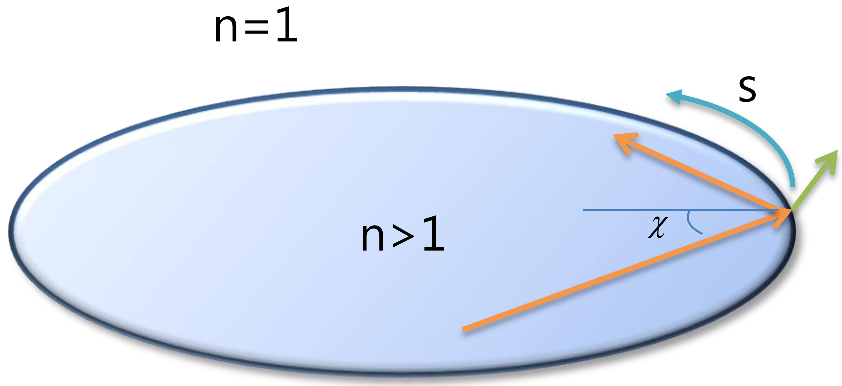


Open wave chaotic systems

Quantum chaos study
- Scarring, Quasiscarring,
- Exceptional point,
- Dynamical tunneling

Lee et al., PRL 93, 164102 (2004)
Lee et al., PRA 72, 061801(R) (2005)
Lee et al., PRL 103, 134101 (2009)
Shinohara et al., PRL 104, 163902 (2010)
Yang et al., PRL 104, 243601 (2010)

Openness of Dielectric Microcavities



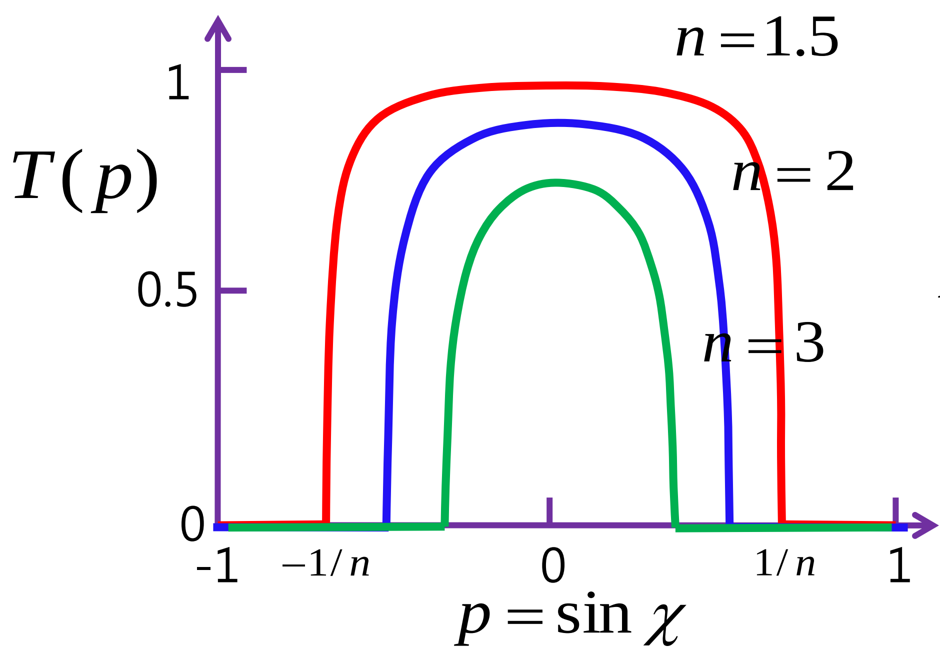
$\chi < \chi_c \equiv \arcsin(1/n)$

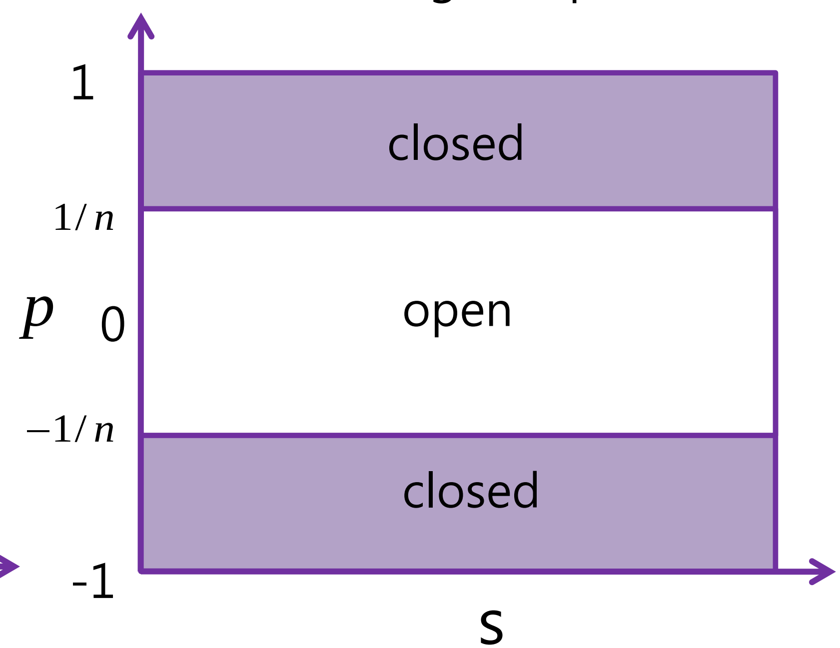
: refractive escape

$\chi > \chi_c$

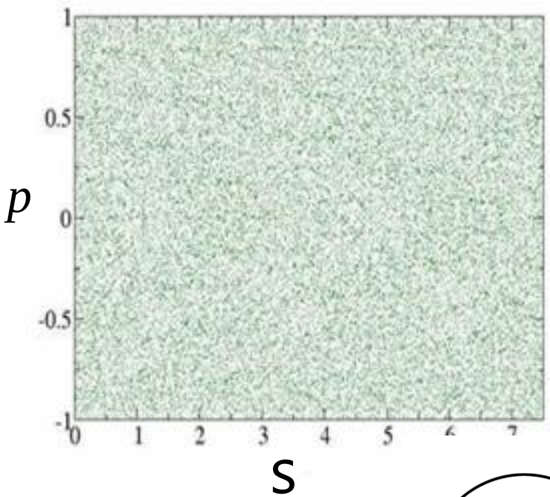
: total internal reflection
tunneling escape

Transmission probability



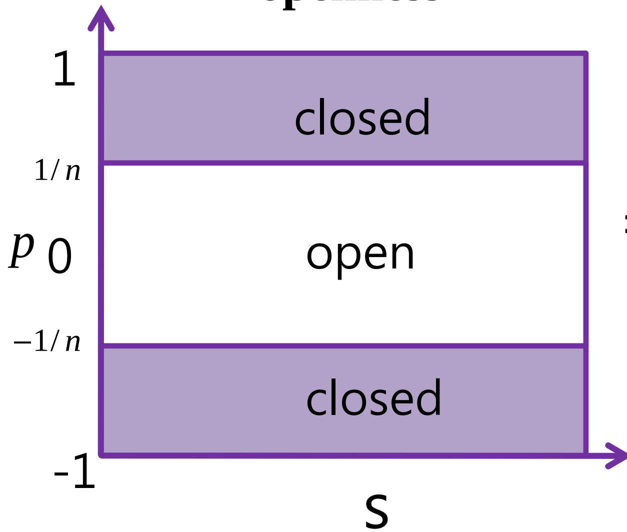


Chaotic dynamics

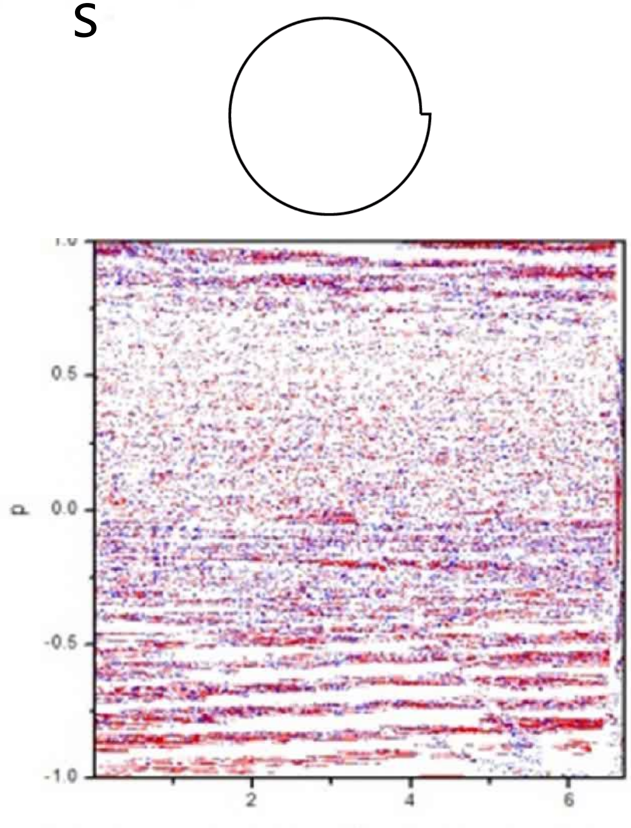


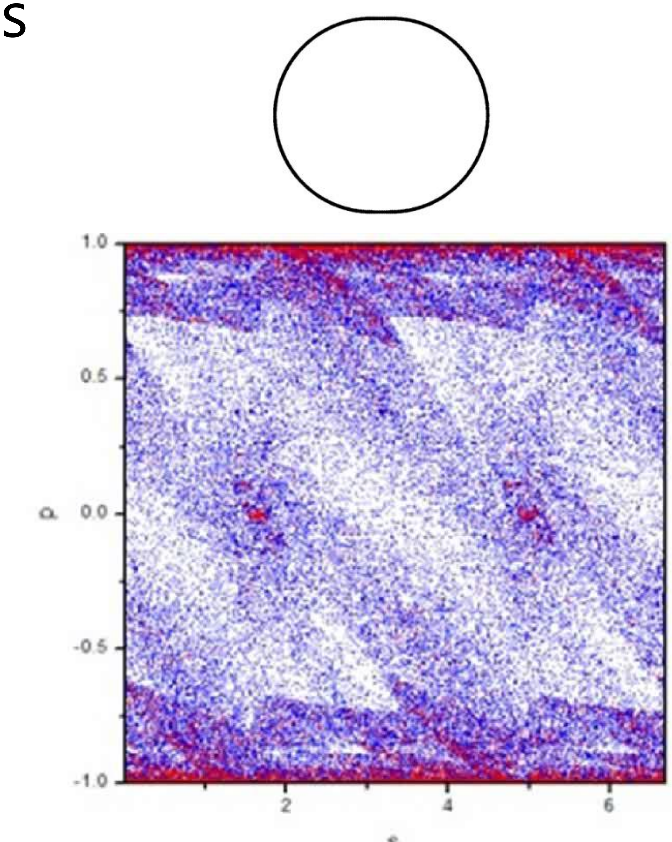
+

openness

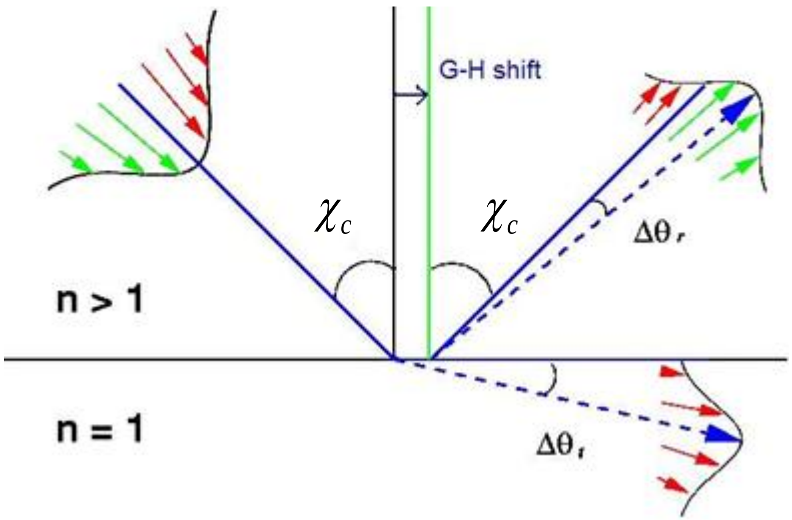


= Steady Prob. Distrib.



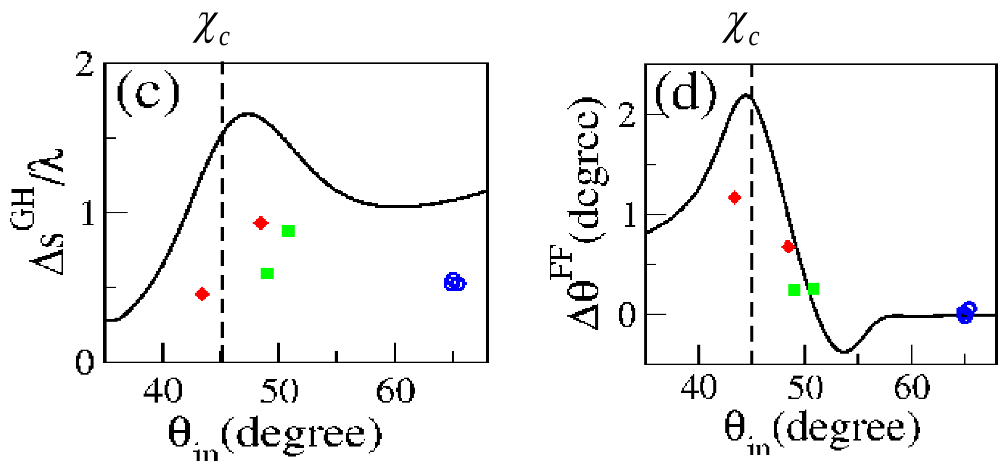


Goos-Hanchen shift and Fresnel filtering effect



Tureci & Stone, Opt. Lett. 27, 7 (2002);
Rex et al. Phys. Rev. Lett. 88, 94102 (2002)

Lai et al. J. Opt. Soc. Am. A, 3, 550 (1996)
Goos-Hanchen lateral shift around the critical angle



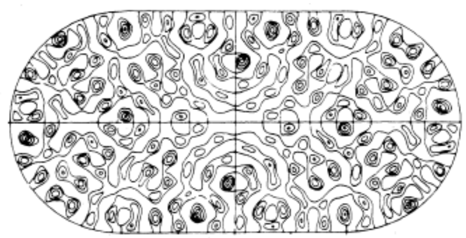
Maximum openness effects near the critical angle

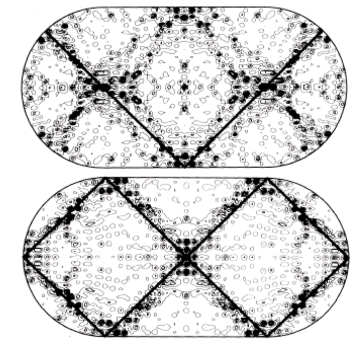
Solid line: Gaussian incident beam at planar interface

Scarred optical modes

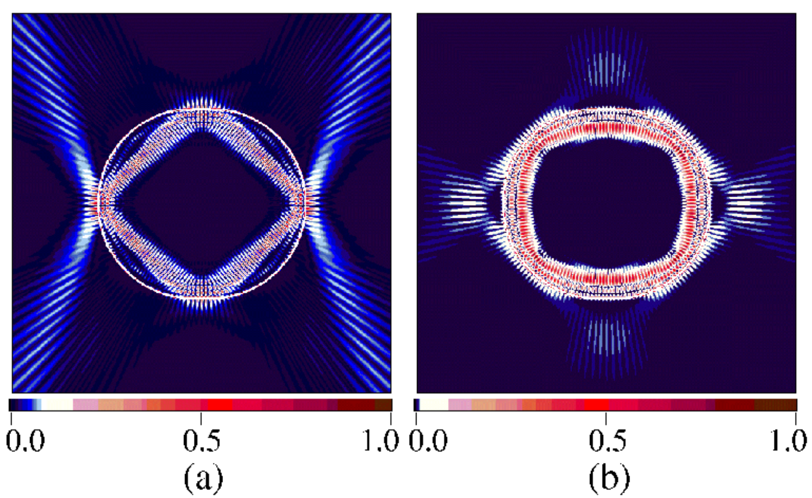
Scarred eigenfunctions in billiards

Heller, Phys. Rev. Lett. 53, 1515 (1984).





Scarred resonances in a stadium-shaped microcavity

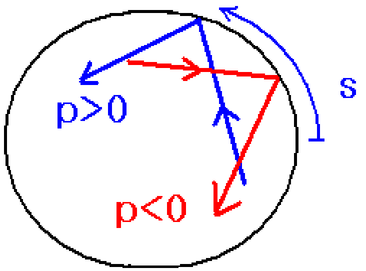


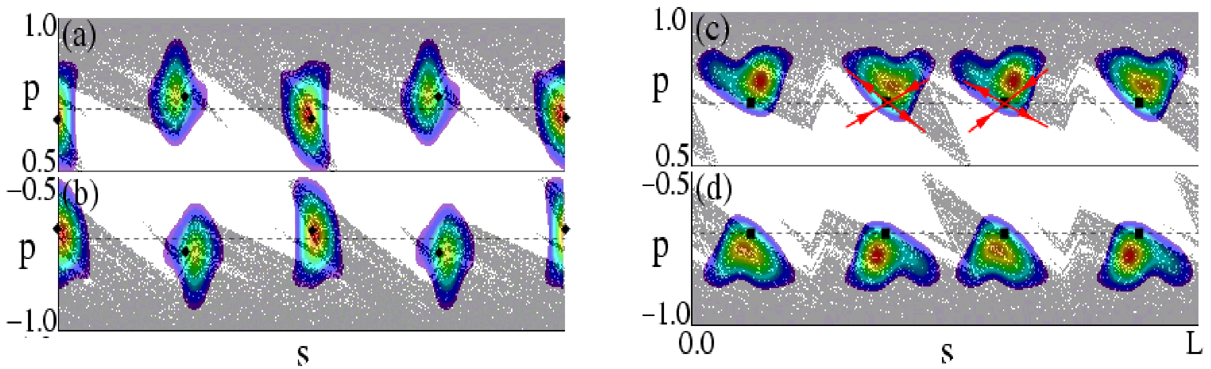
Lee et al., PRA 72, 061801(R) (2005)

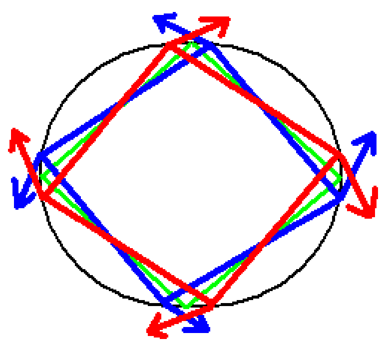
Scarred optical modes

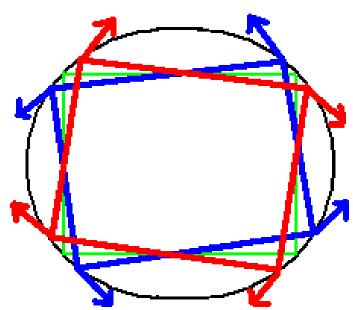
Incident Husimi functions for scarred resonances

Hentschel et al., Europhys. Lett. 62 (2003)



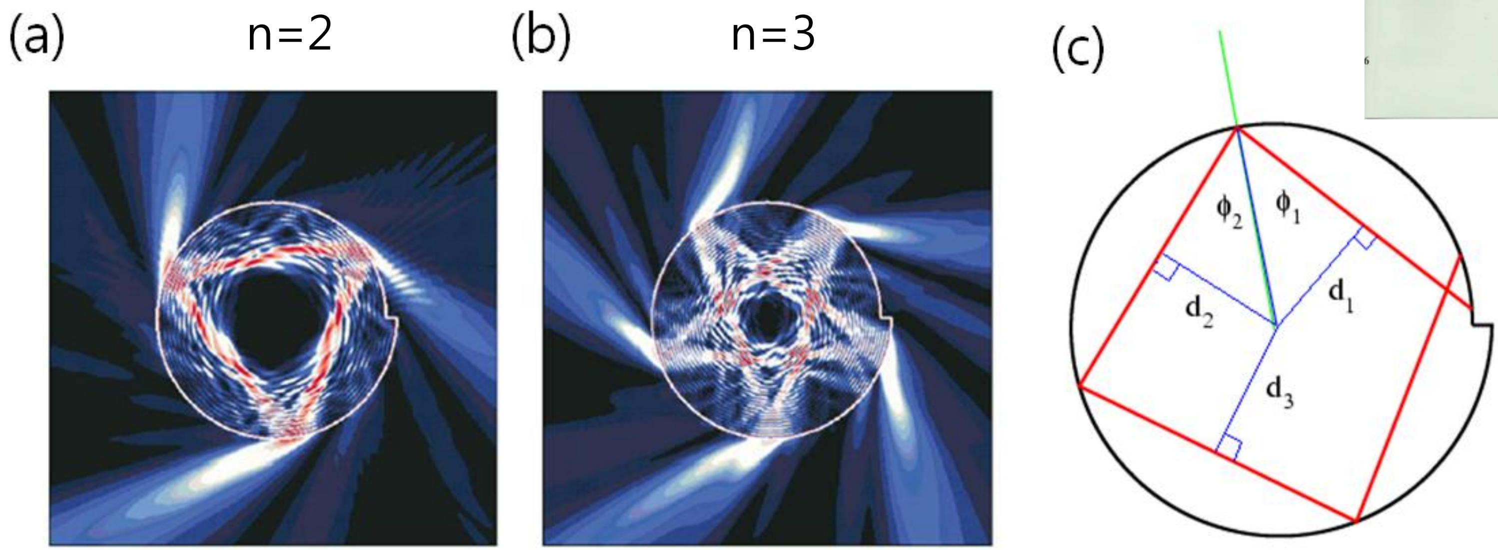






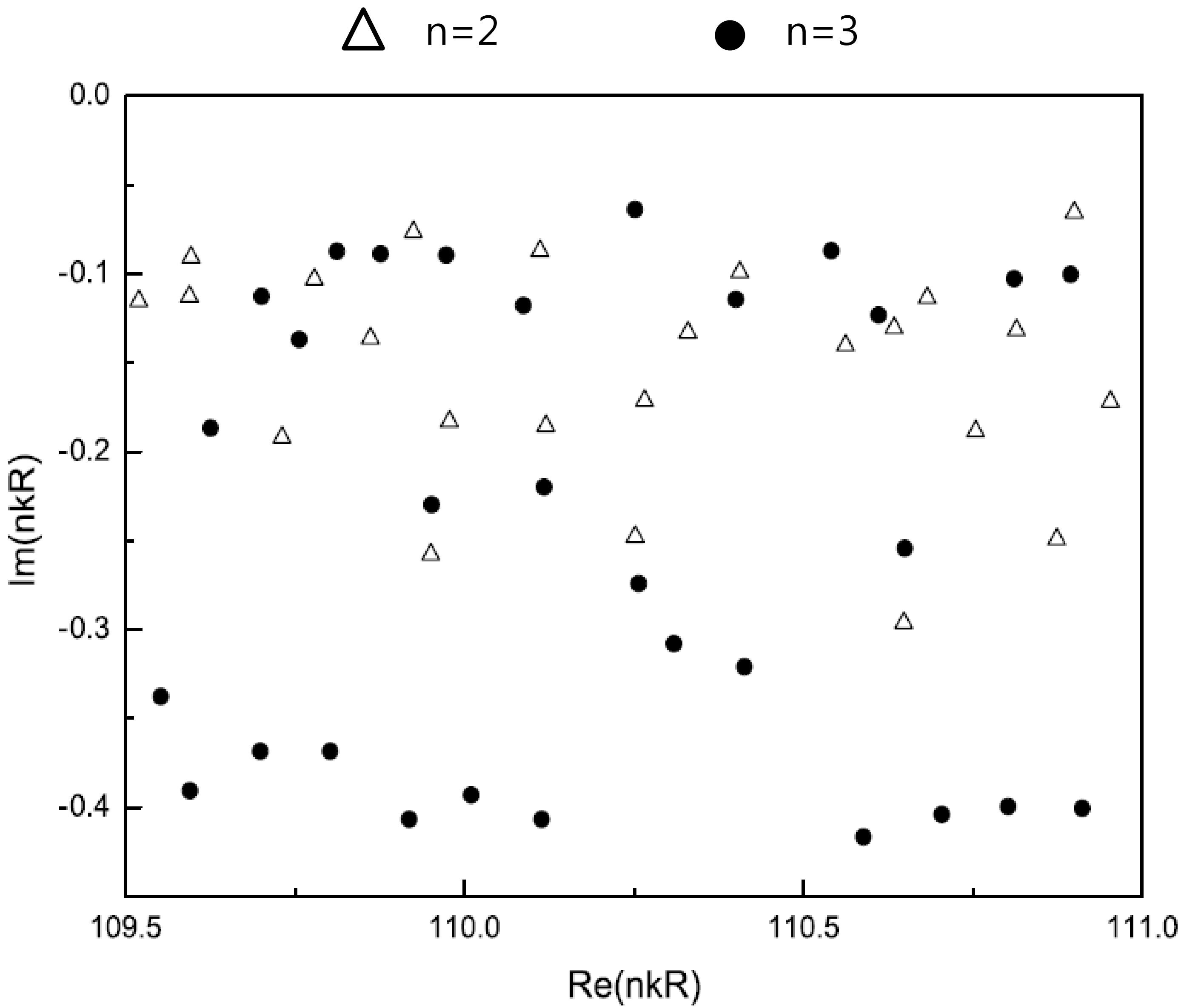
Quasiscarred optical modes

Lee et al., PRL 93, 164102 (2004)



No corresponding periodic orbit!

Resonance modes



Quasiscarred optical modes

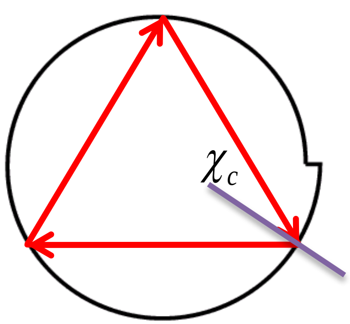
Resonance patterns (n=2)

Critical angle

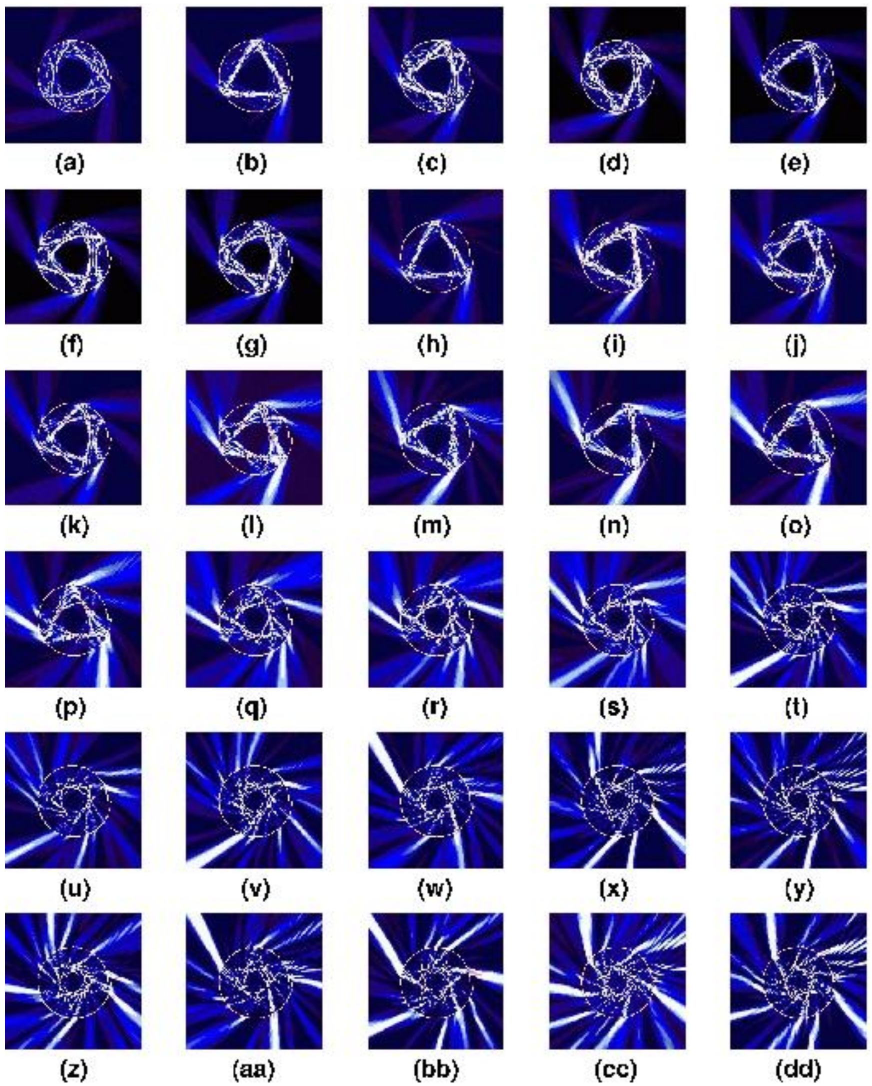
$$\sin \chi_c = 1/n$$

$$\chi_c = \frac{\pi}{6}$$

Incident angle of triangle modes



$$\chi_T \approx \chi_c$$



Quasi-scarred optical modes

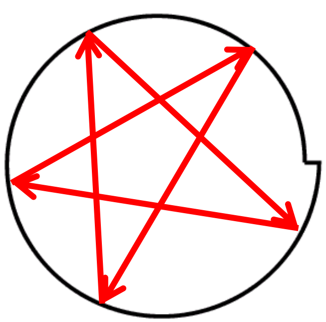
Resonance patterns (n=3)

Critical angle

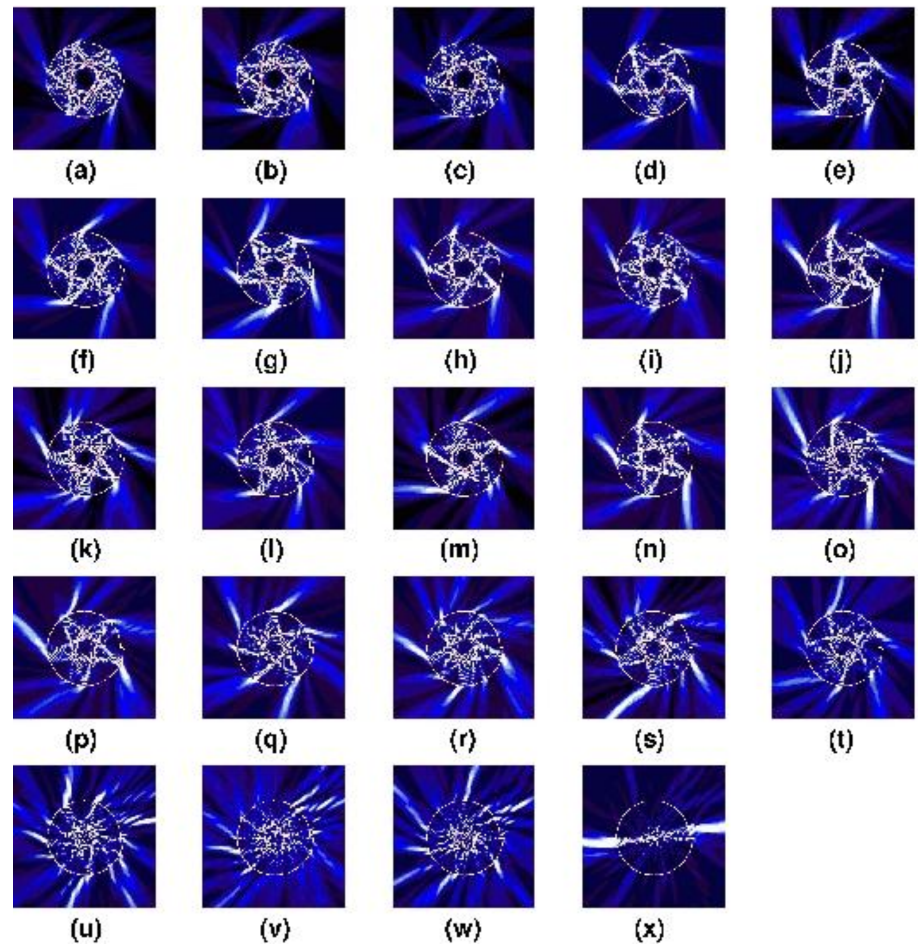
$$\sin \chi_c = 1/n$$

$$\chi_c = \arcsin(1/3) \approx 0.34$$

Incident angle of star modes

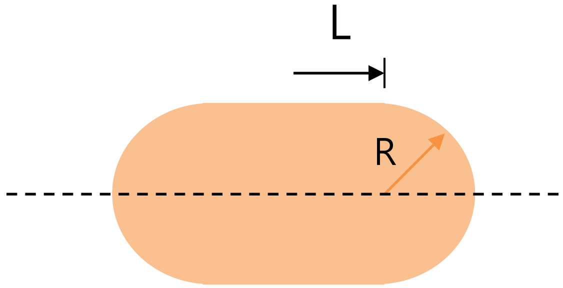


$$\chi_S \approx \frac{\pi}{10} \approx \chi_c$$

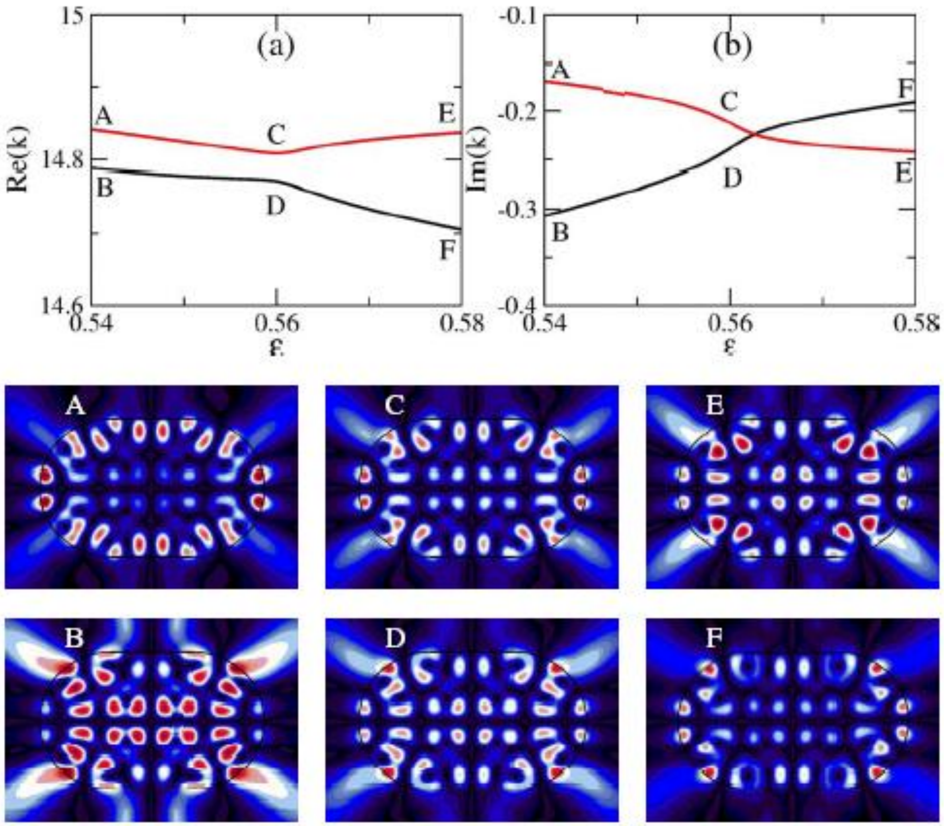


Avoided resonance crossing

Lee et al., PRA 78, 015805 (2008)



ε=L/R : deformation parameter
n : refractive index

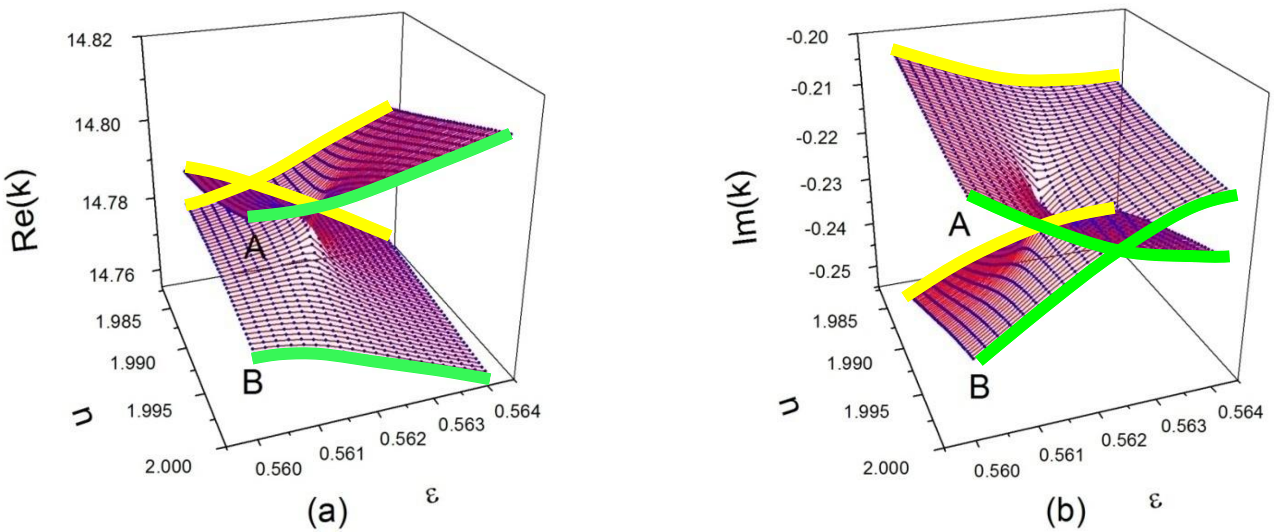


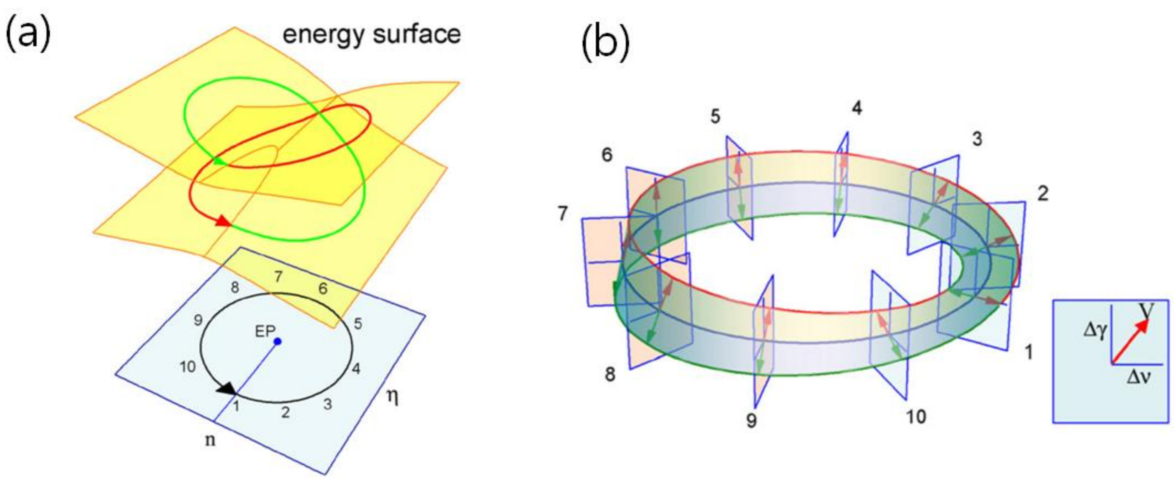
(c)

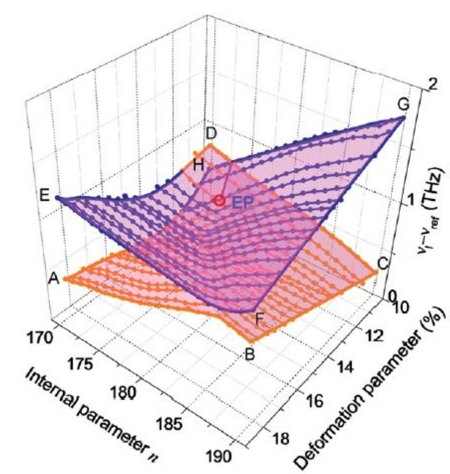
1. Pattern exchange
2. Mixed patterns at ARC

Exceptional point (EP)

Lee et al., PRA 78, 015805 (2008)

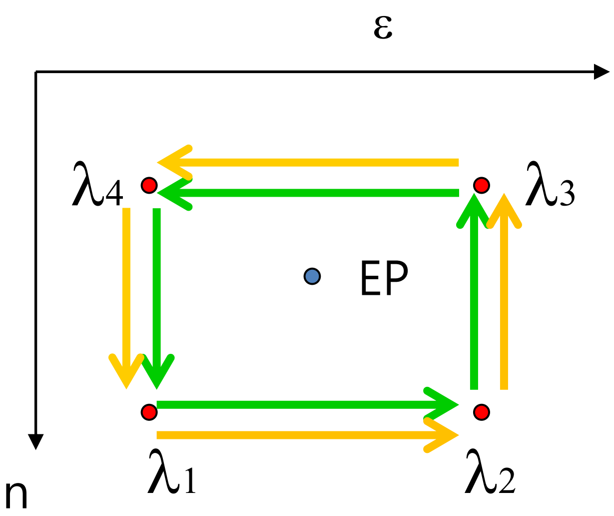


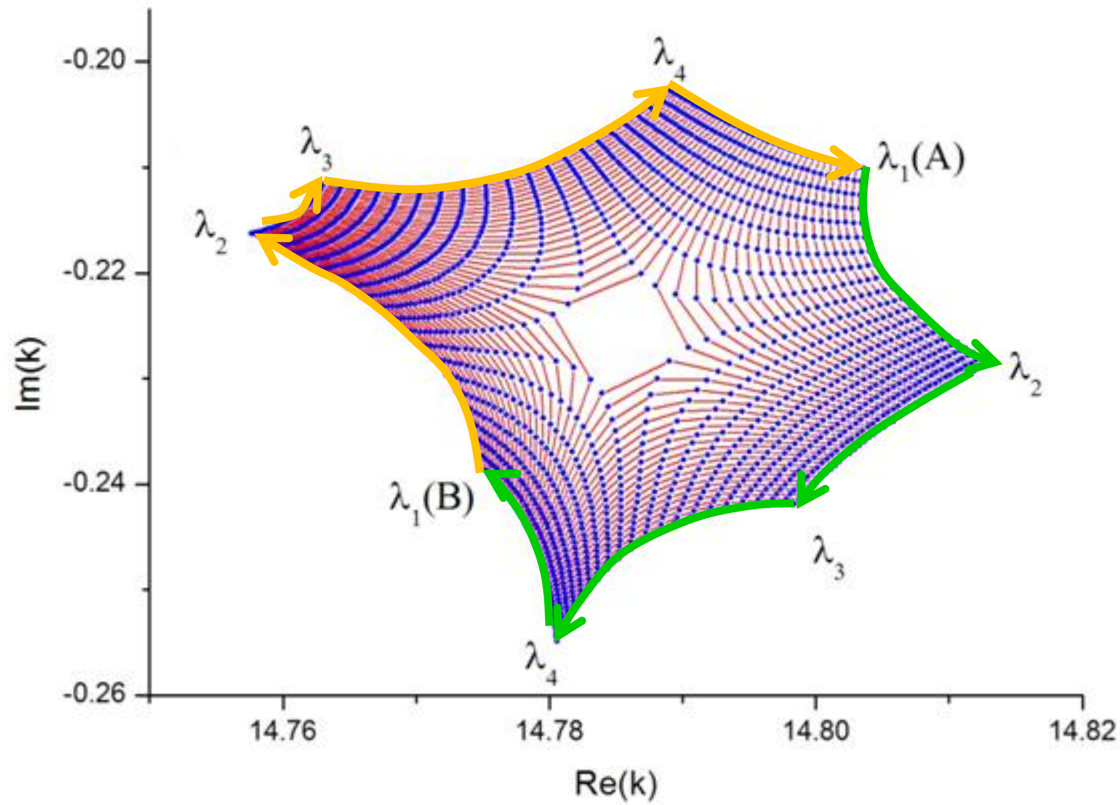




An's group, PRL 103 (2009)

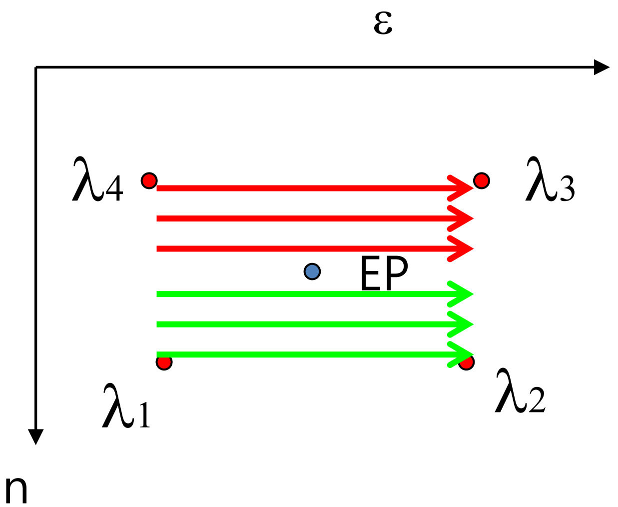
Energy traces around an EP

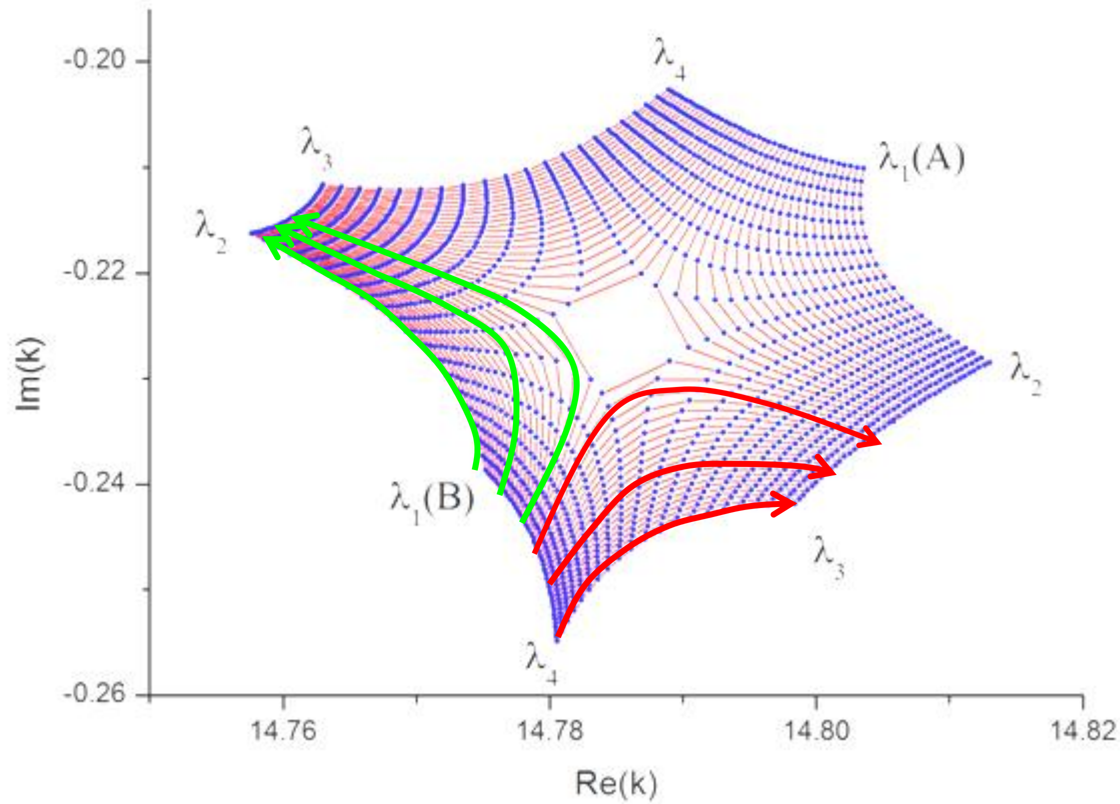




EP is a square root branch point and the two eigenvalues are the values of one analytic function on different Riemann sheets.

Branching by EP



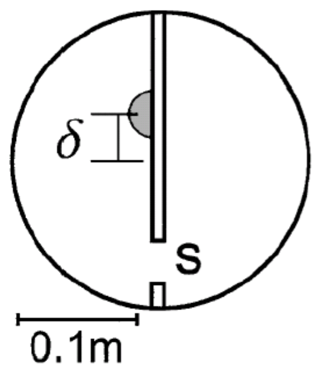


EP is a branching center at which one mode group is branching into two mode groups.

Geometrical phase near an EP

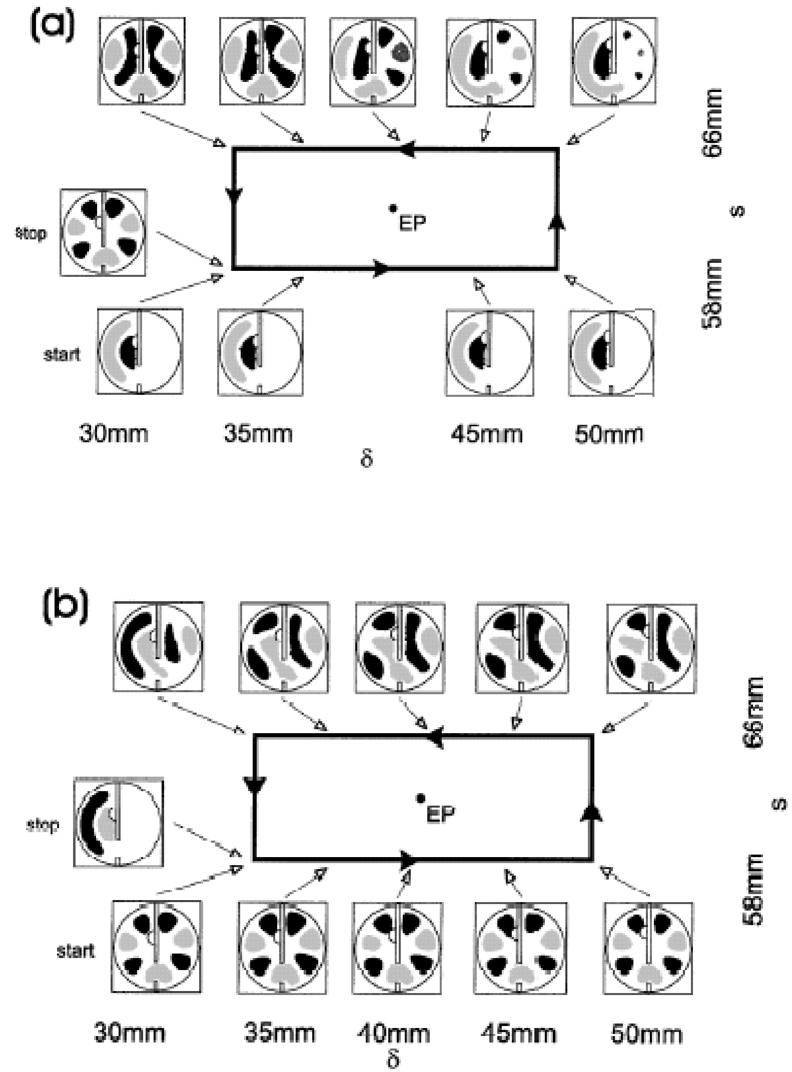
EP in microwave cavity

Dembowski et al. PRL 86 (2001)



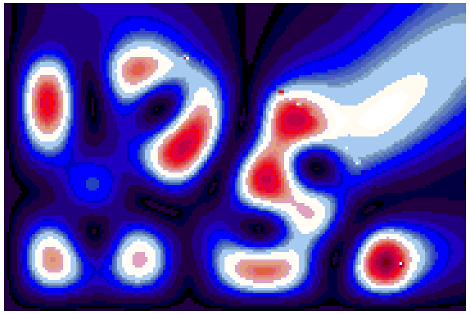
$$\psi_{Berry} = \pm\pi$$

for double cyclic loops



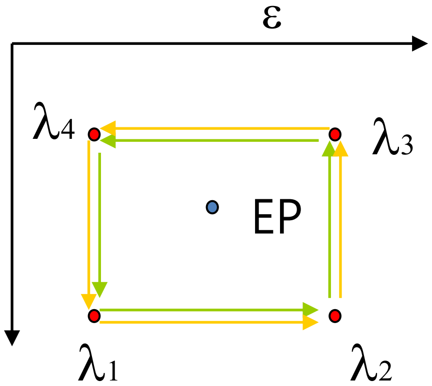
Geometrical phase

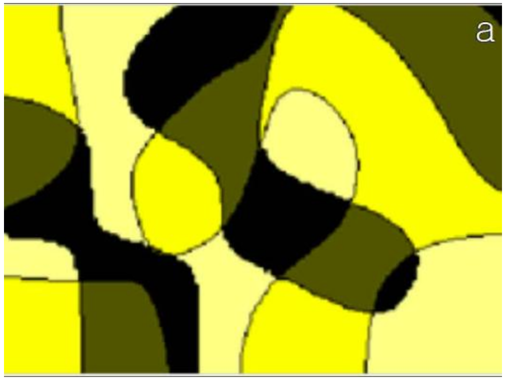
Lee , PRA 82, 064101 (2010)



$$\langle \phi^* | \phi \rangle \cong \int_D \phi^2(\mathbf{x}) d\mathbf{x} \Rightarrow real$$

$$\phi(\mathbf{x}) = r(x, y) \exp i\theta(x, y)$$





Phase plot: $\theta(x, y)$

$\theta(x, y) \pm \pi$ after a double cyclic variation

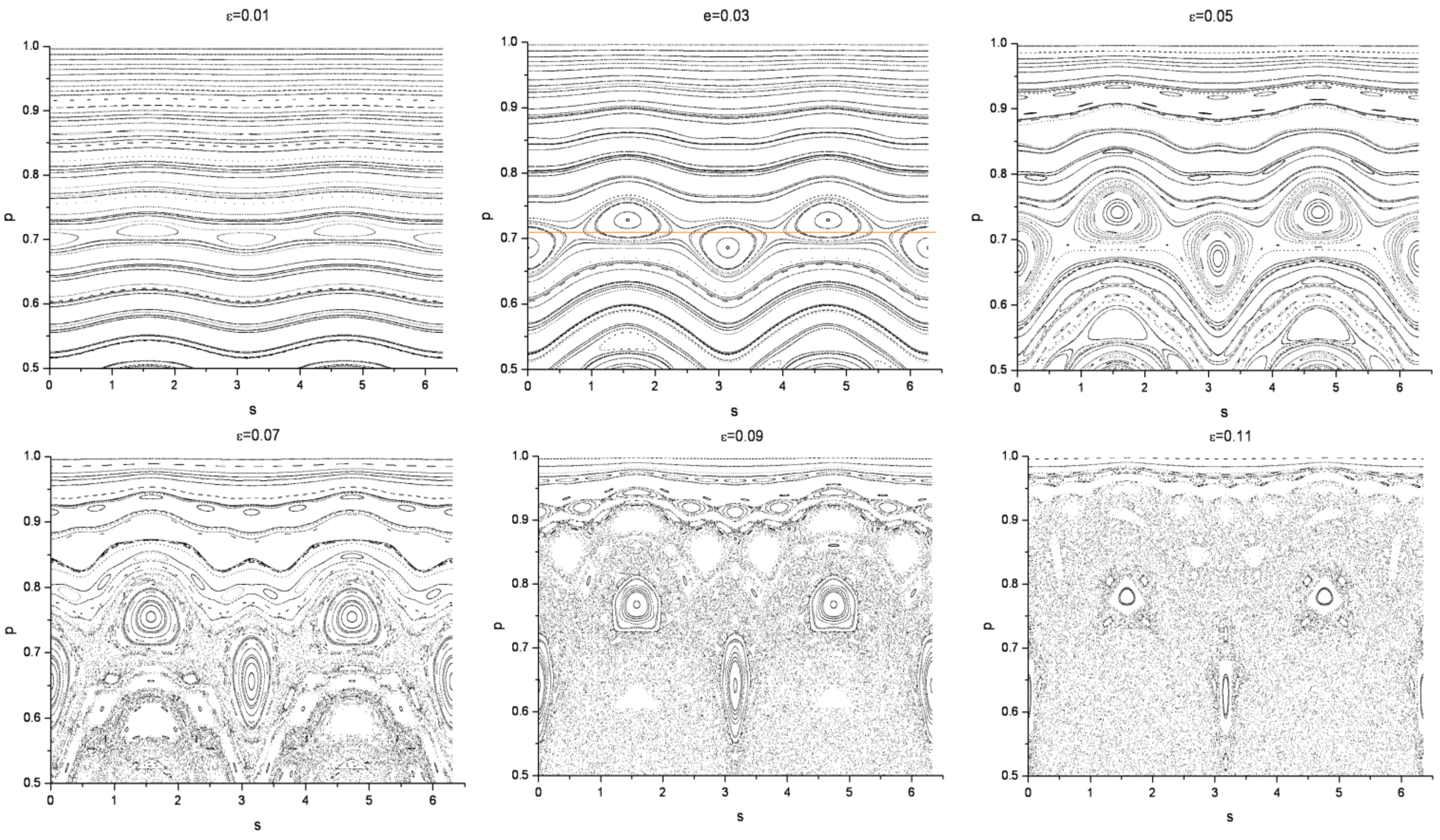
Question

Quasiscarred modes have been found in a spiral-shaped microcavity.

Is the existence of quasiscarred modes exceptional or common?

Quadrupole-deformed microcavities

$$r(\phi) = R_c\left(1 + \varepsilon\cos 2\phi\right)$$

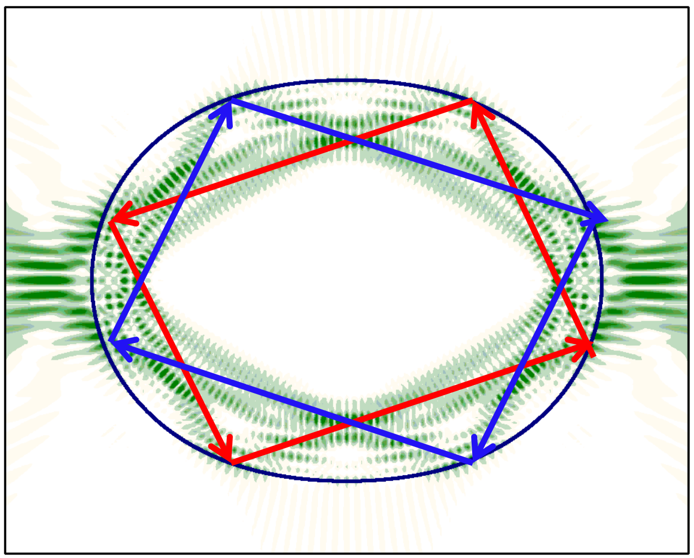


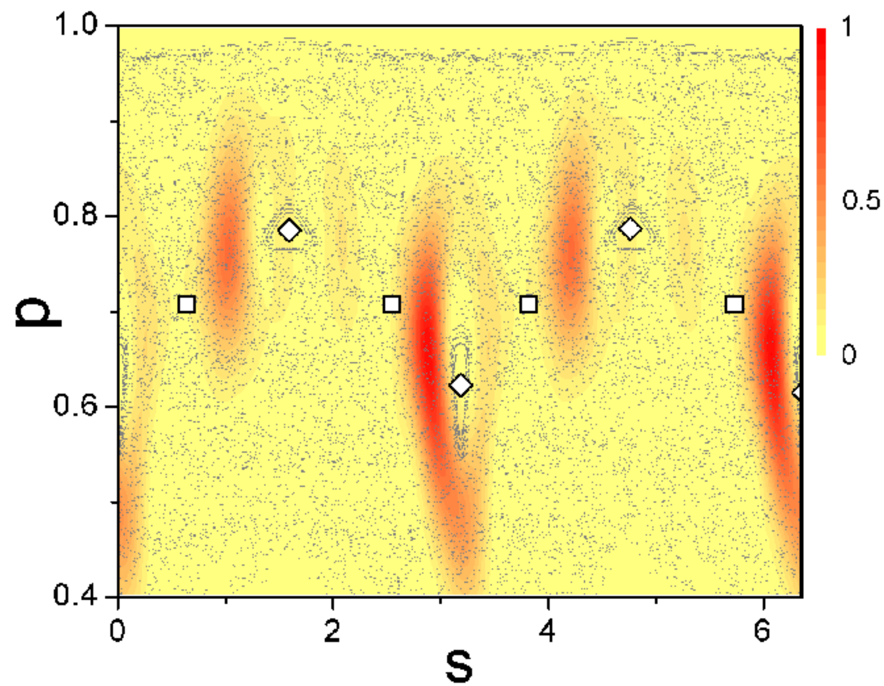
Quasiscarred mode in quadrupole microcavity

Ryu and Lee, PRA 83, 015103(R) (2011)

$$n = \sqrt{2} \qquad \text{Critical angle : } \chi_c = \frac{\pi}{4}$$

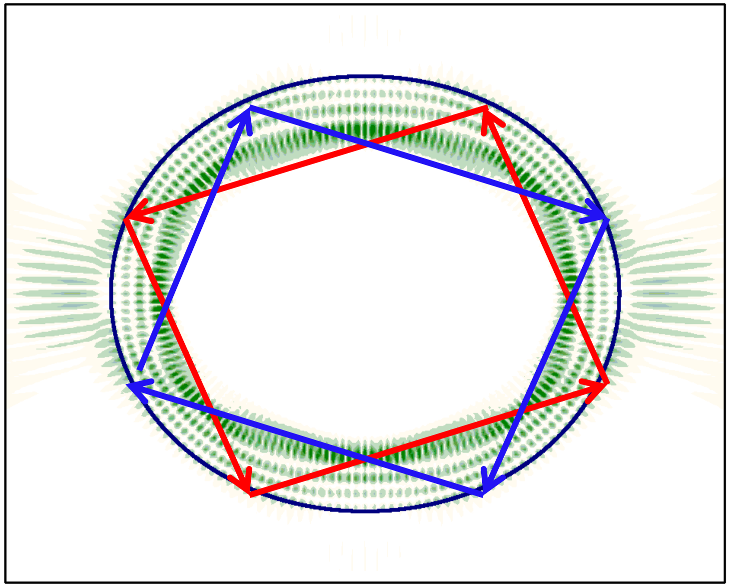
$\varepsilon = 0.12$

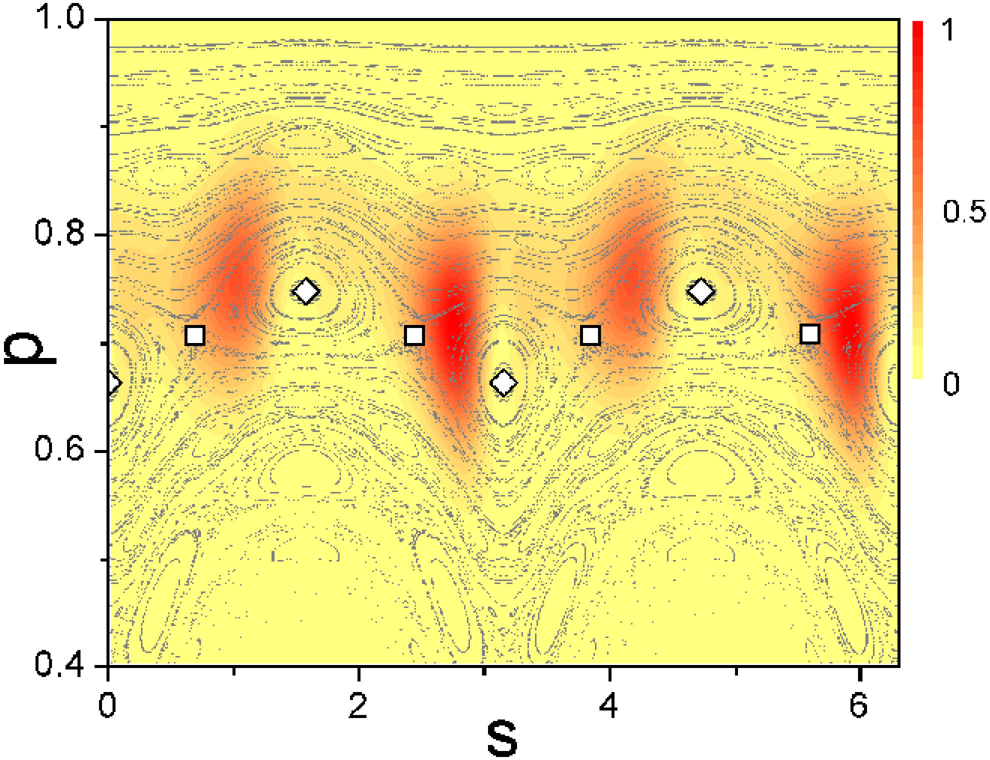




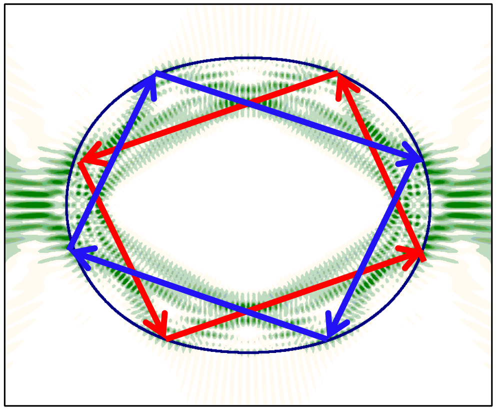
Quasiscarred mode at small deformation

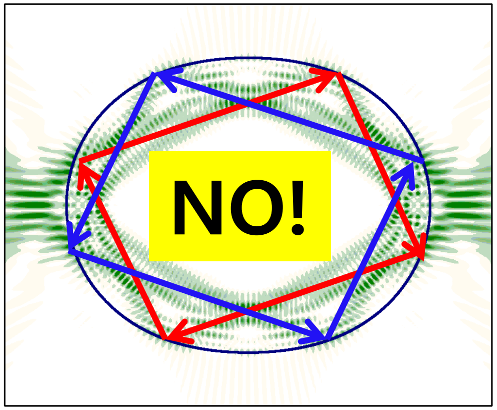
$\varepsilon = 0.06$



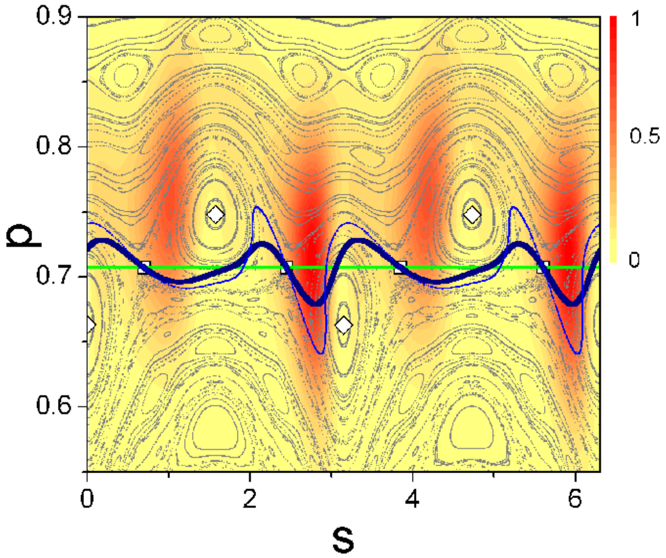


Short time ray dynamics

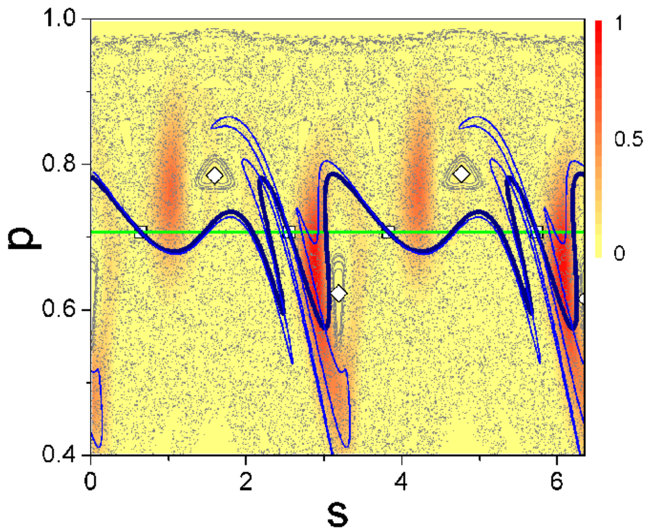




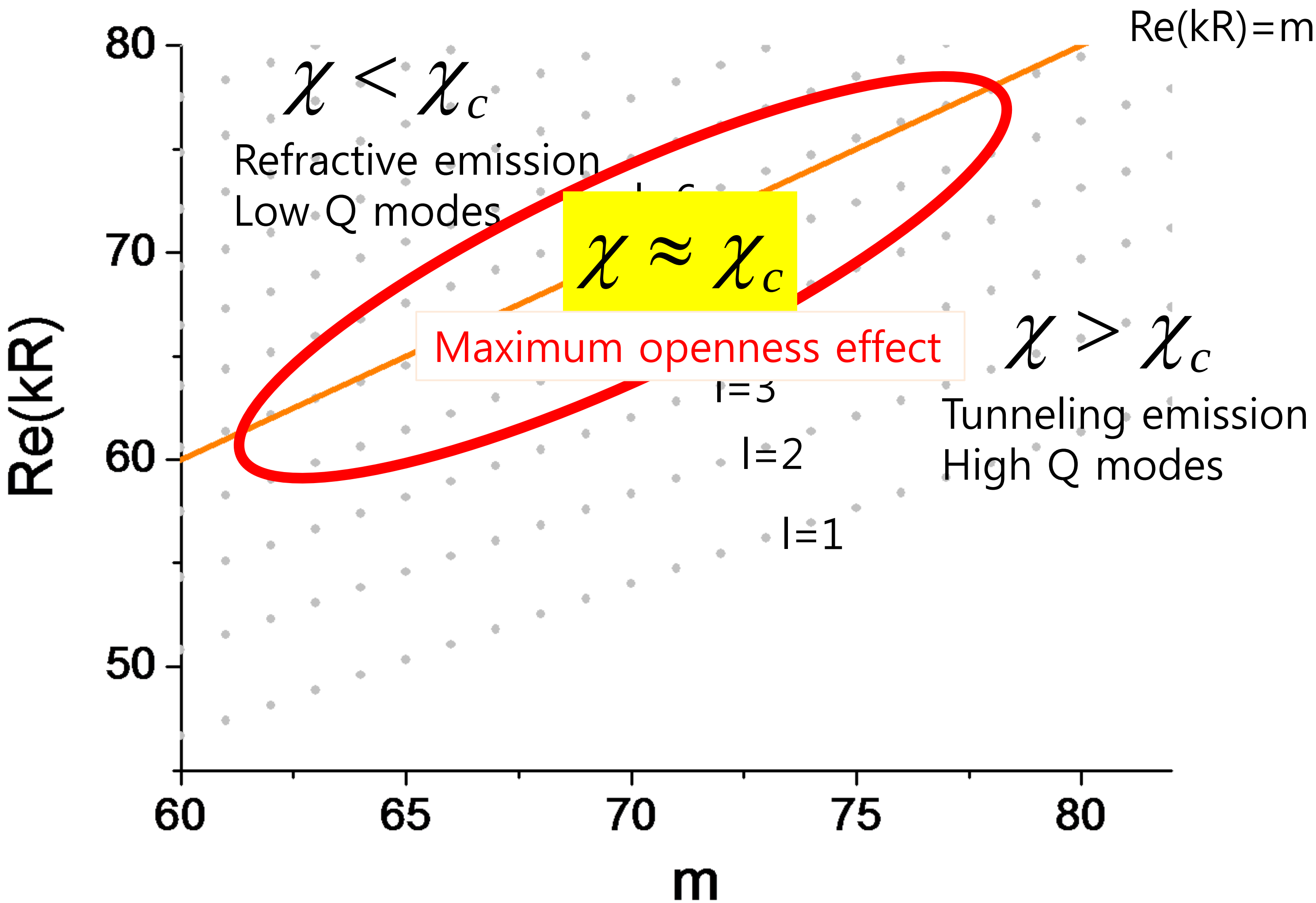
$\varepsilon = 0.06$



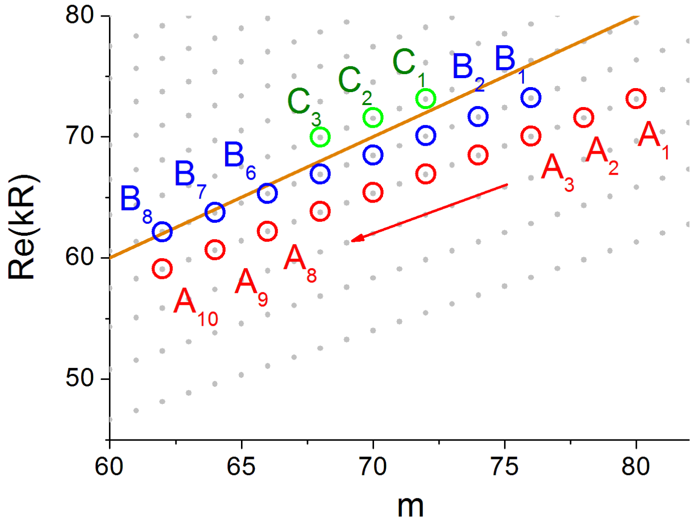
$\varepsilon = 0.12$

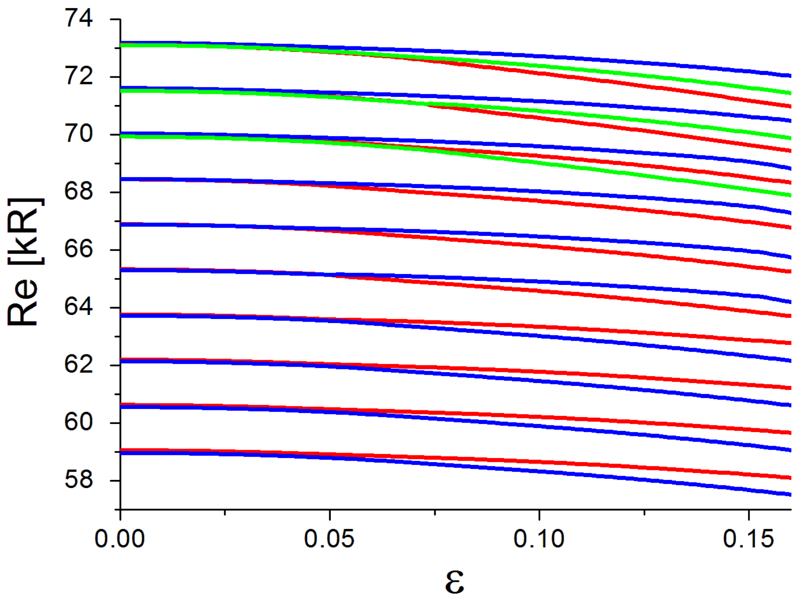


Modes of a circular microcavity



Mode evolution with deformation

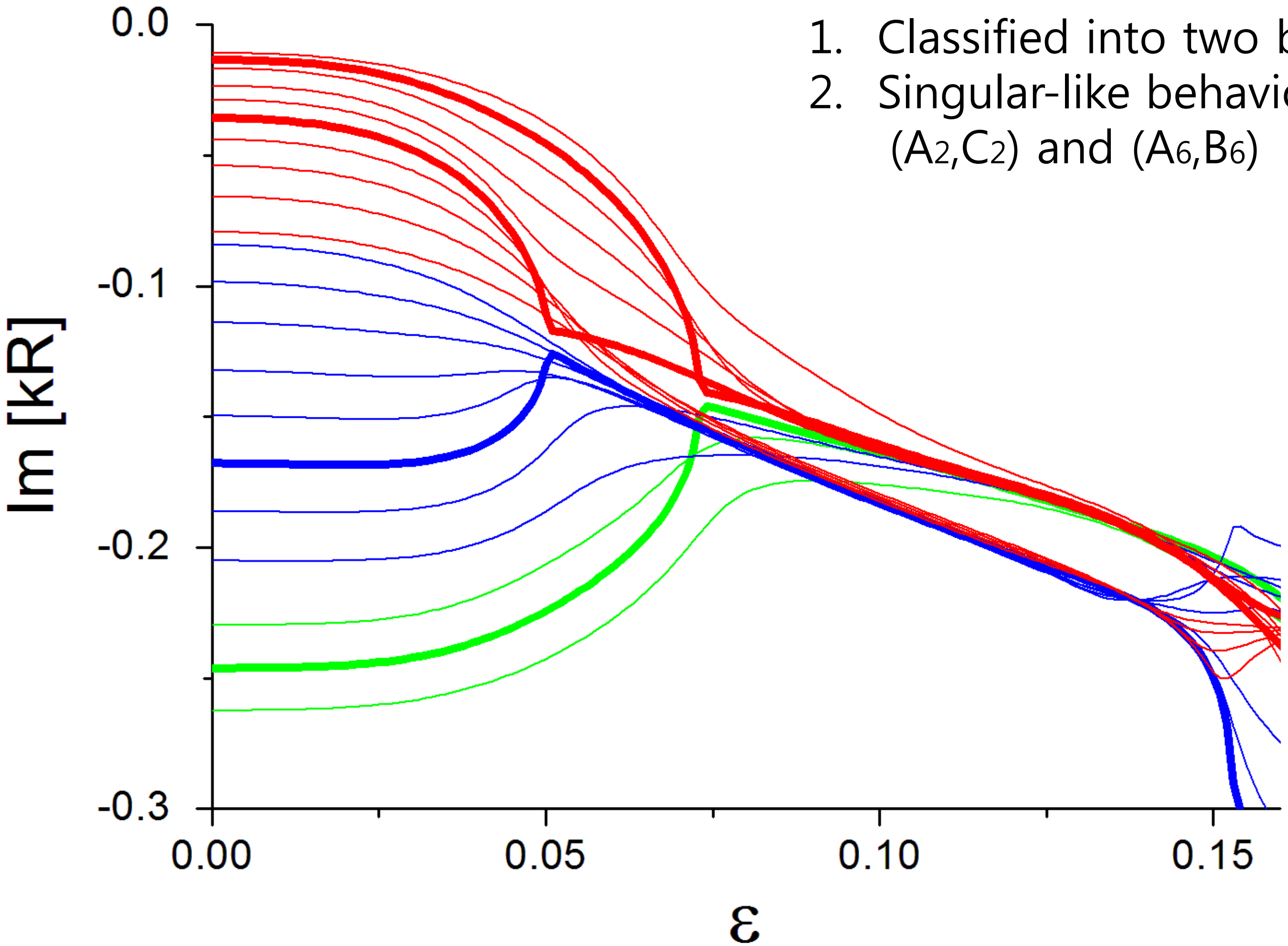




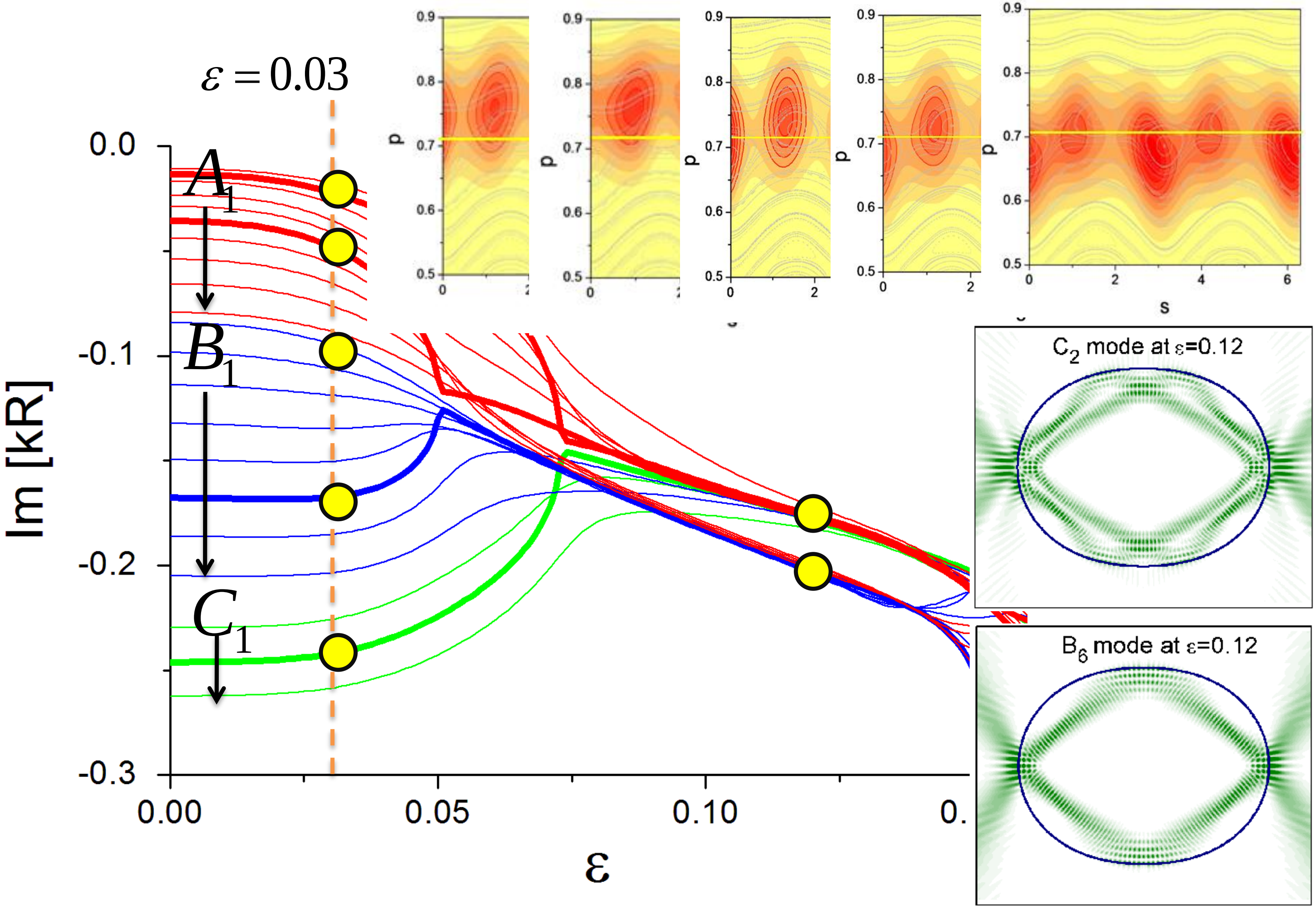
Enhanced mode-mode interaction by chaotic ray dynamics

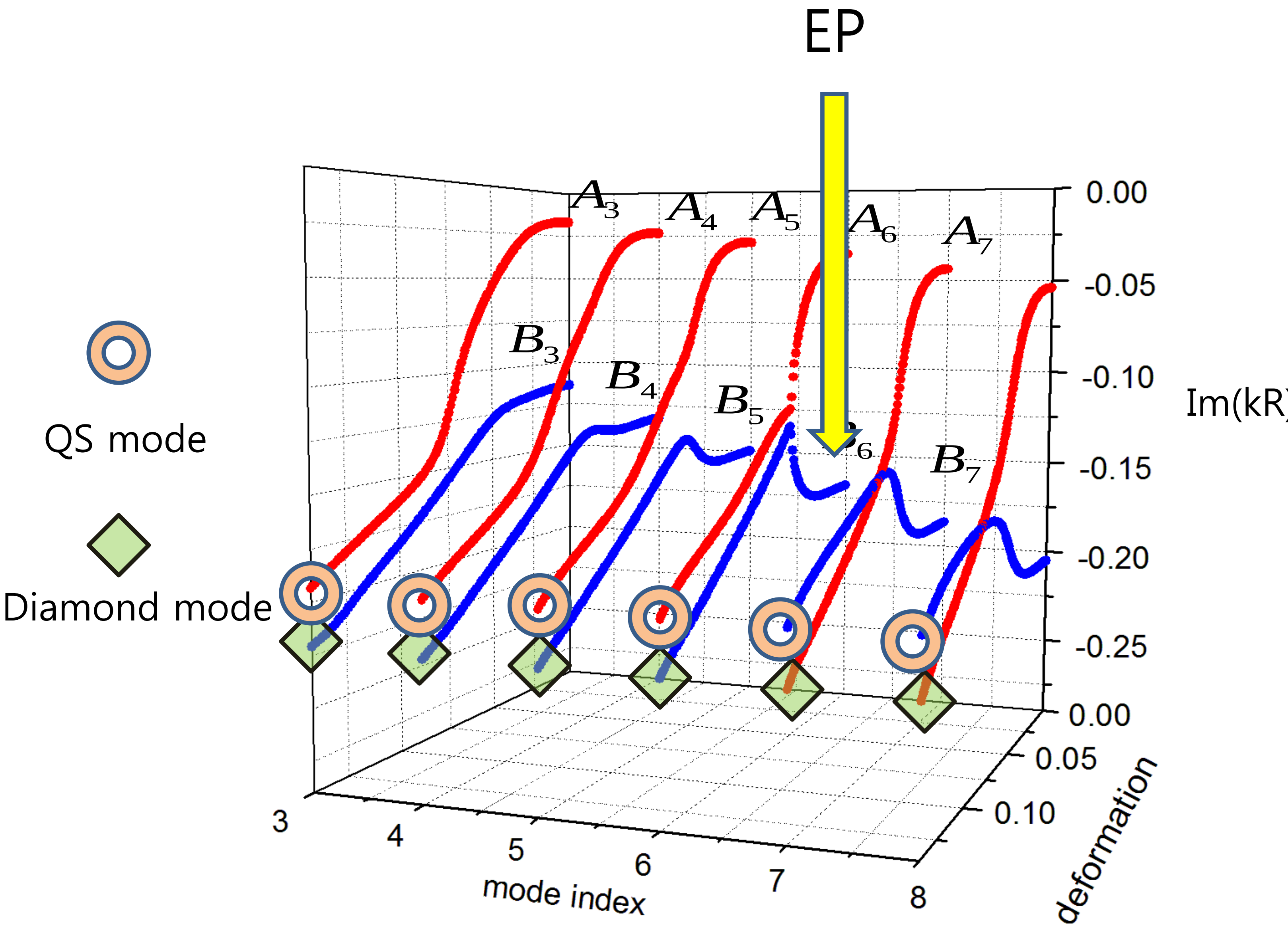
Mode evolution with deformation

1. Classified into two branches
2. Singular-like behavior of some traces (A_2,C_2) and (A_6,B_6)



Mode patterns

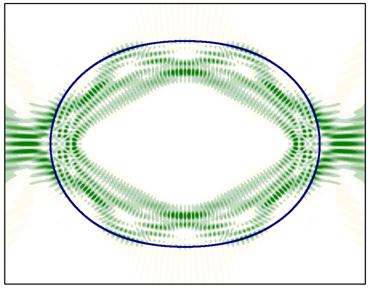




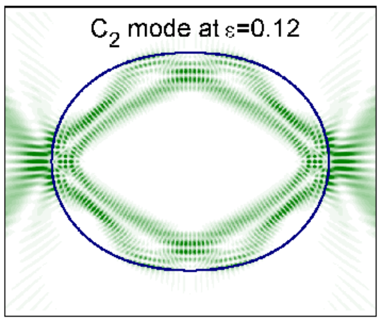
EP
QS mode
Diamond mode
A_3
A_4
A_5
A_6
A_7
B_3
B_4
B_5
B_6
B_7
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
0.00
Im(kR)
0.05
0.10
deformation
3
4
5
6
7
8
mode index

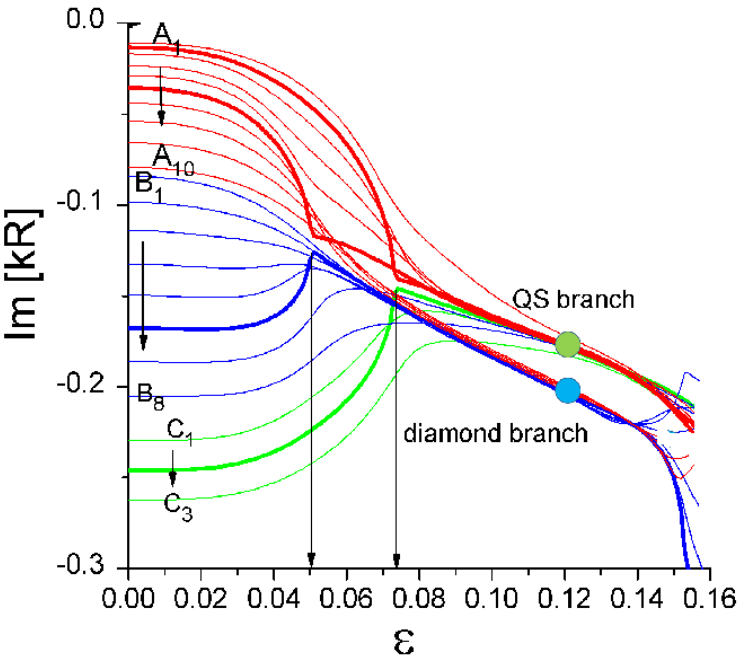
$A_2 \quad \varepsilon = 0.12$

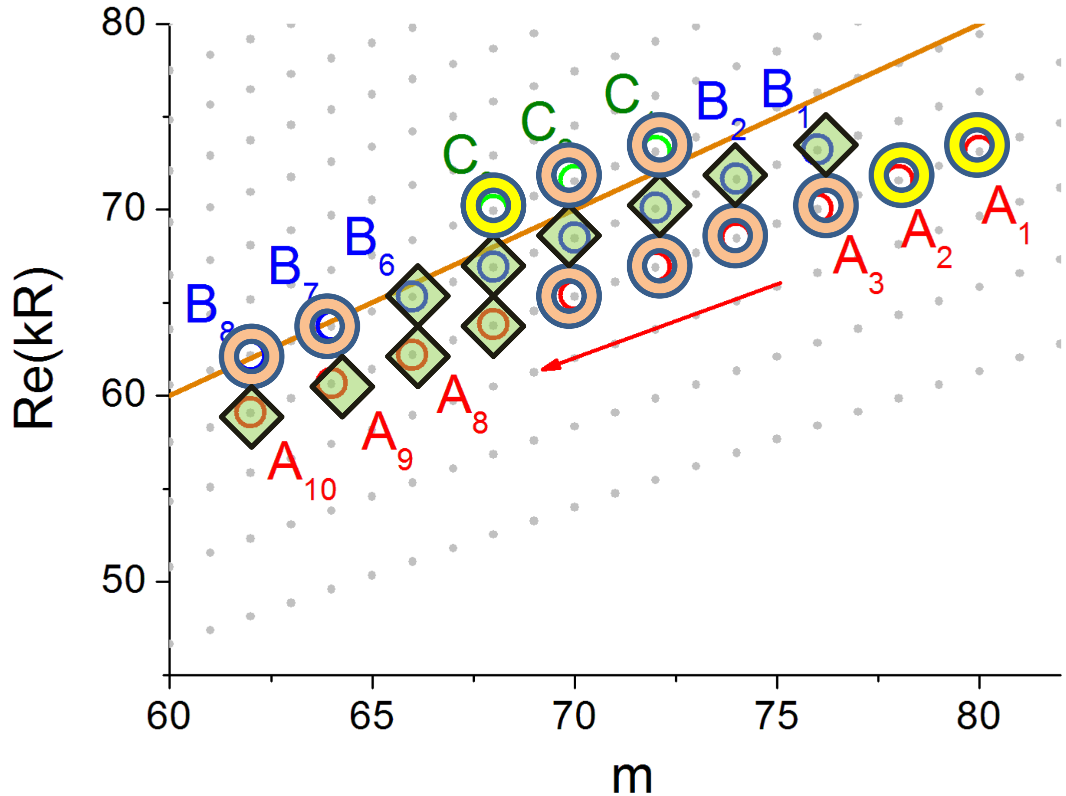
QS-I



QS-II

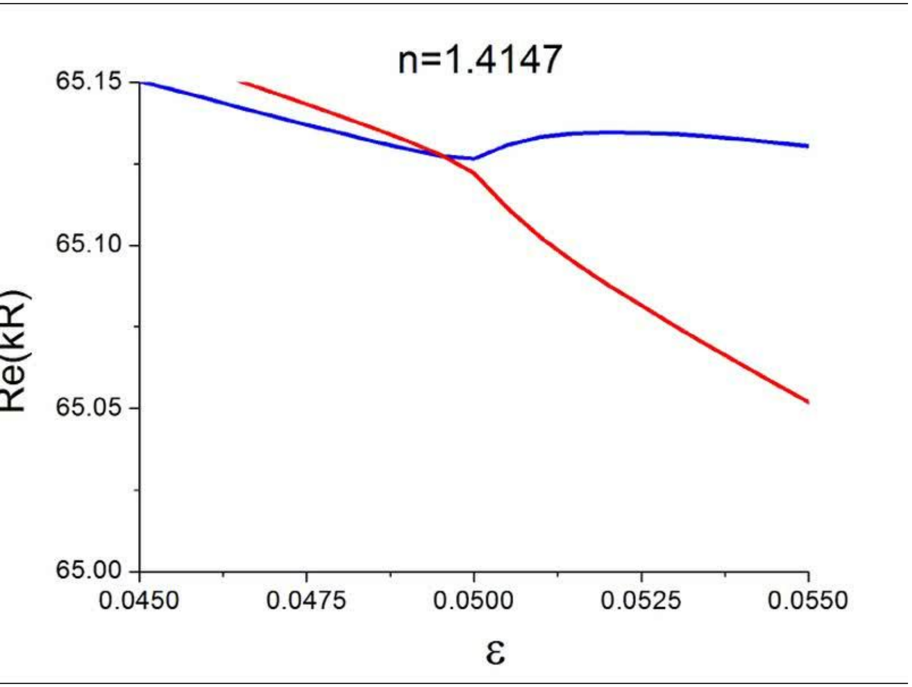


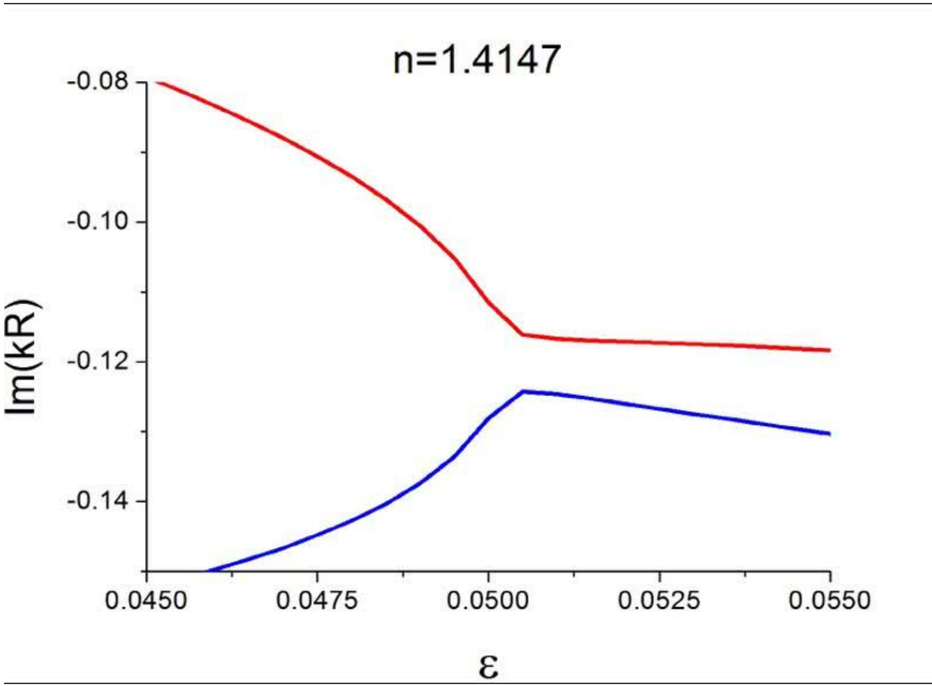




EP 's in (n,ε) parameter space

$$(n_{\mathrm{EP}}, \varepsilon_{\mathrm{EP}}) \simeq (1.4164, 0.050)$$





Question

$$\varepsilon_{EP} \approx 0.05 \quad \mathit{for}\ A_6\ \mathit{and}\ B_6$$

$$\varepsilon_{EP} \approx 0.073 \quad \mathit{for}\ A_2\ \mathit{and}\ C_2$$

Exceptional point (EP) in open system

Non-Hermitial Hamiltonian $\begin{pmatrix} \omega_1 - i\gamma_1 & \Delta \\ \Delta & \omega_2 - i\gamma_2 \end{pmatrix}$

$$E_{\pm} = \frac{\omega_1 + \omega_2}{2} - i\frac{\gamma_1 + \gamma_2}{2} \pm \sqrt{\left(\frac{\omega_1 - \omega_2}{2} - i\frac{\gamma_1 - \gamma_2}{2}\right)^2 + \Delta^2}$$

At $\omega_0 = \omega_1 = \omega_2$ $\quad E_{\pm} = \omega_0 - i\frac{\gamma_1 + \gamma_2}{2} \pm \sqrt{\Delta^2 - \left(\frac{\gamma_1 - \gamma_2}{2}\right)^2}$

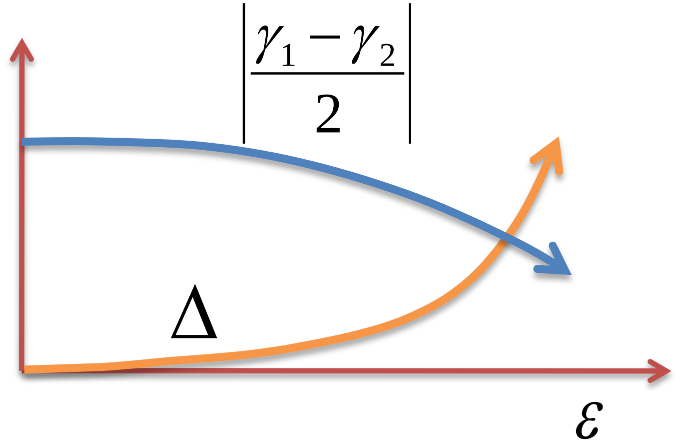
$\Delta > \left|\frac{\gamma_1 - \gamma_2}{2}\right|$: Avoided resonance crossing

$\Delta < \left|\frac{\gamma_1 - \gamma_2}{2}\right|$: Resonance crossing

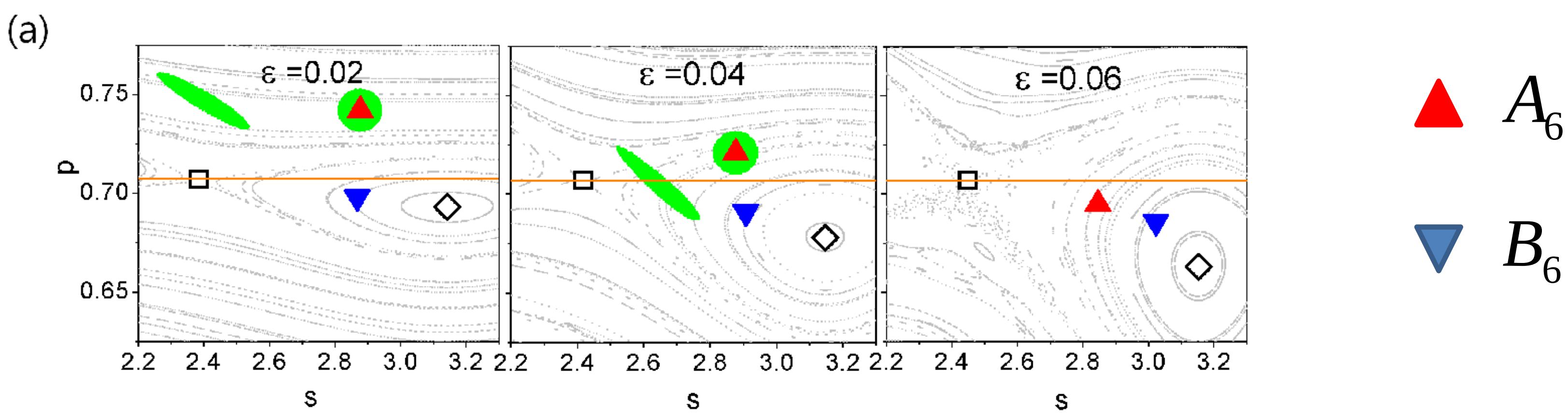
$\Delta = \left|\frac{\gamma_1 - \gamma_2}{2}\right|$: **Exceptional point**
(One egienvalue and one eigenfunction)

Position of EP

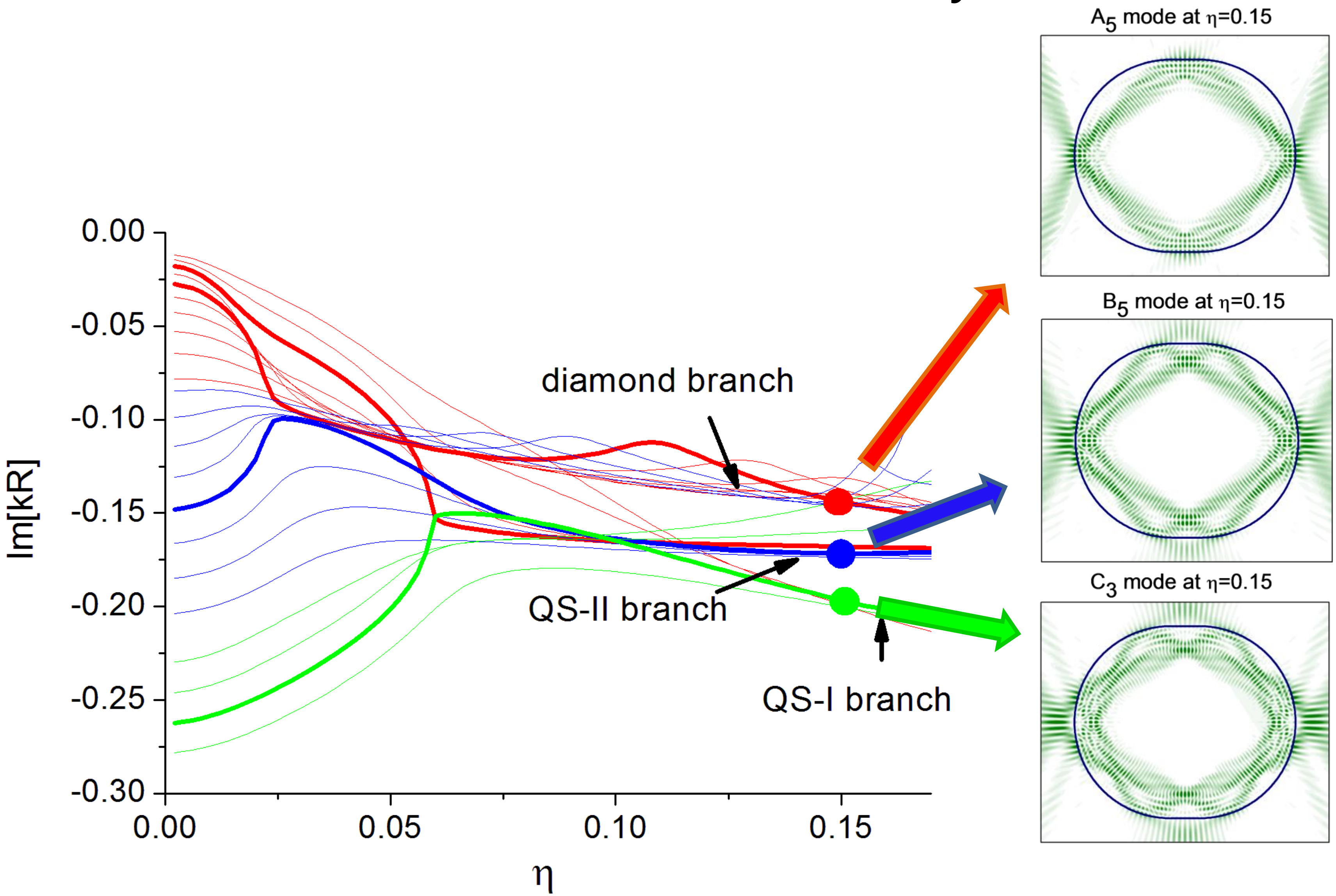
$$\Delta = \left| \frac{\gamma_1 - \gamma_2}{2} \right| \quad \text{at EP}$$



When the coupling strength increases rapidly,
we can expect EP.



Stadium microcavity



Summary

1. We show that quasiscarred modes exist in typically deformed microcavities.

2. The mode pattern of quasiscarred modes can be understood by short time ray dynamics near the critical line.

3. As deformation increases, quasiscarred modes at a low deformation show a branching behavior into robust branch modes at an exceptional point.