Spectral singularity in non-Hermitian Calogero model without confining interactions

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Banaras Hindu University

Nurturing Excellence -2011

- Largest Campus University In Asia
- 135 departments, 20000 students, 2000 faculty members 2500 PhD students all in same campus!!
- Campus area (16 square km) = $\frac{1}{20}$ area of Dresden city

Varanasi City

- one of the oldest city, and of its own character!!
- You are all welcome to visit our university

Collaborators

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- B. Basu-Mallick, (SINP)

Introduction

- Fully consistent, unitary quantum theory with complete set of orthonormal eigenstates is possible for non-Hermitian systems with unbroken PT symmetry if we modify the inner product rule appropriately.
- Obstacles to such formulations are the presence of exceptional points (EP) and spectral singularities (SS) in certain non-Hermitian systems.
- EP is a singularity of non-Hermitian Hamiltonian where eigenvalues and eigenfunctions for the bound states coalesce, whereas SS is characteristic feature of non-Hermitian Hamiltonian possessing continuous spectrum.

Introduction

- There are many signatures to find these singularities
- At the spectral singular point of complex scattering potential the reflection and transmission coefficients trend to diverge.
- Possible presence of a spectral singular point in non-Hermitian Hamiltonian translates in a pronounced resonance in scattering cross section.
- The two eigenfunctions becomes linearly dependent i.e. wronskian of these two eigenfunction vanishes

Plan of the Talk

- Introduction: Spectral Singularity.
- Spectral points in single particle systems.
- Non-Hermitian Calogero-Model with confining potential.
- Non-Hermitian Calogero-Model without confining potential.
- Spectral Singularities in Non-Hermitian Calogero-Model without confining potential.
- Conclusions/Discussions.

Single Particle Systems

$$
\begin{aligned}
H &= \frac{p^2}{2m} + V + iW \\
&= \frac{p^2}{2m} - V_0\theta(\frac{a}{2} - |x|) + i\lambda[\delta(x - \frac{a}{2}) - \delta(x + \frac{a}{2})].
\end{aligned}
$$

Note that the Hamiltonian

- $[H, P] \neq 0 \neq [H, T]$ but PT invariant, $[H, PT] = 0$.
- Obviously it is Parity-Pseudo Hermitian

$$H^\dagger = PHP^{-1}$$

Single Particle Systems

- To discuss the transmission and reflection for such system we consider $S-$ matrix approach where asymptotic channel states $|m,k\rangle$, with $m = R, L$: $\langle x|R,k\rangle = e^{ikx}$ and $\langle x|L,k\rangle = e^{-ikx}$, are introduced where R and L stand for right moving and left moving free particle states and the wave-number $k = \sqrt{\frac{2m}{\hbar^2}E}$.
- The on-shell matrix elements of S operator is $\langle m,k|S|n,k\rangle = S_{m,n}(k)$.
- $S_{m,n}(k)$ is the probability amplitude for a state starting off in the remote past as $|n,k\rangle$, to be found, as a result of evolution through the interaction with the potential, in the state $|m,k\rangle$ in the remote future.

Single Particle Systems

- This 2×2 matrix S would have been unitary (i.e. $S^\dagger S = SS^\dagger = I)$ if the potential was real.
- S matrix elements are related to the familiar transmission (t_R and t_L) and reflection (r_R and r_L) amplitudes for right and left traveling particles:

$$\begin{pmatrix} S_{RR} & S_{RL} \\ S_{LR} & S_{LL} \end{pmatrix} = \begin{pmatrix} t_R & r_L \\ r_R & t_L \end{pmatrix}$$

- With Hermitian Hamiltonian the states evolve in a unitary manner would imply the relation $|t_R|^2 + |r_R|^2 = 1$, $|t_L|^2 + |r_L|^2 = 1$ and $t_R^* r_L + r_R^* t_L = 0$.
- 1st two are conservation of probability, 3rd one implies transmission and reflection are out of phase.

Single Particle Systems

- In the present case the system is parity pseudo-Hermitian and S-operator satisfy a Pseudo-Unitary condition $P^{-1}S^{\dagger}PS = I$, In the basis of S-Matrix.

$$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- Pseudo- Unitary condition of S-matrix operator implies that $t_L^* t_R + r_L^* r_R = 1, \;\; r_L^* t_L + t_L^* r_L = 0$ and $r_R^* t_R + t_R^* r_R = 0$
- Reflection and Transmission for both left and right incident beams are out of phase.However no conservation of probability in non-Hermitian system.

Single Particle Systems

- Transmission amplitudes for left and right incident beams are explicitly found to be the same, and in this example is given by

$$t_R = t_L = \frac{2qke^{-ika}}{2qk\, Cos(qa) - i\, (q^2 + k^2 - \tilde{\lambda}^2)\, Sin(qa)}. \tag{1}$$

- Since the Hamiltonian is PT symmetric, the S-matrix is also PT symmetric, this leads to the fact $t_R = t_L$.

Single Particle Systems

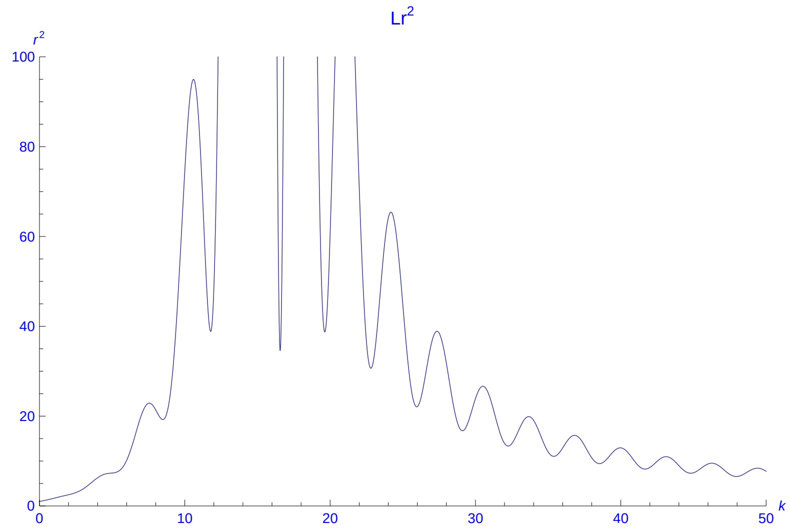
The reflection amplitudes, however, for left and right moving particles are different and are given by

(2) $$r_L = i\,\frac{[q^2-(k+\tilde{\lambda})^2]Sin(qa)\,e^{-ika}}{2qk\,Cos(qa)-i\,(q^2+k^2-\tilde{\lambda}^2)\,Sin(qa)}.$$

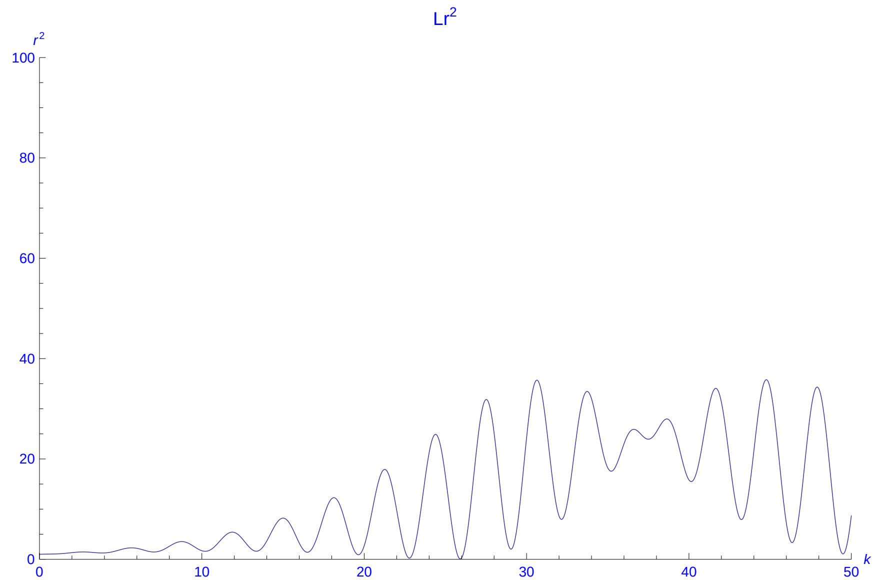
(3) $$r_R = i\,\frac{[q^2-(k-\tilde{\lambda})^2]Sin(qa)\,e^{-ika}}{2kq\,Cos(qa)-i\,(q^2+k^2-\tilde{\lambda}^2)\,Sin(qa)}.$$

Where $q^2=\frac{2m}{\hbar^2}(v_0+E)$ [Deb -Khare-Dutta Roy, PLA 2003]

Observations:SPS

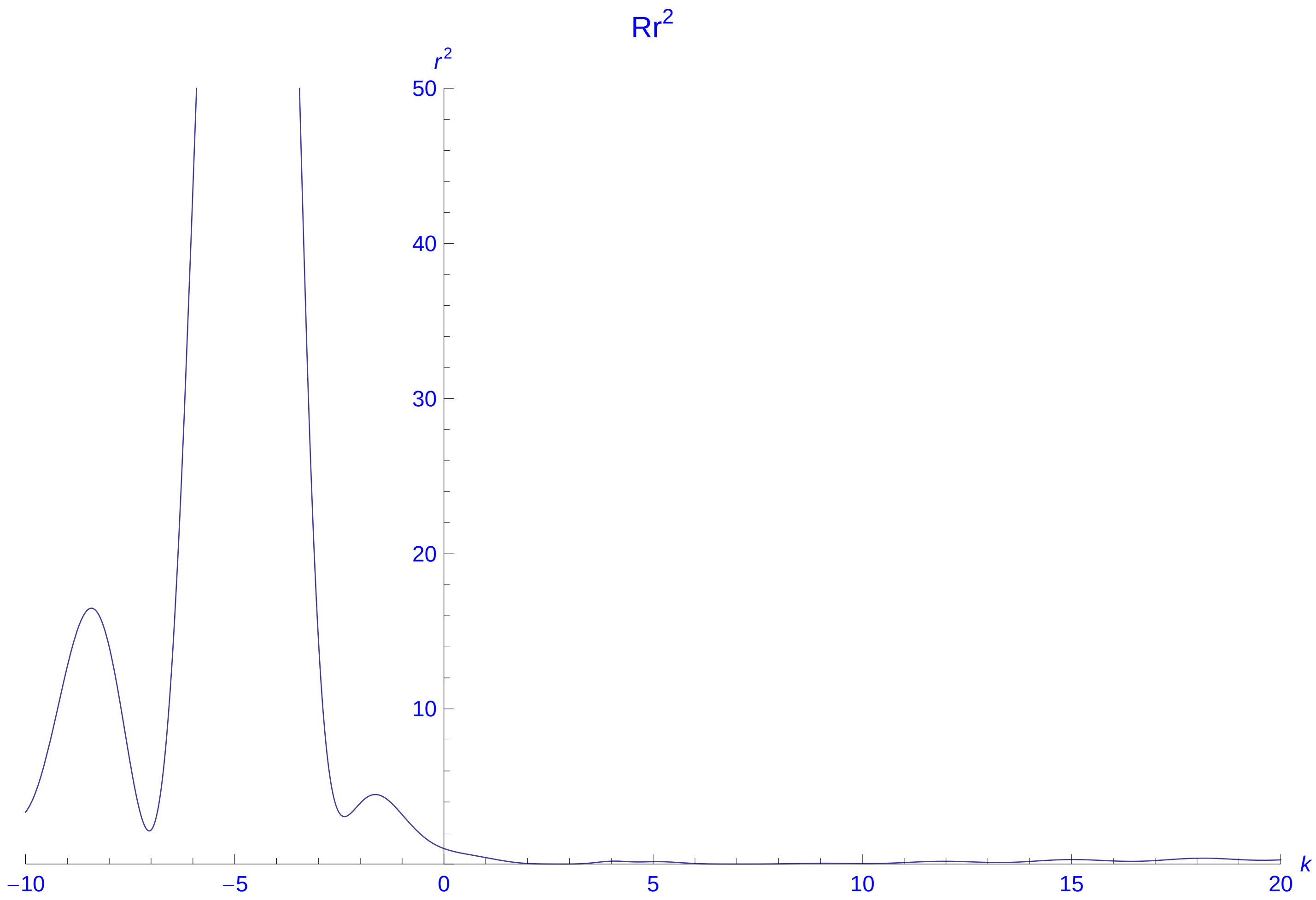


$\lambda = 34, q = 30$



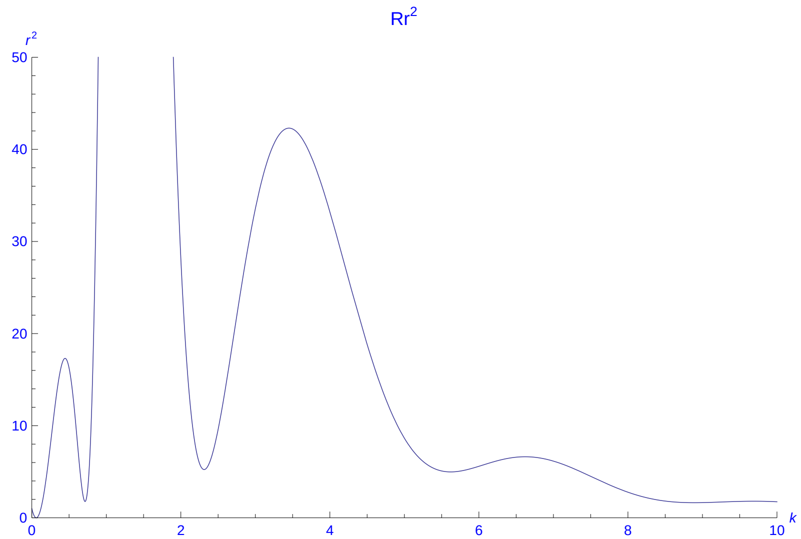
$\lambda = 39, q = 10$

Observations:SPS

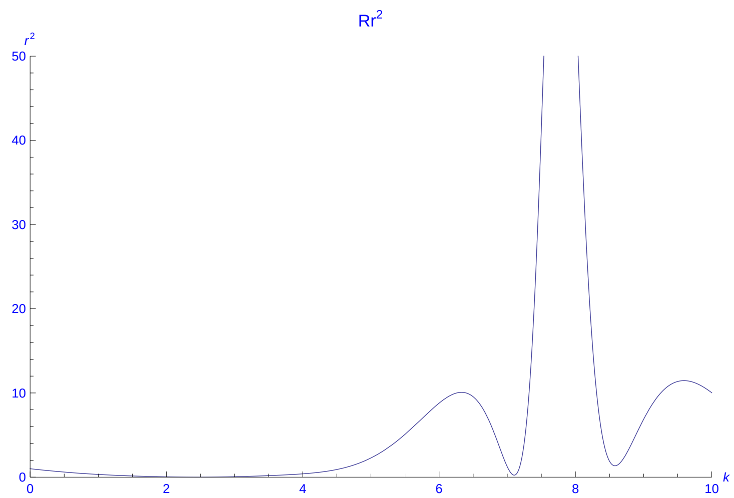


$\lambda = 5, q = 2$

Observations:SPS

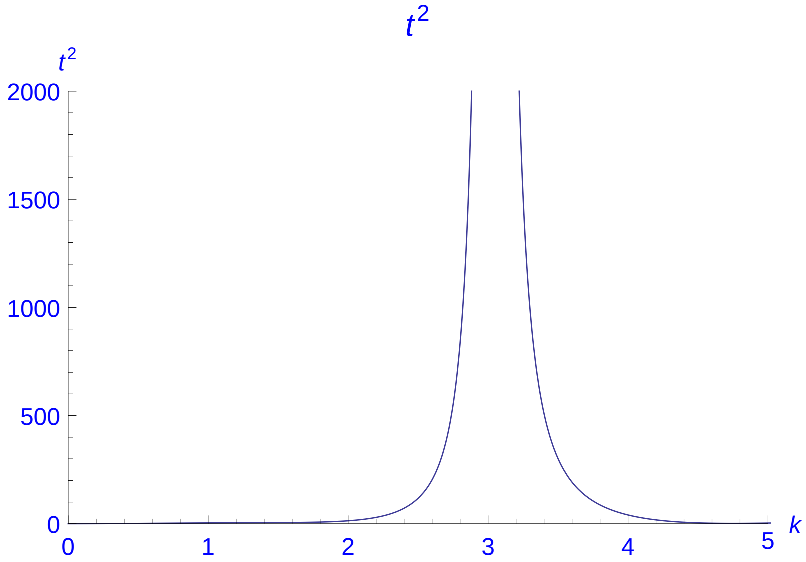


$$\lambda = 11, q = 11$$

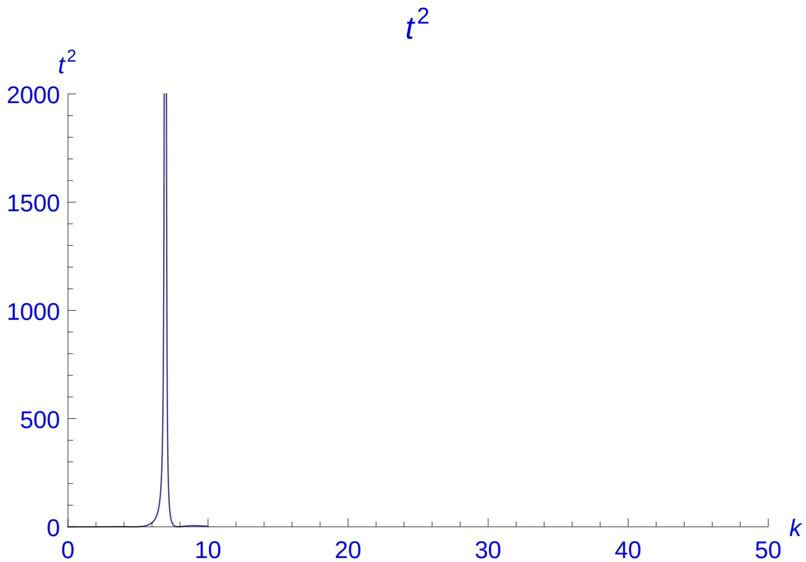


$$\lambda = 13.5, q = 11$$

Observations:SPS

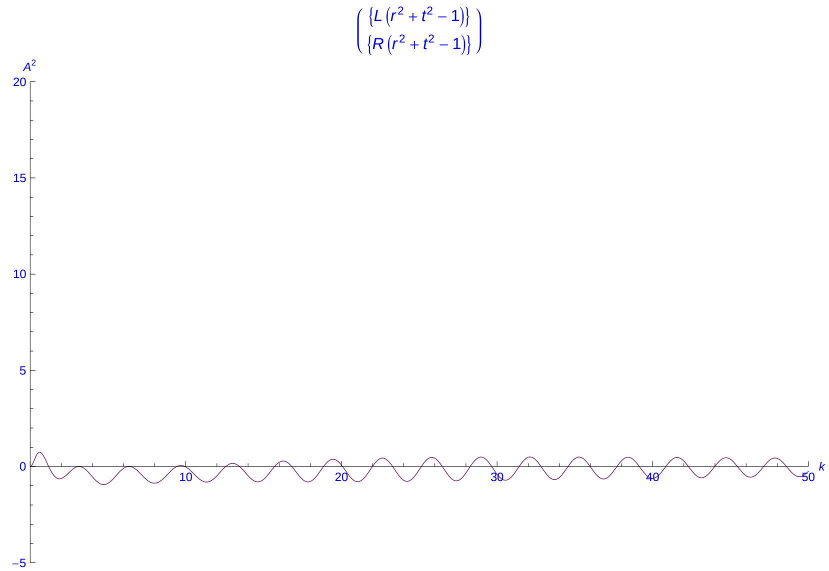


$$\lambda = 8.4, q = 7.85$$

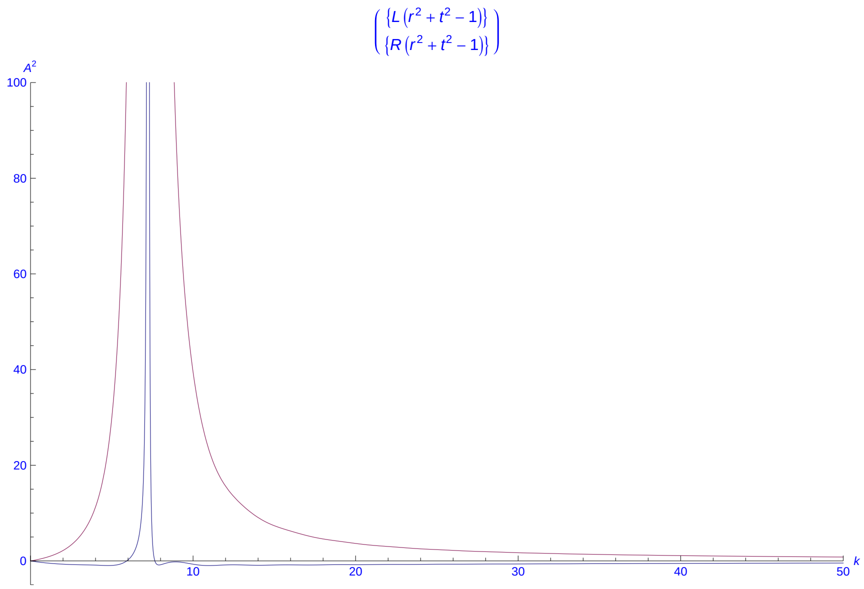


$$\lambda = 8.4, q = 1.58$$

Observations:SPS

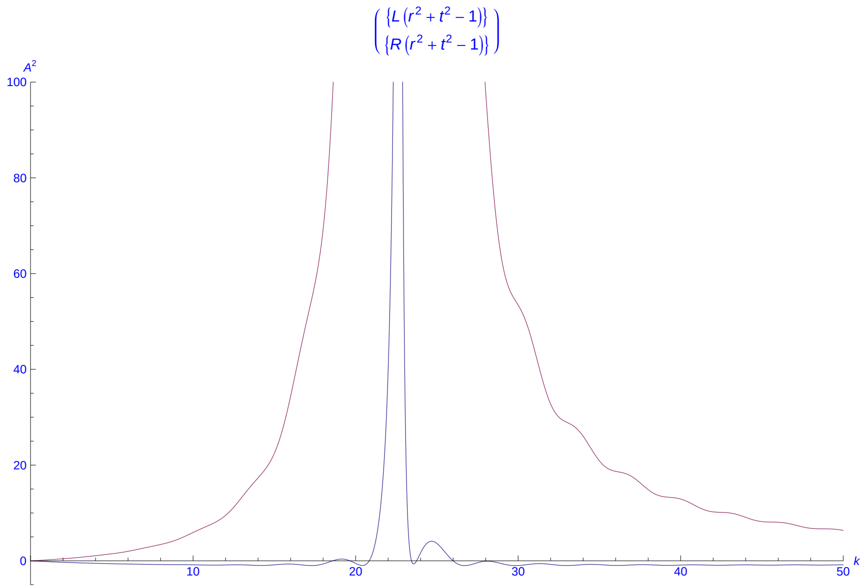


λ=0



λ=7.5, q=1.6

Observations:SPS



λ=23, q=5

Summery of the Observations

Summery of the observations

- r_L become singular in more points if strength of Non-Hermitian increases, but at low energy but no sharp peaks even with extremely high non-Hermiticity
- r_R has one singular point for any nonzero (whatsoever small) value of λ. Singular point shifts depending on the value of λ and q.
- t diverges at one point for particular pair of λ and q. For a fixed λ more q implies less k^*.
- Violation of unitarity diverges at spectral singular points. Precisely this is the reason of obstruction to develop a consistent quantum theory with non-Hermitian Hamiltonain with spectral singular points.

Single Particle Systems-II

More features of spectral singularity can be seen in a second example of a P-Pseudo Hermitian Hamiltonian, [Deb et al PLA 2003]

(4) $$H = \frac{p^2}{2m} - v_0\delta(x) + i\lambda[\delta(x - \frac{a}{2}) - \delta(x + \frac{a}{2})].$$

Here again $H^\dagger = PHP^{-1}$ but unlike case I the imaginary part of the potential is not proportional to the derivative of the real part. Following the method outlined for previous model one can calculate the different coefficients as using notations $\frac{2mv_0}{\hbar^2} = \tilde{\mu}$, $\frac{2m\lambda}{\hbar^2} = \tilde{\lambda}$ and $\frac{2mB}{\hbar^2} = \beta^2$

Single Particle Systems-II

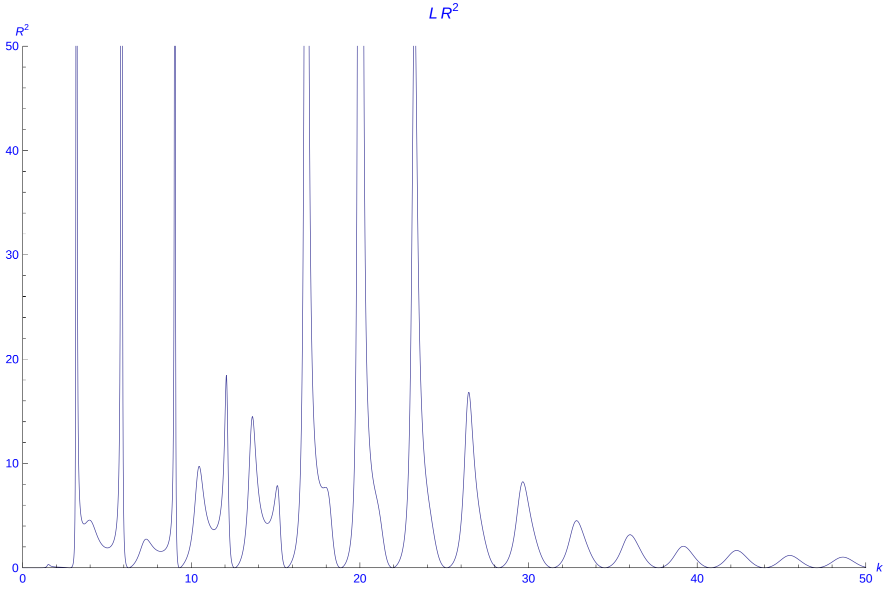
$$t_L = t_R = \frac{1}{(1 - i\frac{\tilde{\mu}}{2k}) + (\frac{\tilde{\lambda}}{2k})^2[-(1 - i\frac{\tilde{\mu}}{2k}) - i\frac{\tilde{\mu}}{k}e^{ika} + (1 + i\frac{\tilde{\mu}}{2k})e^{2ika}]},$$

$$r_L = \frac{i\frac{\tilde{\mu}}{2k}(1 - \frac{\tilde{\lambda}}{k} + \frac{\tilde{\lambda}^2}{2k^2}) + (1 - \frac{\tilde{\lambda}}{2k})[(1 + i\frac{\tilde{\mu}}{2k})\frac{\tilde{\lambda}}{2k}e^{ika} - c.c.]}{(1 - i\frac{\tilde{\mu}}{2k}) + (\frac{\tilde{\lambda}}{2k})^2[-(1 - i\frac{\tilde{\mu}}{2k}) - i\frac{\tilde{\mu}}{k}e^{ika} + (1 + i\frac{\tilde{\mu}}{2k})e^{2ika}]},$$

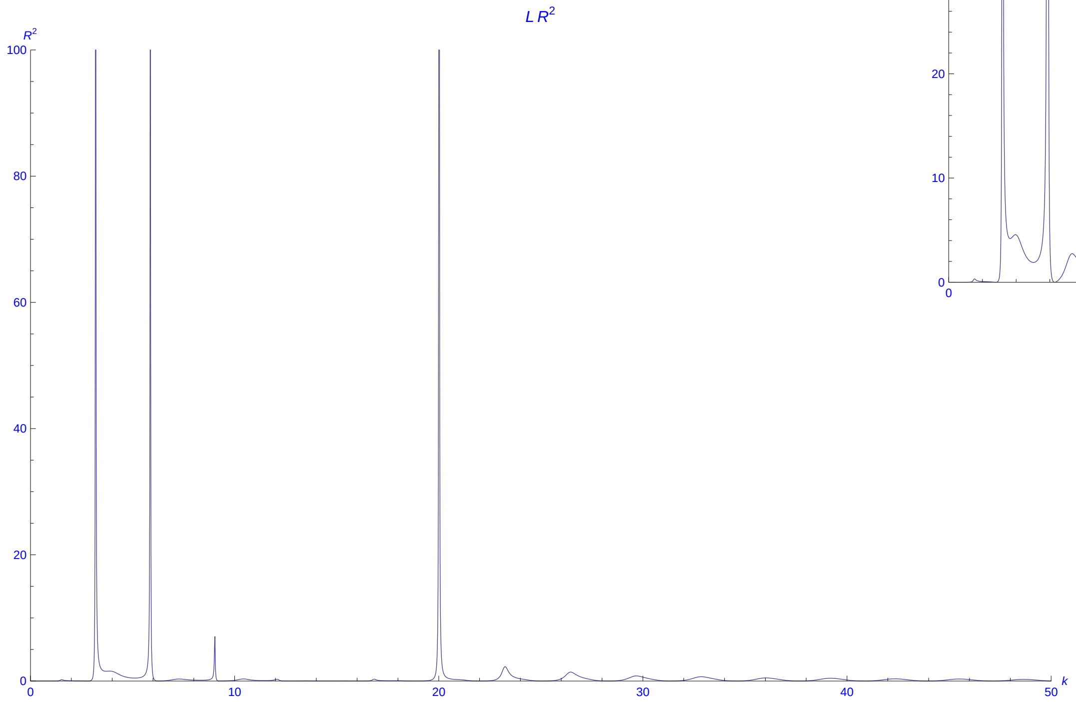
$$r_R = \frac{i\frac{\tilde{\mu}}{2k}(1 + \frac{\tilde{\lambda}}{k} + \frac{\tilde{\lambda}^2}{2k^2}) - (1 + \frac{\tilde{\lambda}}{2k})[(1 + i\frac{\tilde{\mu}}{2k})\frac{\tilde{\lambda}}{2k}e^{ika} - c.c.]}{(1 - i\frac{\tilde{\mu}}{2k}) + (\frac{\tilde{\lambda}}{2k})^2[-(1 - i\frac{\tilde{\mu}}{2k}) - i\frac{\tilde{\mu}}{k}e^{ika} + (1 + i\frac{\tilde{\mu}}{2k})e^{2ika}]},$$

Observations: SPS

$\lambda = -30$

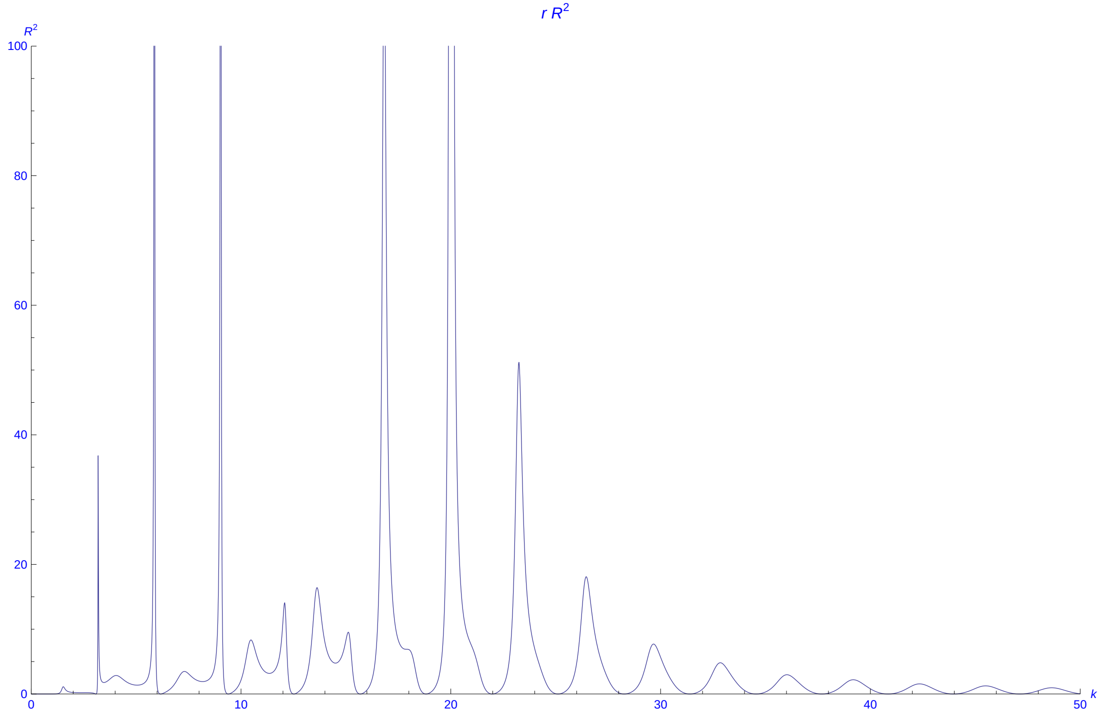


$\lambda = 30$

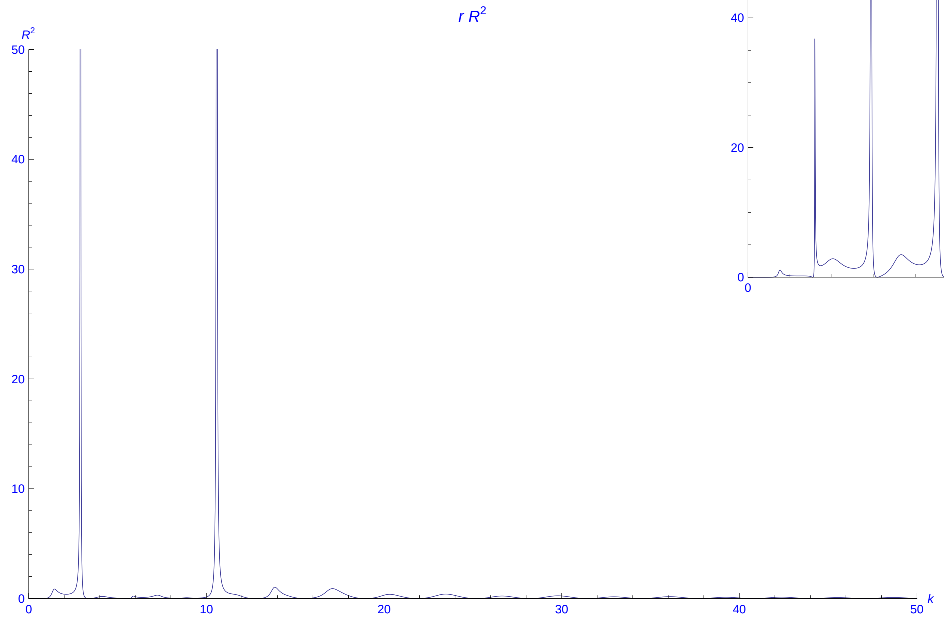


Observations: SPS

$\lambda = -30$



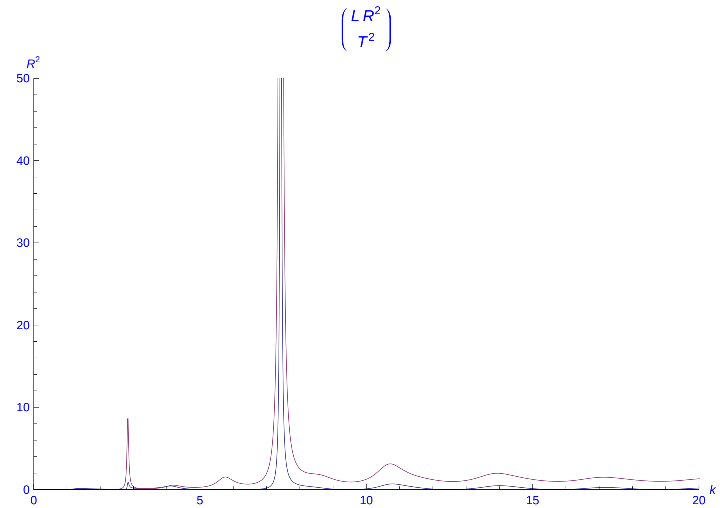
$\lambda = -16.3$

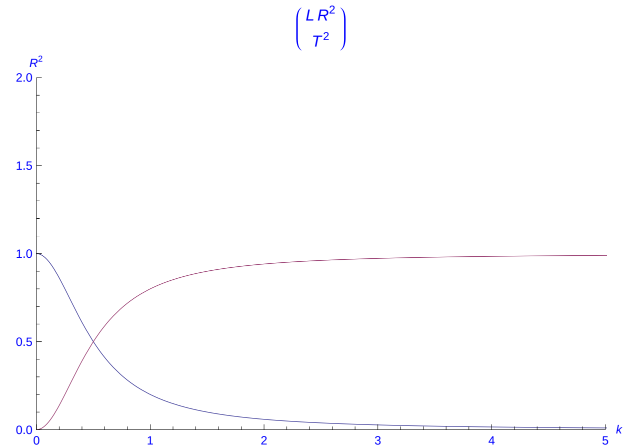


Observations: SPS

$\lambda = 0$

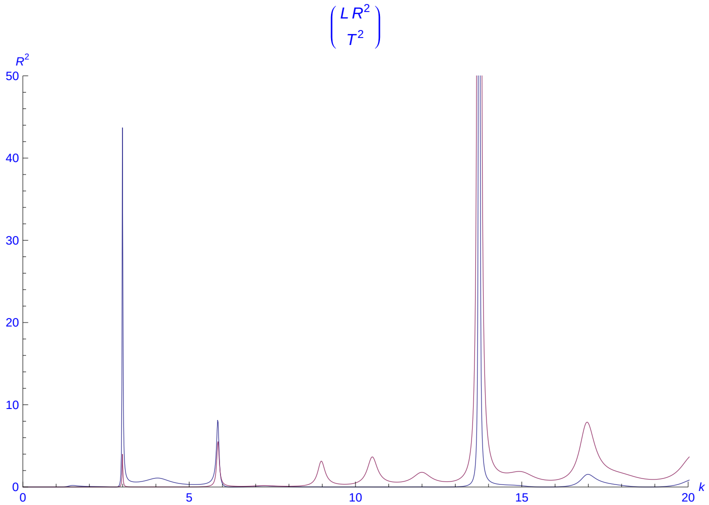
$\lambda = 10.7$





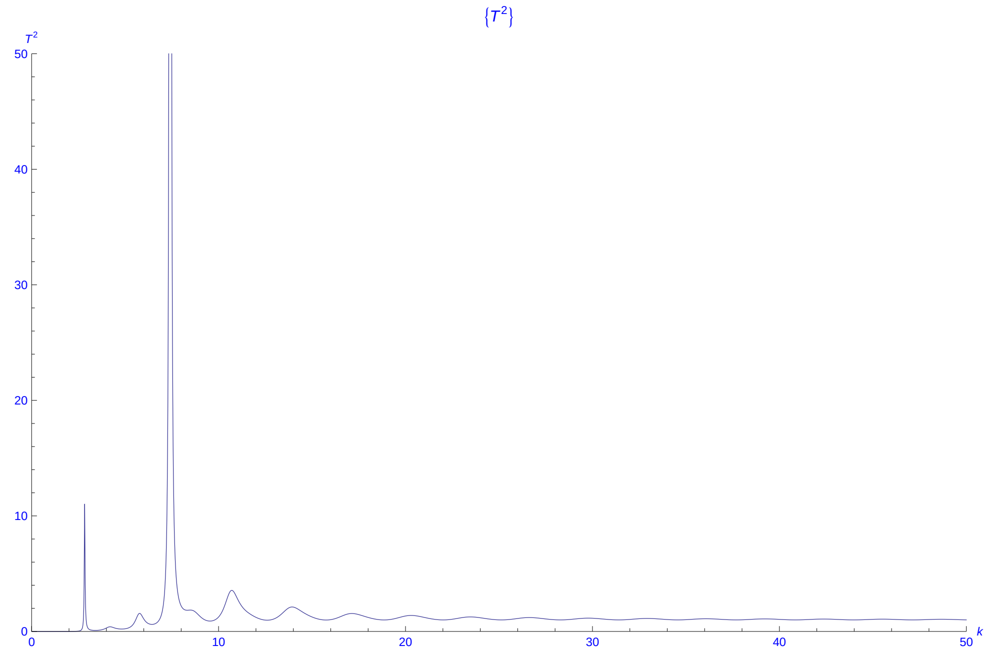
Observations: SPS

$$\lambda = 20.7$$

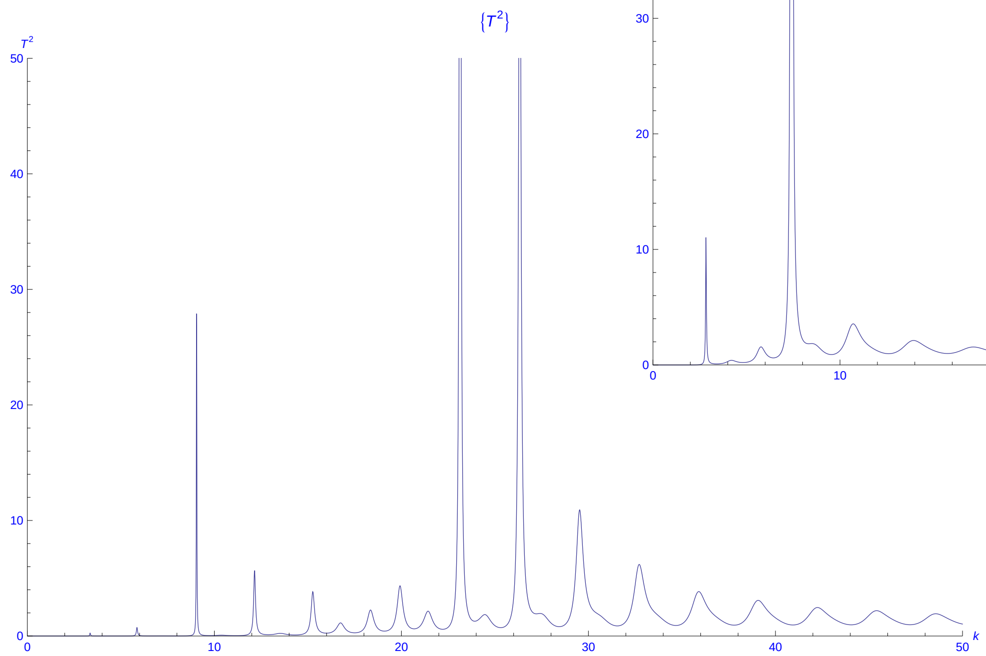


Observations: SPS

$\lambda = 11$

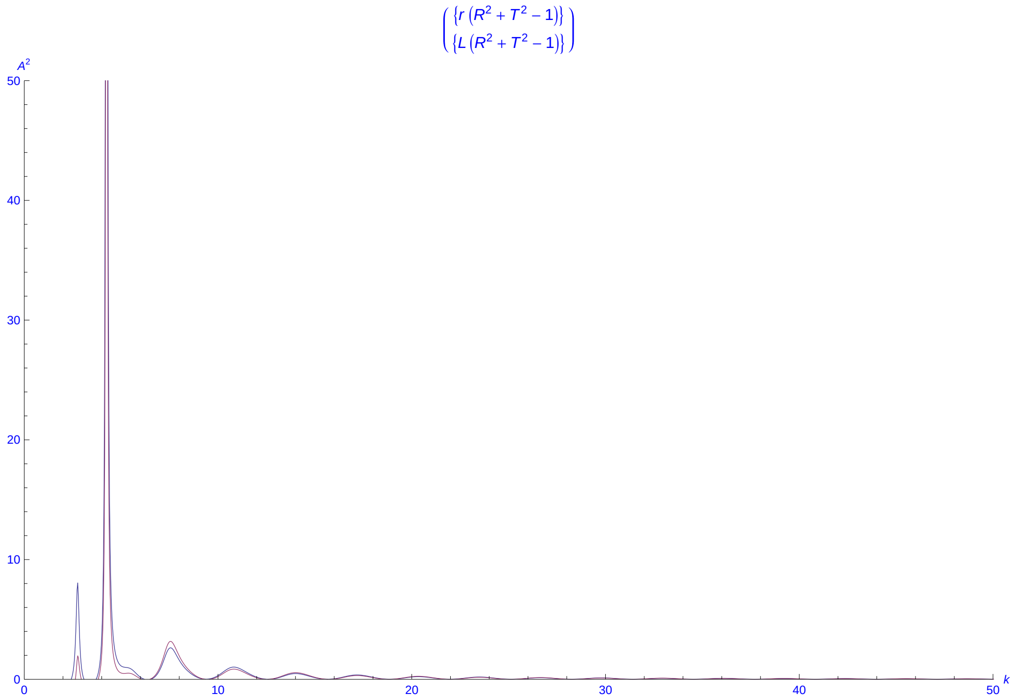


$\lambda = 37$

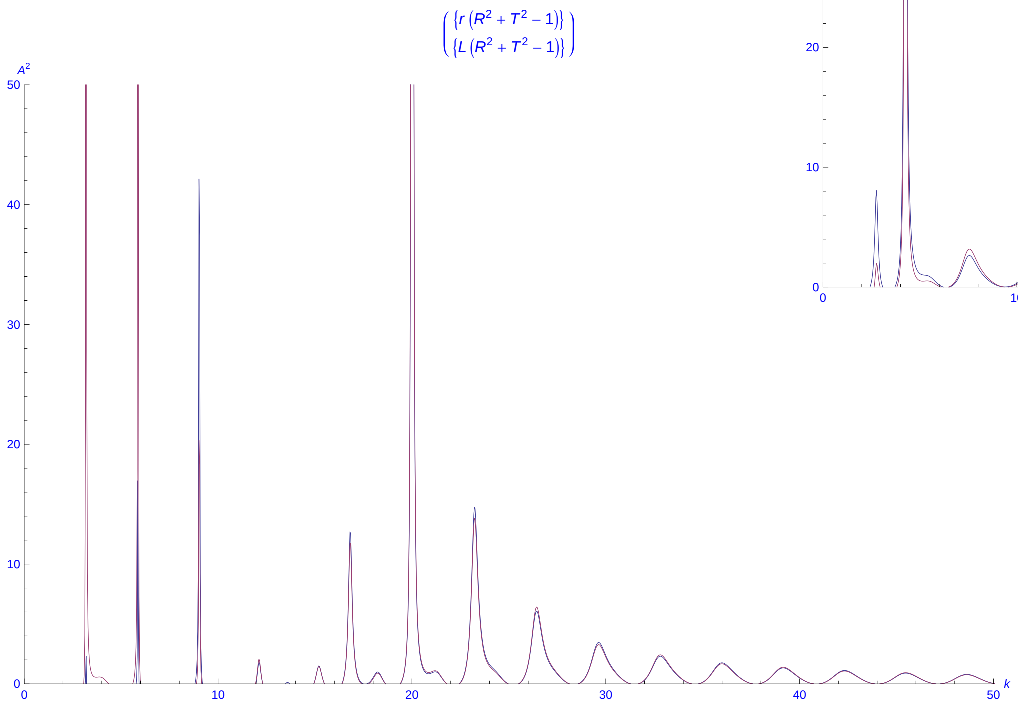


Observations: SPS

$\lambda = 7.3$

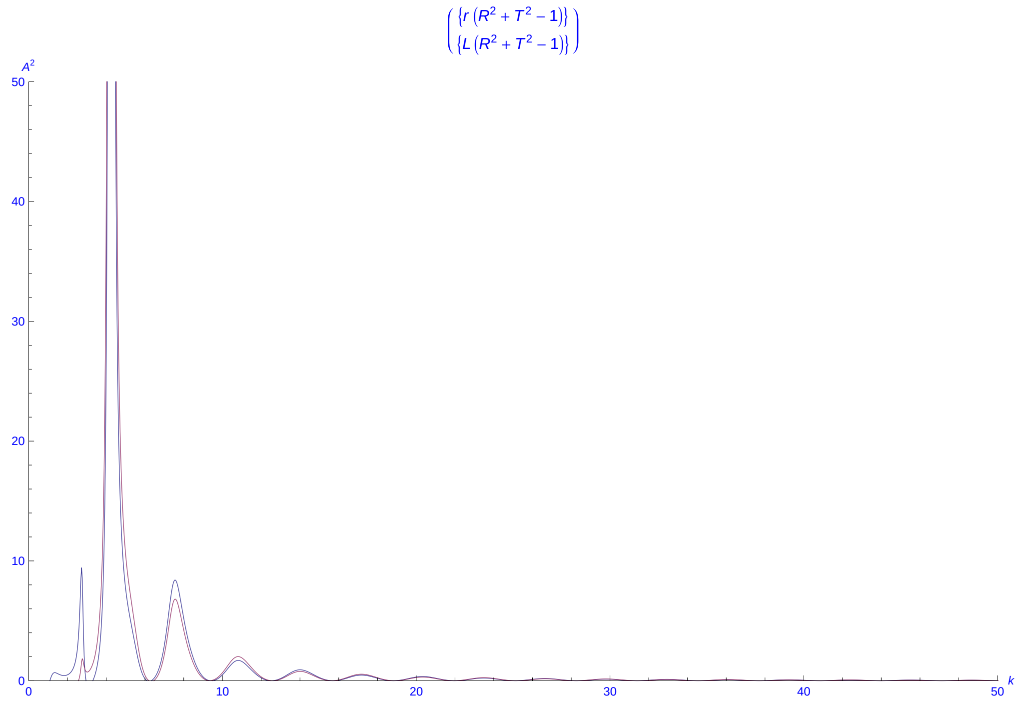


$\lambda = 30$

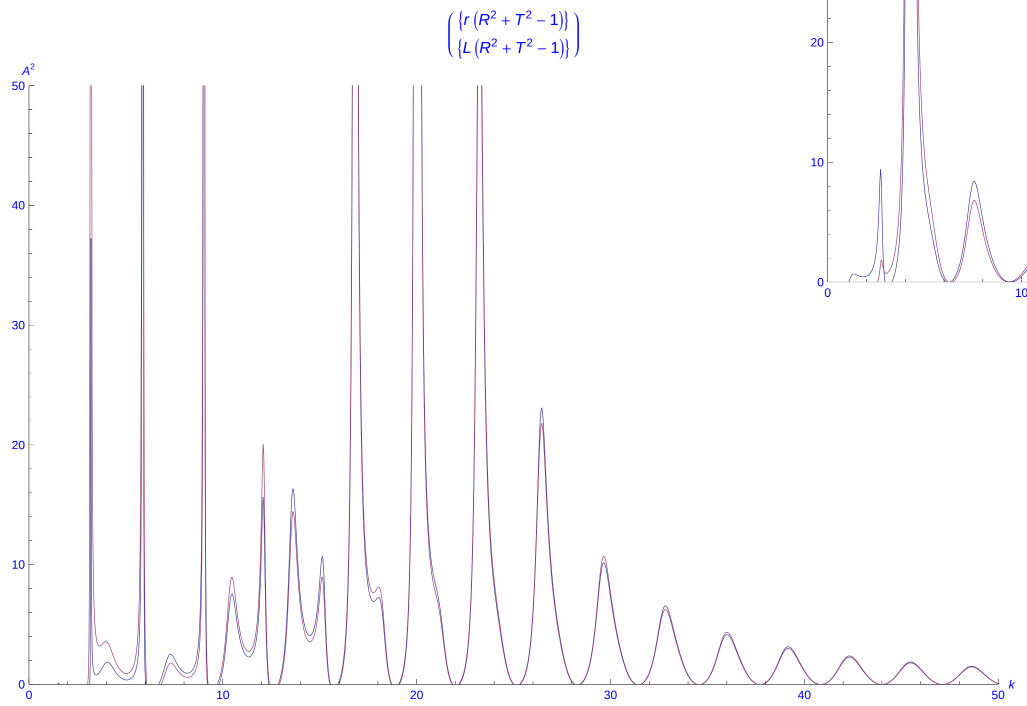


Observations: SPS

$\lambda = -7.2$



$\lambda = -30$



Single Particle Systems

Summery of the Observations

- Divergence structure changes under $\lambda \rightarrow -\lambda$.
- Behavior of unitary violation is independent of the fact whether beam is left moving or right moving.
- t has one singularity point for any nonzero (whatsoever small) value of λ. More singular points when non-hermiticity strength increases.
- Violation of unitarity diverges at spectral singular points. Precisely this is the reason of obstruction to develop a consistent quantum theory with non-Hermitian Hamiltonain with spectral singular points.

Many Particles

The A_{N-1} Calogero model (related to A_{N-1} Lie algebra) is the simplest example of such a dynamical model, describing N particles on a line and with Hamiltonian given by,

$$H = -\frac{1}{2}\sum_{j=1}^{N}\frac{\partial^2}{\partial x_j^2} + \frac{g}{2}\sum_{j\neq k}\frac{1}{(x_j - x_k)^2} + \frac{\omega^2}{2}\sum_{j=1}^{N} x_j^2 \,,$$

where g is the coupling of long-range interaction and ω is the coupling of harmonic confining interaction. This A_{N-1} Calogero model is solved exactly to obtain the complete set of discrete energy eigenvalues and corresponding bound state eigenfunctions.

Usual Calogero Model

The complete set of energy eigenvalues can be written as

$$E_{n_1,n_2,\cdots n_N} = \frac{N\omega}{2}[1 + (N-1)\nu] + \omega \sum_{j=1}^{N} n_j \ ,$$

where n_js are non-negative integer valued quantum numbers with $n_j \leq n_{j+1}$ and ν is a real positive parameter related to the coupling constant of long range interaction as

$$g = \nu^2 - \nu.$$

This spectrum is same as that for a N number of free bosonic oscillators apart from a overall shift for all levels.

Extended Calogero Model

An extension of A_{N-1} Calogero model with confining term is proposed by adding a momentum dependent long-range interaction $(\delta \sum_{j\neq k} \frac{1}{(x_j - x_k)} \frac{\partial}{\partial x_j})$

$$H_{ext} = -\frac{1}{2}\sum_{j=1}^{N}\frac{\partial^2}{\partial x_j^2} + \frac{g}{2}\sum_{j\neq k}\frac{1}{(x_j - x_k)^2} + \delta\sum_{j\neq k}\frac{1}{(x_j - x_k)}\frac{\partial}{\partial x_j} + \frac{\omega^2}{2}\sum_{j=1}^{N} x_j^2 \,,$$

The PT transformation for such N-particle system can be written as

$$i \to -i, \quad x_j \to -x_j, \quad p_j \to p_j$$

Extended Calogero Model

where $j \in [1, 2 \cdots N]$ and x_j $(p_j \equiv -i\frac{\partial}{\partial x_j})$ denotes coordinate (momentum) operator of the $j-$th particle. We have shown that this nonhermitian, PT invariant model can be solved exactly and within certain range of the related parameters it yields a real spectrum

$$E_{n_1 n_2 \cdots n_N} = \frac{N\omega}{2}[1 + (N-1)\tilde{\nu} + \omega \sum_{j=1}^{N} n_j \,.$$

Here $\tilde{\nu} = \nu' - \delta$ and ν' is a real positive parameter which is related to the coupling constants g and δ as

(5) $$g = {\nu'}^2 - \nu' \,(1 + 2\delta)\,.$$

Extended Calogero Model

It will be interesting to consider complex extension of A_{N-1} Calogero model without confining term as in such cases will have scattering states. Before going to discuss that let us quickly go through the usual A_{N-1} Calogero model without confining interaction.

CM without confining term

A_{N-1} Calogero model in the absence of confining interaction is described by the Hamiltonian

$$\mathcal{H}_0 = -\frac{1}{2}\sum_{j=1}^{N}\frac{\partial^2}{\partial x_j^2} + \frac{g}{2}\sum_{j\neq k}\frac{1}{(x_j - x_k)^2}\,.$$

The above system is solved to obtain scattering states within a sector of configuration space corresponding to a definite ordering of particles like $x_1 \geq x_2 \geq \cdots \geq x_N$. The zero energy ground state wave function of this model is given by

$$\psi_{gr} = \prod_{j<k}(x_j - x_k)^{\nu},$$

CM without confining term

we consider the general eigenvalue equation associated with the above Hamiltonian

(6) $$\mathcal{H}_0\psi = p^2\psi,$$

where p is real and positive. Solutions of this eigenvalue equation can be written in the form

$$\psi = \psi_{gr}\tau(x_1, x_2, \cdots x_N)\,,$$

where $\tau(x_1, x_2, \cdots x_N)$ satisfies the following differential equation,

$$-\frac{1}{2}\sum_{j=1}^{N}\frac{\partial^2\tau}{\partial x_j^2} - \nu\sum_{j\neq k}\frac{1}{(x_j - x_k)}\frac{\partial\tau}{\partial x_j} = p^2\tau\,.$$

CM without confining term

to separate the 'radial' and 'angular' part of the eigenfunction, one assumes that $\tau(x_1, x_2, \cdots x_N) = P_{k,q}(x)\chi(r)$, where the radial variable r is defined as

$$r^2 = \frac{1}{N} \sum_{i \neq j} (x_i - x_j)^2$$

and substitute in the above Schrodinger Eqn. to obtain

$$\sum_{j=1}^{N} \frac{\partial^2 P_{k,q}(x)}{\partial x_j^2} + \nu \sum_{j \neq k} \frac{1}{(x_j - x_k)} \left(\frac{\partial}{\partial x_j} - \frac{\partial}{\partial x_k} \right) P_{k,q}(x) = 0\,.$$

CM without confining term

$P_{k,q}(x)$s are translationally invariant, symmetric, k-th order homogeneous polynomials also satisfy the following relations,

$$\sum_{j=1}^{N} \frac{\partial P_{k,q}(x)}{\partial x_j} = 0\,, \qquad \sum_{j=1}^{N} x_j \frac{\partial P_{k,q}(x)}{\partial x_j} = k P_{k,q}(x)\,.$$

Note that the index q in $P_{k,q}(x)$ can take any integral value ranging from 1 to $g(N,k)$, where $g(N,k)$ is the number of independent polynomials The radial part satisfies

$$-\frac{d^2\chi(r)}{dr^2} - \frac{(1+2b)}{r}\frac{d\chi(r)}{dr} = p^2\chi(r)\,,$$

where $b = \frac{N-3}{2} + k + \frac{N(N-1)\nu}{2}$

CM without confining term

The above equation admits a solution of the form $\chi(r) = r^{-b} J_b(pr)$, where $J_b(pr)$ denotes the Bessel function. the scattering state eigenfunctions of $\mathcal{H}_0$ with eigenvalue p^2 are finally obtained as

$$\psi = \prod_{j<k} (x_j - x_k)^\nu r^{-b} J_b(pr) P_{k,q}(x) \, .$$

To find the scattering phase shift for the above model we need to construct a more general eigenfunction which in the asymptotic limit (i.e., $r \to \infty$ limit) can be expressed in terms of an incoming free particle wave function (ψ_+)

CM without confining term

The outgoing free particle wave function (ψ_-), where the incoming wave function will be of the form

$$\psi_+ = \exp[\,i \sum_{j=1}^{N} p_j x_j\,]\,,$$

This can be achieved by taking appropriate linear superposition of all degenerate eigenfunctions (with eigenvalue p^2)

$$\psi_{gen} = \prod_{j<k} (x_j - x_k)^\nu \sum_{k=0}^{\infty} \sum_{q=1}^{g(N,k)} C_{kq} r^{-b} J_b(pr) P_{k,q}(x)\,,$$

where C_{kq}s are expansion coefficients.

CM without confining term

By using this relation for C_{kq} and also the asymptotic properties of Bessel function at $r \to \infty$ limit,

$$J_b(pr) \to \frac{1}{\sqrt{2\pi pr}} \{ e^{-i(n+\frac{1}{2})\frac{\pi}{2}+ipr} + e^{i(n+\frac{1}{2})\frac{\pi}{2}-ipr} \} ,$$

one can write down the asymptotic from of ψ_{gen} as

$$\psi_{gen} \sim \psi_+ + \psi_- ,$$

where

$$\psi_\pm = (2\pi r)^{-\frac{1}{2}} p^{(n-\frac{1}{2})} \prod_{j<k} (x_j - x_k)^\nu r^{-A} \sum_{k=0}^{\infty} \sum_{q=1}^{g(N,k)} \tilde{C}_{kq}(\alpha_i) r^{-k} P_{k,q}(x)$$

CM without confining term

with $A = b - k = \frac{N-3}{2} + \frac{N(N-1)\nu}{2}$ and $n = \frac{3-N}{2}$.

Extended CM without confining ...

Now we have appropriate platform to consider PT symmetric nonhermitian extension of the Calogero model without confining interaction. The extended Calogero model which we will be considering here is described by the Hamiltonian

$$\mathcal{H}_{ext} = -\frac{1}{2}\sum_{j=1}^{N}\frac{\partial^2}{\partial x_j^2} + \frac{g}{2}\sum_{j\neq}\frac{1}{(x_j - x_k)^2} + \delta\sum_{j\neq k}\frac{1}{(x_j - x_k)}\frac{\partial}{\partial x_j}.$$

Extended CM without confining · · ·

We start by observing that the zero energy ground state wave function of this Hamiltonian is quite similar with the ground state wave function of the original Calogero model:

$$\psi_{gr} = \prod_{j<k} (x_j - x_k)^{\nu'} \ ,$$

where the modified exponent ν' is related to the coupling constants g and δ. For the purpose of obtaining non-singular ground state eigenfunction at the limit $x_i \to x_j$, ν' should be a non-negative exponent. This restricts the ranges of coupling constants g and δ as (i)$\delta \geq -\frac{1}{2}, \ 0 > g \geq -(\delta + \frac{1}{2})^2$, and (ii) $g \geq 0$ with arbitrary value of δ.

Extended CM without confining $\cdots$

The general eigenvalue equation associated the extended Hamiltonian given by

(7) $$\mathcal{H}_{ext}\,\psi \;=\; p^2\,\psi,$$

where p is a real positive parameter. As in the case of usual Calogero model the solutions of this eigenvalue equation can be written in the form $\psi = \psi_{gr}\tau'(x_1, x_2 \cdots x_N)$. $\tau'(x_1, x_2 \cdots x_N)$ satisfies a differential equation like

$$-\frac{1}{2}\sum_{j=1}^{N}\frac{\partial^2\tau'}{\partial x_j^2} - (\nu' - \delta)\sum_{j\neq k}\frac{1}{(x_j - x_k)}\frac{\partial\tau'}{\partial x_j} = p^2\tau'\,.$$

Extended CM without confining $\cdots$

we assume that $\tau'(x_1, x_2 \cdots x_N)$ can be factorized as

$$\tau'(x_1, x_2 \cdots x_N) = P'_{k,q}(x)\chi'(r),$$

where r is the radial variable and $P'_{k,q}(x)$s are translationally invariant, symmetric, k-th order homogeneous polynomials satisfying the differential equations

$$\sum_{j=1}^{N} \frac{\partial^2 P'_{k,q}(x)}{\partial x_j^2} + (\nu' - \delta) \sum_{j \neq k} \frac{1}{(x_j - x_k)} \left(\frac{\partial}{\partial x_j} - \frac{\partial}{\partial x_k} \right) P'_{k,q}(x) = 0 \, .$$

Extended CM without confining ···

Note that the form of solution eqn. is same as apart from the fact that here ν is replaced by $\nu' - \delta$. Hence $P'_{k,q}(x)$ can be obtained from any given expression of $P_{k,q}(x)$ by simply substituting the parameter ν with $\nu' - \delta$. So the index q in $P'_{k,q}(x)$ can also take values ranging from 1 to $g(N, k)$. Substituting the factorized form of $\tau(x_1, x_2 \cdots x_N)$ and making use of the properties of $P'_{k,q}(x)$ we obtain the equation satisfied by the 'radial' part of the wave function as

$$-\frac{\partial^2 \chi'(r)}{\partial r^2} - \frac{1 + 2b'}{r} \frac{\partial \chi'(r)}{\partial r} = p^2 \chi'(r)$$

Extended CM without confining $\cdots$

with $b' = \frac{N-3}{2} + k + (\nu' - \delta)\frac{N(N-1)}{2}$. The solution of the above eqn. can be expressed through the Bessel function: $\chi'(r) = r^{-b'} J_{b'}(pr)$. Hence the scattering state eigenfinctions of $\mathcal{H}_{ext}$ with real positive eigenvalue p^2 are obtained as

$$\psi = \prod_{j<k} (x_j - x_k)^{\nu'} r^{-b'} J_{b'}(pr) P'_{k,q}(x) \, .$$

Extended CM without confining $\cdots$

$$\psi_{gen} = \prod_{j<k} (x_j - x_k)^{\nu'} \sum_{k=0}^{\infty} \sum_{q=1}^{g(N,k)} C'_{kq} r^{-b'} J_{b'}(pr) P'_{k,q}(x) \,,$$

where C'_{kq}s are expansion coefficients which are functions of particle monenta. Once again by doing dimensional analysis, we obtain

$$C'_{kq} \, = \, p^{\frac{(3-N)}{2}+\frac{N(N-1)\delta}{2}} \, \tilde{C}'_{kq}(\alpha_i) \,,$$

where $\tilde{C}'_{kq}(\alpha_i)$ depends only on the angular parts of the momenta.

Extended CM without confining ...

Using explicit form of C'_{kq} and asymptotic properties of Bessel function we obtain the asymptotic form of ψ_{gen} as $\psi_{gen} \sim \psi_+ + \psi_-,$ where

$$\psi_\pm = (2\pi r)^{-\frac{1}{2}} p^{(n'-\frac{1}{2})} \prod_{j<k} (x_j - x_k)^{\nu'} r^{-A'} \sum_{k=0}^{\infty} \sum_{q=1}^{g(N,k)} \tilde{C}'_{kq}(\alpha_i) r^{-k} P'_{k,q}(x$$

In the above expression $A' = b' - k = \frac{N-3}{2} + (\nu' - \delta)\frac{N(N-1)}{2}$ and $n' = \frac{3-N}{2} + \frac{N(N-1)\delta}{2}$.

Extended CM without confining ···

the outgoing wave function (ψ_-) further can be written as

$$\begin{aligned}\psi_- &= e^{-i\pi(A'+n')}\psi_+\Big(x \to -Tx,\ p \to -p\Big) \\ &= e^{-i\pi\nu'\frac{N(N-1)}{2}}\exp[\,i\sum_{j=1}^{N} x_j\, p_{N+1-j}\,]\,.\end{aligned}$$

We can expect spectral singularity for certain value of p_i where wronskian of the asymptotic solutions, $\psi_\pm$ vanish. Recall, $H\psi(x) = p^2\psi$, satisfying the asymptotic boundary condition

$$\psi_{k\pm}(x) \to e^{\pm ikx}\text{as}\, x \to \pm\infty$$

Extended CM without confining ...

will have spectral singularity at $k = k_*$ if $\psi_{k_*\pm}(x)$ are linearly dependent, that they will have vanishing Wronskian

$$\psi_{k_*+}\psi'_{k_*-} - \psi_{k_*-}\psi'_{k_*+} = 0$$

Vanishing Wronskian leads to the condition for spectral singularity to exists for PT symmetric non-Hermitian extension of A_{N-1} Calogero model without confining interaction is

$$|p_j| = |p_{N+1-j}|$$

subjected to the other restrictions on particles momentum.

This result is far from completion and further detail investigations are required,

Conclusions & Discussions

- It is very important to know the Exceptional points and spectral singular points in non-Hermitian theories as these are the obstacles to have a consistent unitary quantum theory with complete set of orthonormal eigenstates.
- We consider two 1-dimensional PT symmetric non-Hermitian system and study the spectral singularities in different situations by changing the strength of non-Hermitian interactions.
- We observe in both the model that maximal violation of Unitarity i.e. $|t|^2 + |r|^2 - 1$ is the signature for spectral singularity. To make a general statement one needs to be confirmed from other models.

Conclusions & Discussions

- Non-Hermitian PT symmetric A_{N-1} Calogero models have been considered with and without confining interaction. We have discreet energy levels when the confining interaction is present. On the other hand we have continuous scattering for the system with out confining interaction.
- We obtain an condition for spectral singularity in the case of non-Hermitian A_{N-1} Calogero model without confining potential.
- Further investigation regarding transmission and refection coefficient are required to find the spectral singular points explicitly.

THANK YOU