OBSERVATIONAL EVIDENCE FOR THE NON-DIAGONALIZABLE HAMILTONIAN OF CONFORMAL GRAVITY

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On **January 19, 2000** a paper by P. D. Mannheim and A. Davidson,"Fourth order theories without ghosts" appeared on the arXiv as hep-th/0001115. The paper was later published as P. D. Mannheim and A. Davidson, "Dirac quantization of the Pais-Uhlenbeck fourth order oscillator", Physical Review A 71, 042110 (2005). (hep-th/0408104). The abstract of the 2000 paper read:

Using the Dirac constraint method we show that the pure fourth-order Pais-Uhlenbeck oscillator model is free of observable negative norm states. Even though such ghosts do appear when the fourth order theory is coupled to a second order one, **the limit in which the second order action is switched off is found to be a highly singular one in which these states move off shell**. Given this result, construction of a fully unitary, renormalizable, gravitational theory based on a purely fourth order action in 4 dimensions now appears feasible.

Then in the 2000 paper it read:

"We see that the complete spectrum of eigenstates of $H(\epsilon = 0) = 8\gamma\omega^4(2b^\dagger b + a^\dagger b + b^\dagger a) + \omega$ is the set of all states $(b^\dagger)^n|\Omega\rangle$, a spectrum whose dimensionality is that of a one rather than a two-dimensional harmonic oscillator, even while the complete Fock space has the dimensionality of the two-dimensional oscillator."

With a pure fourth-order equation of motion being of the form

$$(\partial_t^2 - \nabla^2)^2\phi(x) = 0, \tag{1}$$

the solutions are of the form

$$\phi(x) = e^{i\bar{k}\cdot\bar{x}-i\omega t}, \qquad \phi(x) = t\, e^{i\bar{k}\cdot\bar{x}-i\omega t}. \tag{2}$$

With this latter solution not being stationary, the Hamiltonian lacks energy eigenstates, and is thus non-diagonalizable, and hence non-Hermitian. However, with stationary solutions having real energies, the Hamiltonian is thus PT symmetric, with the one-particle sector of the Hamiltonian for instance being of the form:

$$M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} p+q \\ q \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}, \qquad q = 0, \qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3}$$

with only one eigenstate. **This is one of the first examples of a *PT* symmetric Hamiltonian at an exceptional point – a point at which there is no equivalent Hermitian Hamiltonian. Except I only found out that the theory was a *PT* theory when Carl Bender told me in 2007 — Working with Carl has pluses and minuses — you learn wonderful things about physics, and you learn how dumb you had previously been.**

THINGS WE TAKE FOR GRANTED IN QUANTUM MECHANICS.....

(1) The Hamiltonian must be Hermitian.

(2) The momentum operator must be Hermitian.

(3) Self-adjoint is the same as Hermitian.

(4) The classical limit of a quantum theory must be based on real numbers.

(5) In the $[x,p]=i\hbar$ commutator the momentum operator can always be represented by $p=-i\hbar\frac{\partial}{\partial x}$.

(6) States such as energy eigenstates must form a complete set.

(7) To be complete states must be normalizable.

(8) The scalar product must be given as $\langle m|n\rangle=\delta_{m,n}$.

(9) The completeness relation must be given by $\sum|n\rangle\langle n|=1$.

(10) Theories in which $\langle n|n\rangle$ is negative are unphysical and cannot be formulated in Hilbert space.

(11) The Hamiltonian must be diagonalizable.

.....AIN'T NECESSARILY SO

AND FOR THEORIES BASED ON FOURTH-ORDER DERIVATIVES....
ALL THESE THINGS ARE NECESSARILY NOT SO,......

AND CAN ENABLE FOURTH-ORDER DERIVATIVE CONFORMAL GRAVITY TO BE A CONSISTENT THEORY OF QUANTUM GRAVITY IN FOUR SPACETIME DIMENSIONS

GHOST PROBLEM AND UNITARITY

1. C. M. Bender and P. D. Mannheim, *No-ghost theorem for the fourth-order derivative Pais-Uhlenbeck oscillator model*, Phys. Rev. Lett. **100**, 110402 (2008). (0706.0207 [hep-th]).
2. P. D. Mannheim, *Conformal gravity challenges string theory*, Pascos-07, July 2007 (0707.2283 [hep-th]).
3. C. M. Bender and P. D. Mannheim, *Giving up the ghost*, Jour. Phys. A **41**, 304018 (2008). (0807.2607 [hep-th])
4. C. M. Bender and P. D. Mannheim, *Exactly solvable PT-symmetric Hamiltonian having no Hermitian counterpart*, Phys. Rev. D **78**, 025022 (2008). (0804.4190 [hep-th])

PT QUANTUM MECHANICS

5. C. M. Bender and P. D. Mannheim, *PT symmetry and necessary and sufficient conditions for the reality of energy eigenvalues*, Phys. Lett. A 374, 1616 (2010). (0902.1365 [hep-th])
6. P. D. Mannheim, *PT symmetry as a necessary and sufficient condition for unitary time evolution*, 0912.2635 [hep-th], December 2009.

PAIS-UHLENBECK FOURTH-ORDER OSCILLATOR

7. P. D. Mannheim and A. Davidson, *Fourth order theories without ghosts*, January 2000 (hep-th/0001115).
8. P. D. Mannheim and A. Davidson, *Dirac quantization of the Pais-Uhlenbeck fourth order oscillator*, Phys. Rev. A **71**, 042110 (2005). (hep-th/0408104).
9. P. D. Mannheim, *Solution to the ghost problem in fourth order derivative theories*, Found. Phys. **37**, 532 (2007). (hep-th/0608154).

CONFORMAL GRAVITY AND THE COSMOLOGICAL CONSTANT AND DARK MATTER PROBLEMS

10. P. D. Mannheim, *Alternatives to dark matter and dark energy*, Prog. Part. Nuc. Phys. **56**, 340 (2006). (astro-ph/0505266).
11. P. D. Mannheim, *Dynamical symmetry breaking and the cosmological constant problem*, September 2008 (0809.1200 [hep-th]). Proceedings of ICHEP08, eConf C080730.
12. P. D. Mannheim, *Comprehensive solution to the cosmological constant, zero-point energy, and quantum gravity problems*, Gen. Rel. Gravit. **43**, 703 (2011). (0909.0212 [hep-th])
13. P. D. Mannheim, *Intrinsically quantum-mechanical gravity and the cosmological constant problem*, May 2010 (1005.5108 [hep-th]).
14. P. D. Mannheim and J. G. O'Brien, *Impact of a global quadratic potential on galactic rotation curves*, Phys. Rev. Lett. **106**, 121101 (2011), (arXiv:1007.0970 [astro-ph.CO]).
15. P. D. Mannheim and J. G. O'Brien, *Fitting galactic rotation curves with conformal gravity and a global quadratic potential*, November 2010. (arXiv: 1011.3495 [astro-ph.CO])
16. P. D. Mannheim, *Making the case for conformal gravity*, January 2011. (arXiv: 1101.2186 [hep-th]).

THE QUANTUM GRAVITY UNITARITY PROBLEM

In four spacetime dimensions invariance under

$$g_{\mu\nu}(x) \to e^{2\alpha(x)} g_{\mu\nu}(x) \tag{4}$$

leads to a unique gravitational action

$$I_W = -\alpha_g \int d^4x(-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} = -2\alpha_g \int d^4x(-g)^{1/2} \left[R^{\mu\nu}R_{\mu\nu} - \frac{1}{3}(R^{\alpha}{}_{\alpha})^2\right] \tag{5}$$

where α_g is dimensionless and

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} + \frac{1}{6}R^{\alpha}{}_{\alpha}\left[g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu}\right] - \frac{1}{2}\left[g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}\right] \tag{6}$$

is the conformal Weyl tensor. The associated gravitational equations of motion are the **fourth-order derivative**:

$$4\alpha_g[2C^{\mu\lambda\nu\kappa}{}_{;\lambda;\kappa} - C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}] = T^{\mu\nu}. \tag{7}$$

Conformal gravity is thus a **power-counting renormalizable theory of gravity** since α_g is dimensionless. Moreover, conformal gravity **controls the cosmological constant** since no $I = \int d^4x(-g)^{1/2}\Lambda$ term is allowed. Moreover, in a Robertson-Walker cosmology we have $C^{\mu\lambda\nu\kappa} = 0$, to yield

$$T^{\mu\nu} = 0, \tag{8}$$

so unlike the double-well Higgs potential, conformal gravity knows where the zero of energy is. However, since the field equations are fourth-order derivative equations, the theory is thought to have negative norm ghost states and not be unitary.

To illustrate the issues involved, consider the typical second- plus fourth-order derivative theory:

$$I = \frac{1}{2}\int d^4x \left[\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi - M^2\partial_\mu\phi\partial^\mu\phi\right] \tag{9}$$

$$(\partial_t^2 - \nabla^2)(\partial_t^2 - \nabla^2 + M^2)\phi(\bar{x}, t) = 0 \tag{10}$$

$$D^{(4)}(k^2 = -k_0^2 + \vec{k}^2) = \frac{1}{k^2(k^2 + M^2)} = \frac{1}{M^2}\left(\frac{1}{k^2} - \frac{1}{k^2 + M^2}\right). \tag{11}$$

Does the relative minus sign in propagator mean ghost states with negative norm and loss of unitarity, since anticipate that one can write the propagator as

$$D(\bar{x}, \bar{x}', E) = \sum \frac{\psi_n(\bar{x})\psi_n^*(\bar{x}')}{E - E_n} - \sum \frac{\psi_m(\bar{x})\psi_m^*(\bar{x}')}{E - E_m}, \tag{12}$$

and the completeness relation as

$$\sum |n\rangle\langle n| - \sum |m\rangle\langle m| = \mathbf{1}. \tag{13}$$

PAIS-UHLENBECK OSCILLATOR

$$\phi(\bar{x},t) \sim z(t)e^{i\bar{k}\cdot\bar{x}}, \quad \omega_1 = (\bar{k}^2 + M^2)^{1/2}, \quad \omega_2 = |\bar{k}| \tag{14}$$

$$\frac{d^4 z}{dt^4} + (\omega_1^2 + \omega_2^2)\frac{d^2 z}{dt^2} + \omega_1^2\omega_2^2 z = 0 \tag{15}$$

$$I_{\mathrm{PU}} = \frac{\gamma}{2}\int dt[\ddot{z}^2 - (\omega_1^2 + \omega_2^2)\dot{z}^2 + \omega_1^2\omega_2^2 z^2] \tag{16}$$

$$H_{\mathrm{PU}} = \frac{p_x^2}{2\gamma} + p_z x + \frac{\gamma}{2}\left(\omega_1^2 + \omega_2^2\right)x^2 - \frac{\gamma}{2}\omega_1^2\omega_2^2 z^2, \qquad x = \dot{z} \tag{17}$$

$$[x, p_x] = i, \qquad [z, p_z] = i \tag{18}$$

$$\begin{aligned} z &= a_1 + a_1^\dagger + a_2 + a_2^\dagger, \\ p_z &= i\gamma\omega_1\omega_2^2(a_1 - a_1^\dagger) + i\gamma\omega_1^2\omega_2(a_2 - a_2^\dagger), \\ x &= -i\omega_1(a_1 - a_1^\dagger) - i\omega_2(a_2 - a_2^\dagger), \\ p_x &= -\gamma\omega_1^2(a_1 + a_1^\dagger) - \gamma\omega_2^2(a_2 + a_2^\dagger) \end{aligned} \tag{19}$$

$$H_{\mathrm{PU}} = 2\gamma(\omega_1^2 - \omega_2^2)(\omega_1^2 a_1^\dagger a_1 - \omega_2^2 a_2^\dagger a_2) + (\omega_1 + \omega_2)/2 \tag{20}$$

$$[a_1, a_1^\dagger] = \frac{1}{2\gamma\omega_1\left(\omega_1^2 - \omega_2^2\right)}, \qquad [a_2, a_2^\dagger] = -\frac{1}{2\gamma\omega_2\left(\omega_1^2 - \omega_2^2\right)} \tag{21}$$

To avoid negative norm states, try

$$a_1|\Omega\rangle = a_2^\dagger|\Omega\rangle = 0, \quad \langle\Omega|a_2^\dagger a_2|\Omega\rangle > 0, \quad H_{\mathrm{PU}}|\Omega\rangle = \frac{1}{2}(\omega_1 - \omega_2)|\Omega\rangle \tag{22}$$

Energy spectrum is unbounded below. So instead try

$$a_1|\Omega\rangle = a_2|\Omega\rangle = 0, \quad \langle\Omega|a_2 a_2^\dagger|\Omega\rangle < 0, \quad H_{\mathrm{PU}}|\Omega\rangle = \frac{1}{2}(\omega_1 + \omega_2)|\Omega\rangle \tag{23}$$

Negative norm state problem looks insurmountable, but.....

QUANTUM MECHANICS IS A GLOBAL THEORY. NEED TO SUPPLY GLOBAL INFORMATION. NEED TO LOOK AT WAVE FUNCTIONS. FIND THAT $H_{\rm PU}$, z, p_z ARE NOT HERMITIAN

$$[x,p_x]=i, \qquad p_x=-i\frac{\partial}{\partial x}, \qquad [z,p_z]=i, \qquad p_z=-i\frac{\partial}{\partial z} \tag{24}$$

$$\psi_0(z,x)=\exp\Big[\frac{\gamma}{2}(\omega_1+\omega_2)\omega_1\omega_2 z^2+i\gamma\omega_1\omega_2 zx-\frac{\gamma}{2}(\omega_1+\omega_2)x^2\Big] \tag{25}$$

The states of negative norm are also states of INFINITE norm since $\int dxdz\psi_0^*(z,x)\psi_0(z,x)$ **is divergent, and when acting on such states, one CANNOT set** $p_z=-i\partial/\partial z$.

$$\left[e^{i\theta}z,-\frac{i}{e^{i\theta}}\frac{\partial}{\partial z}\right]\psi(e^{i\theta}z)=i\psi(e^{i\theta}z), \qquad z\rightarrow -iz, \qquad p_z\rightarrow \frac{\partial}{\partial z} \tag{26}$$

p_z **and** z **not Hermitian – they are anti-Hermitian.**

$$y=e^{\pi p_z z/2}ze^{-\pi p_z z/2}=-iz, \qquad q=e^{\pi p_z z/2}p_z e^{-\pi p_z z/2}=ip_z \tag{27}$$

$$H=\frac{p^2}{2\gamma}-iqx+\frac{\gamma}{2}\left(\omega_1^2+\omega_2^2\right)x^2+\frac{\gamma}{2}\omega_1^2\omega_2^2 y^2\neq H^{\dagger}, \qquad p=p_x \tag{28}$$

$$\textbf{Hermitian} \quad [x,p]=i, \quad \textbf{Hermitian} \quad [y,q]=i, \quad \textbf{non-Hermitian} \;\; H \tag{29}$$

The Hamiltonian is not Hermitian – but it is PT symmetric, and thus still has real eigenvalues. Bender and collaborators showed that $H = p^2 + ix^3$ has a completely real energy spectrum.

$$C^2 = 1, \qquad [C, PT] = 0, \qquad [C, H] = 0, \qquad C = e^Q P \tag{30}$$

$$Q = \alpha[pq + \gamma^2\omega_1^2\omega_2^2 xy], \qquad \alpha = \frac{1}{\gamma\omega_1\omega_2}\log\left(\frac{\omega_1 + \omega_2}{\omega_1 - \omega_2}\right) \tag{31}$$

$$\tilde{H} = e^{-Q/2} H e^{Q/2} = \frac{p^2}{2\gamma} + \frac{q^2}{2\gamma\omega_1^2} + \frac{\gamma}{2}\omega_1^2 x^2 + \frac{\gamma}{2}\omega_1^2\omega_2^2 y^2 \tag{32}$$

Hamiltonian can be diagonalized by a similarity transformation which is non-unitary since Q is Hermitian rather than anti-Hermitian. Original Hamiltonian H is thus a Hermitian Hamiltonian as written in a skew basis. The eigenstates of H and $\tilde{H}$ are not unitarily equivalent, and thus ....

....THE NORM IS NOT THE DIRAC NORM

$$\tilde{H} = e^{-Q/2} H e^{Q/2} = \frac{p^2}{2\gamma} + \frac{q^2}{2\gamma\omega_1^2} + \frac{\gamma}{2}\omega_1^2 x^2 + \frac{\gamma}{2}\omega_1^2\omega_2^2 y^2 \tag{33}$$

$$\tilde{H}|\tilde{n}\rangle = E_n|\tilde{n}\rangle, \qquad H|n\rangle = E_n|n\rangle, \qquad |n\rangle = e^{Q/2}|\tilde{n}\rangle \tag{34}$$

$$\langle\tilde{n}|\tilde{H} = E_n\langle\tilde{n}|, \qquad \langle n| \equiv \langle\tilde{n}|e^{Q/2}, \qquad \langle n|e^{-Q}H = \langle n|e^{-Q}E_n \tag{35}$$

The energy eigenbra $\langle n|e^{-Q} = \langle n|PC$ is not the Dirac conjugate of the energy eigenket $|n\rangle$, since $\langle n|H^{\dagger} = \langle n|E_n$ is not an eigenvalue equation for H.

$$\langle\tilde{n}|\tilde{m}\rangle = \delta_{m,n}, \qquad \Sigma|\tilde{n}\rangle\langle\tilde{n}| = \mathbf{1}, \qquad \tilde{H} = \Sigma|\tilde{n}\rangle E_n\langle\tilde{n}| \tag{36}$$

$$\langle n|e^{-Q}|m\rangle = \delta_{m,n}, \qquad \Sigma|n\rangle\langle n|e^{-Q} = \mathbf{1}, \qquad H = \Sigma|n\rangle E_n\langle n|e^{-Q} \tag{37}$$

The e^{-Q} norm is positive and so theory is unitary. Since $C^2 = 1$, its eigenvalues are ± 1, with the relative plus and minus signs in the fourth-order propagator being due to the fact that the two poles have opposite-signed eigenvalues of C.

NON-DIAGONALIZABILITY AND UNITARITY THE SINGULAR EQUAL-FREQUENCY LIMIT

In equal frequency limit the diagonalizing operator Q becomes singular and partial fraction decomposition of propagator becomes undefined.

$$\begin{aligned}\psi_0(x,y,t) &= \exp\left[-\frac{\gamma}{2}(\omega_1+\omega_2)(x^2+\omega_1\omega_2 y^2)-\gamma\omega_1\omega_2 yx\right]\exp(-iE_0 t),\\ E_0 &= (\omega_1+\omega_2)/2\end{aligned} \tag{38}$$

$$\begin{aligned}\psi_1(x,y,t) &= (x+\omega_2 y)\psi_0(x,y,t)e^{-i\omega_1 t}, \qquad E_1 = E_0+\omega_1\\ \psi_2(x,y,t) &= (x+\omega_1 y)\psi_0(x,y,t)e^{-i\omega_2 t}, \qquad E_2 = E_0+\omega_2\end{aligned} \tag{39}$$

$$\hat{\psi}_0(x,y,t) = \exp\left[-\gamma\omega^3 y^2-\gamma\omega^2 yx-\gamma\omega x^2-i\omega t\right], \qquad \hat{E}_0=\omega \tag{40}$$

$$\hat{\psi}_1(x,y,t) = (x+\omega y)\hat{\psi}_0(x,y,t)e^{-i\omega t}, \qquad \hat{E}_1=\hat{E}_0+\omega \tag{41}$$

TWO one-particle states have collapsed into ONE state.

THE MISSING ENERGY EIGENSTATES....

$$H_{1\mathrm{P}}(\epsilon) = \frac{1}{2\omega}\begin{pmatrix} 4\omega^2+\epsilon^2 & 4\omega^2-\epsilon^2 \\ \epsilon^2 & 4\omega^2-\epsilon^2 \end{pmatrix}, \qquad \omega_1 \equiv \omega+\epsilon, \quad \omega_2 \equiv \omega-\epsilon \tag{42}$$

$$|2\omega+\epsilon\rangle \equiv \begin{pmatrix} 2\omega+\epsilon \\ \epsilon \end{pmatrix}, \qquad |2\omega-\epsilon\rangle \equiv \begin{pmatrix} 2\omega-\epsilon \\ -\epsilon \end{pmatrix} \tag{43}$$

$$S^{-1}\left(\frac{1}{2\omega}\right)\begin{pmatrix} 4\omega^2+\epsilon^2 & 4\omega^2-\epsilon^2 \\ \epsilon^2 & 4\omega^2-\epsilon^2 \end{pmatrix} S = \begin{pmatrix} 2\omega+\epsilon & 0 \\ 0 & 2\omega-\epsilon \end{pmatrix} \tag{44}$$

$$S = \frac{1}{2\epsilon\omega^{1/2}(2\omega+\epsilon)^{1/2}}\begin{pmatrix} 2\omega+\epsilon & -(4\omega^2-\epsilon^2)\epsilon \\ \epsilon & (2\omega+\epsilon)\epsilon^2 \end{pmatrix} \tag{45}$$

$$H_{1P}(\epsilon=0) = 2\omega\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \tag{46}$$

Non-diagonalizable, Jordan-block matrix with TWO eigenvalues ($\lambda_1 = 1,\ \lambda_2 = 1$ since $\mathrm{Tr} = 1,\ \mathrm{Det} = 1$), but only ONE eigenvector.

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} c+d \\ d \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix}, \qquad d=0 \tag{47}$$

... BECAME NONSTATIONARY

$$\begin{aligned}
\hat{\psi}_1(x,y,t) &= \lim_{\epsilon\to 0}\frac{\psi_2(x,y,t)+\psi_1(x,y,t)}{2} = (x+\omega y)\hat{\psi}_0(x,y,t)e^{-i\omega t}\\
\hat{\psi}_{1a}(x,y,t) &= \lim_{\epsilon\to 0}\frac{\psi_2(x,y,t)-\psi_1(x,y,t)}{2\epsilon} = \left[(x+\omega y)it+y\right]\hat{\psi}_0(x,y,t)e^{-i\omega t}\\
\hat{\psi}_2(x,y,t) &= \lim_{\epsilon\to 0}\frac{\psi_3(x,y,t)+\psi_4(x,y,t)+\psi_5(x,y,t)}{3} = \left[(x+\omega y)^2-1/2\gamma\omega\right]\hat{\psi}_0(x,y,t)e^{-2i\omega t}\\
\hat{\psi}_{2a}(x,y,t) &= \lim_{\epsilon\to 0}\frac{\psi_5^{\rm R}(x,y,t)-\psi_3^{\rm R}(x,y,t)}{2\epsilon}\\
&= \left[\left((x+\omega y)^2-\frac{1}{2\gamma\omega}\right)2it+2xy+2\omega y^2-\frac{1}{2\gamma\omega^2}\right]\hat{\psi}_0^{\rm R}(x,y,t)e^{-2i\omega t}\\
\hat{\psi}_{2b}(x,y,t) &= \lim_{\epsilon\to 0}\frac{2\psi_4^{\rm R}(x,y,t)-\psi_3^{\rm R}(x,y,t)-\psi_5^{\rm R}(x,y,t)}{2\epsilon^2}\\
&= \left[\left((x+\omega y)^2-\frac{1}{2\gamma\omega}\right)2t^2-\left(2xy+2\omega y^2-\frac{1}{2\gamma\omega^2}\right)2it-2y^2+\frac{1}{2\gamma\omega^3}\right]\hat{\psi}_0^{\rm R}(x,y,t)e^{-2i\omega t}
\end{aligned} \tag{48}$$

$$i\frac{\partial}{\partial t}\hat{\psi}(x,y,t) = \left(-\frac{1}{2\gamma}\frac{\partial^2}{\partial x^2}-x\frac{\partial}{\partial y}+\gamma\omega^2x^2+\frac{\gamma}{2}\omega^4y^2\right)\hat{\psi}(x,y,t) \tag{49}$$

Stationary plus non-stationary together are complete since just the right number of independent polynomial functions of x and y.

$$i\frac{\partial}{\partial t}\int dx\,dy\,\hat{\psi}_B^{\rm c}(x,y,t)\hat{\psi}_A(x,y,t) = -\int dx\,dy\,x\frac{\partial}{\partial y}\left[\hat{\psi}_B^{\rm c}(x,y,t)\hat{\psi}_A(x,y,t)\right] = 0 \tag{50}$$

Norm preserved in time so time evolution is UNITARY.

Figure 1: Face on view of a spiral galaxy

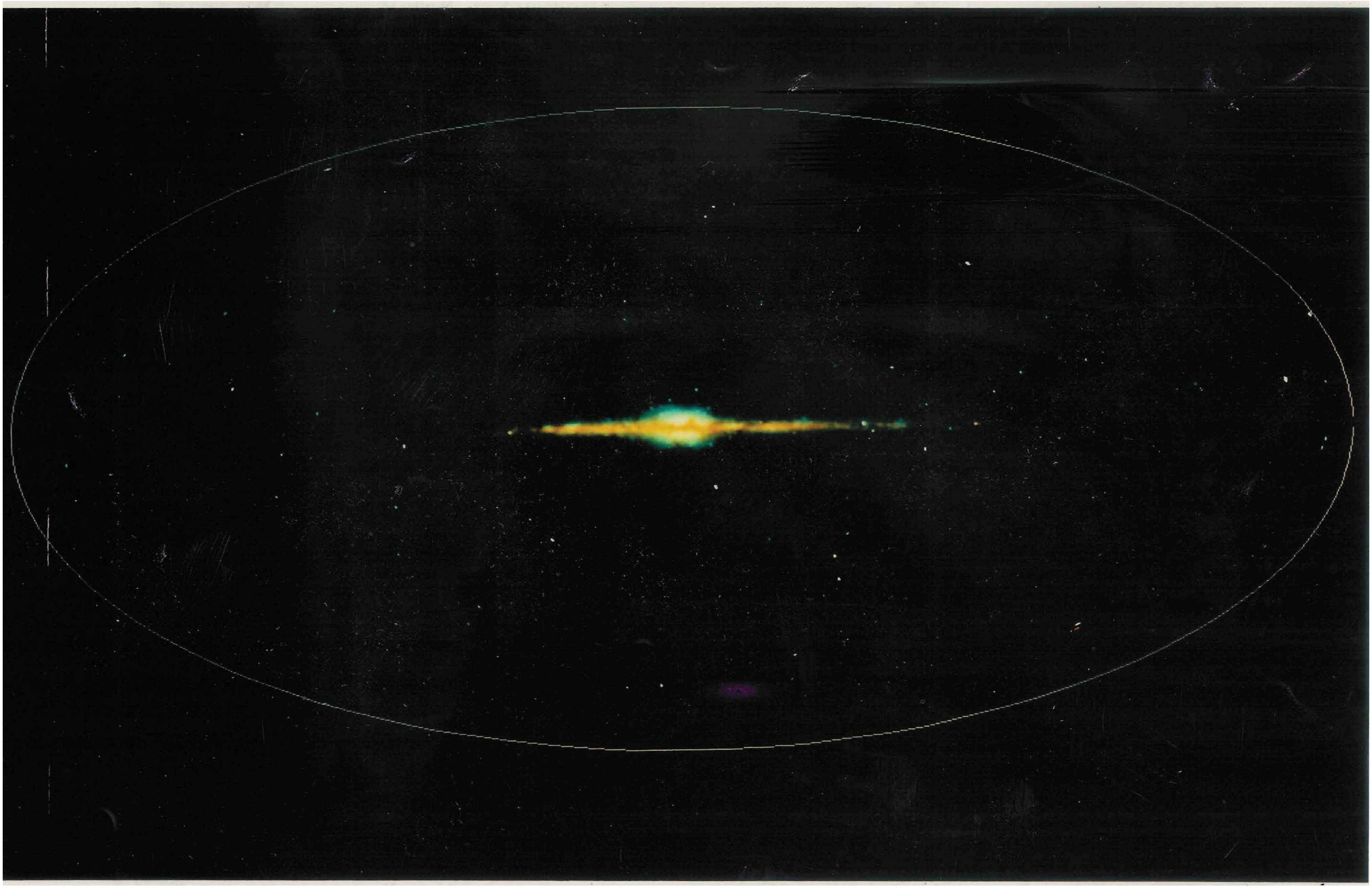


Figure 2: Edge on view of a spiral galaxy

IMPACT OF GLOBAL LINEAR AND QUADRATIC POTENTIALS ON GALACTIC ROTATION CURVES

Mannheim and Kazanas, Astrophysical Journal 342, 635 (1989):

$$ds^2 = -B(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega_2, \qquad B(r > r_0) = 1 - \frac{2\beta}{r} + \gamma r - kr^2. \tag{51}$$

$$\frac{3}{B(r)}(W^0{}_0 - W^r{}_r) = \nabla^4 B = B'''' + \frac{4B'''}{r} = \frac{(rB)''''}{r} = f(r) = \frac{3}{4\alpha_g B(r)}(T^0{}_0 - T^r{}_r), \tag{52}$$

$$\gamma = -\frac{1}{2}\int_0^{r_0} dr' r'^2 f(r'), \qquad 2\beta = \frac{1}{6}\int_0^{r_0} dr' r'^4 f(r'). \tag{53}$$

$$\nabla^4\phi = \rho, \qquad \phi = -\frac{\beta}{r} + \frac{\gamma r}{2} \qquad \textbf{Second order Poisson equation not unique} \tag{54}$$

$$V^*(r) = -\frac{\beta^* c^2}{r} + \frac{\gamma^* c^2 r}{2} \tag{55}$$

Typically for spiral galaxies $\Sigma(R) = \Sigma_0 e^{-R/R_0}$

$$\begin{aligned}\frac{v^2_{\rm LOC}}{R} &= \frac{N^*\beta^* c^2 R}{2R_0^3}\left[I_0\left(\frac{R}{2R_0}\right)K_0\left(\frac{R}{2R_0}\right) - I_1\left(\frac{R}{2R_0}\right)K_1\left(\frac{R}{2R_0}\right)\right] \\ &\quad + \frac{N^*\gamma^* c^2 R}{2R_0} I_1\left(\frac{R}{2R_0}\right)K_1\left(\frac{R}{2R_0}\right)\end{aligned} \tag{56}$$

$$\frac{v^2_{\rm LOC}}{R} \to \frac{N^*\beta^* c^2}{R^2}\left(1 + \frac{9R_0^2}{2R^2}\right) + \frac{N^*\gamma^* c^2}{2}\left(1 - \frac{3R_0^2}{2R^2} - \frac{45R_0^4}{8R^4}\right) \to \frac{N^*\beta^* c^2}{R^2} + \frac{N^*\gamma^* c^2}{2}, \tag{57}$$

Comparing second order and fourth order

$$\phi(r) = -\frac{1}{r}\int_0^r dr' r'^2 g(r') - \int_r^\infty dr' r' g(r'), \tag{58}$$

$$\frac{d\phi(r)}{dr} = \frac{1}{r^2}\int_0^r dr' r'^2 g(r'). \tag{59}$$

Newtonian Gravity is LOCAL

$$\phi(r) = -\frac{r}{2}\int_0^r dr' r'^2 h(r') - \frac{1}{6r}\int_0^r dr' r'^4 h(r') - \frac{1}{2}\int_r^\infty dr' r'^3 h(r') - \frac{r^2}{6}\int_r^\infty dr' r' h(r'). \tag{60}$$

$$\frac{d\phi(r)}{dr} = -\frac{1}{2}\int_0^r dr' r'^2 h(r') + \frac{1}{6r^2}\int_0^r dr' r'^4 h(r') - \frac{r}{3}\int_r^\infty dr' r' h(r'), \qquad h(r) = \frac{f(r)c^2}{2} \tag{61}$$

Conformal Gravity is GLOBAL

So cannot ignore the rest of the universe

Rest of universe has homogeneous and inhomogeneous matter

For homogeneous matter have Roberston-Walker Hubble flow

$$\rho = \frac{4r}{2(1+\gamma_0 r)^{1/2} + 2 + \gamma_0 r}, \qquad \tau = \int dt R(t) \tag{62}$$

$$
\begin{aligned}
&-(1+\gamma_0 r)c^2dt^2 + \frac{dr^2}{(1+\gamma_0 r)} + r^2 d\Omega_2 = \\
&\frac{1}{R^2(\tau)}\left(\frac{1+\gamma_0\rho/4}{1-\gamma_0\rho/4}\right)^2\left[-c^2d\tau^2 + \frac{R^2(\tau)}{[1-\gamma_0^2\rho^2/16]^2}\left(d\rho^2+\rho^2 d\Omega_2\right)\right], \qquad K = -\frac{\gamma_0^2}{4}
\end{aligned} \tag{63}
$$

In rest frame an open ($K < 0$) RW geometry acts like a universal linear potential

$$
\frac{v_{\rm TOT}^2}{R} = \frac{v_{\rm LOC}^2}{R} + \frac{\gamma_0 c^2}{2}. \tag{64}
$$

$$
\frac{v_{\rm TOT}^2}{R} \to \frac{N^*\beta^* c^2}{R^2} + \frac{N^*\gamma^* c^2}{2} + \frac{\gamma_0 c^2}{2}. \tag{65}
$$

$$
\begin{aligned}
&N^* = \text{visible mass in solar mass units}, \qquad \beta^* = 1.48\times 10^5 \text{ cm}, \\
&\gamma^* = 5.42\times 10^{-41}\text{cm}^{-1}, \qquad \gamma_0 = 3.06\times 10^{-30}\text{cm}^{-1} \quad .
\end{aligned} \tag{66}
$$

But also inhomogeneous matter – clusters and superclusters of galaxies

$$
\frac{v_{\rm TOT}^2}{R} = \frac{v_{\rm LOC}^2}{R} + \frac{\gamma_0 c^2}{2} - \kappa c^2 R, \tag{67}
$$

$$
\frac{v_{\rm TOT}^2}{R} \to \frac{N^*\beta^* c^2}{R^2} + \frac{N^*\gamma^* c^2}{2} + \frac{\gamma_0 c^2}{2} - \kappa c^2 R, \qquad \kappa = 9.54\times 10^{-54}\text{cm}^{-2}. \tag{68}
$$

Mannheim and O'Brien fit 111 galaxies with VISIBLE N^* of each galaxy as only variable, β^*, γ^*, γ_0^* and κ are all universal, and NO DARK MATTER.

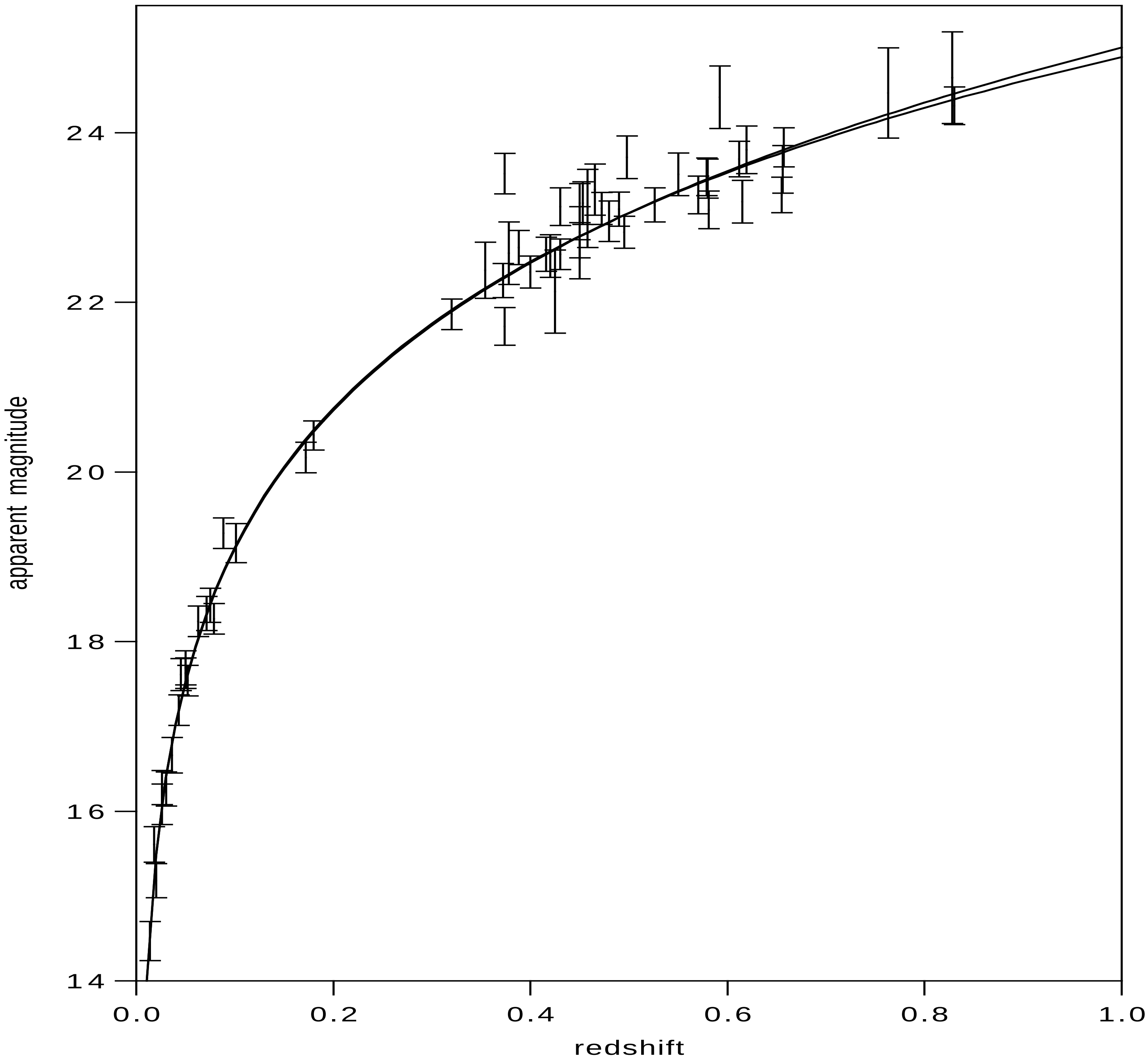


Figure 3: The $q_0 = -0.37$ conformal gravity fit (upper curve) and the $\Omega_M(t_0) = 0.3$, $\Omega_\Lambda(t_0) = 0.7$ standard model fit (lower curve) to the $z < 1$ supernovae Hubble plot data.

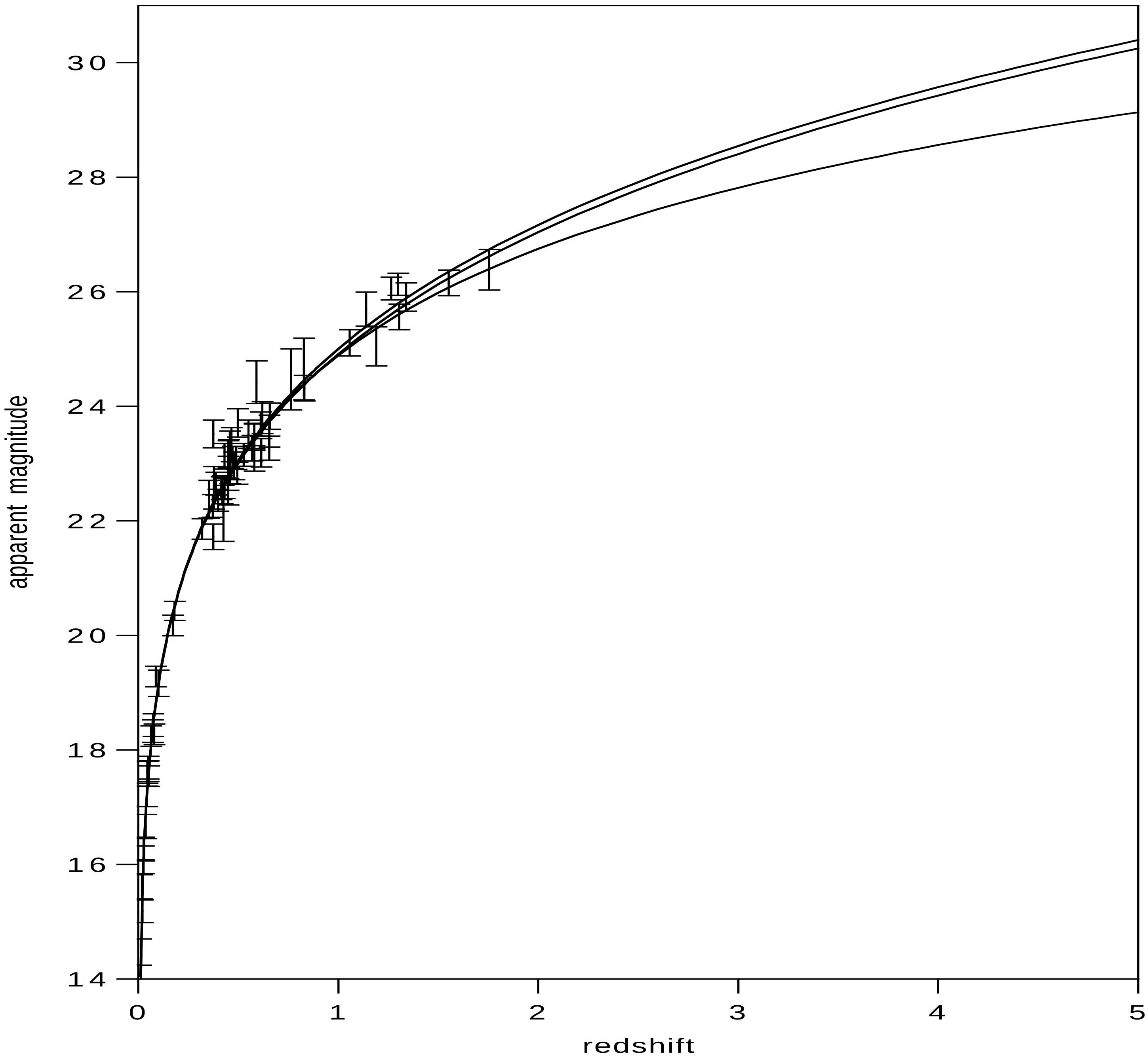
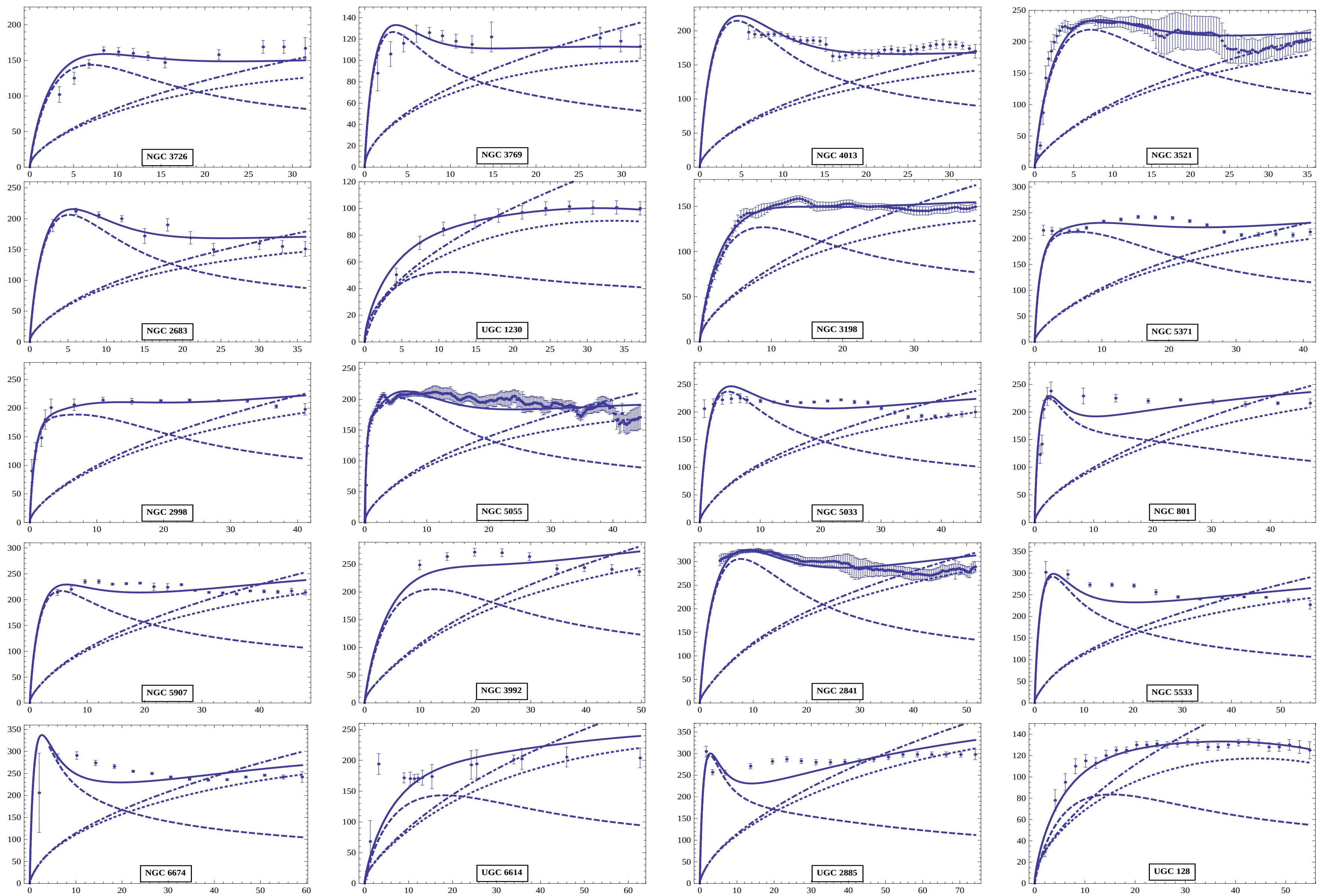


Figure 4: Hubble plot expectations for $q_0 = -0.37$ (highest curve) and $q_0 = 0$ (middle curve) conformal gravity and for $\Omega_M(t_0) = 0.3$, $\Omega_\Lambda(t_0) = 0.7$ standard gravity (lowest curve).

Table 1: Properties of the 20 Large Galaxy Sample

Galaxy	Type	Distance (Mpc)	L_B (10^{10}L$_\odot$)	R_0 (kpc)	$R_{\rm last}$ (kpc)	$M_{\rm HI}$ ($10^{10}M_\odot$)	$M_{\rm disk}$ ($10^{10}M_\odot$)	$(M/L)_{\rm stars}$ ($M_\odot L_\odot^{-1}$)	$(v^2/c^2R)_{\rm last}$ (10^{-30}cm^{-1})
NGC 3726	HSB	17.4	3.34	3.2	31.5	0.60	3.82	1.15	3.19
NGC 3769	HSB	15.5	0.68	1.5	32.2	0.41	1.36	1.99	1.43
NGC 4013	HSB	18.6	2.09	2.1	33.1	0.32	5.58	2.67	3.14
NGC 3521	HSB	12.2	4.77	3.3	35.3	1.03	9.25	1.94	4.21
NGC 2683	HSB	10.2	1.88	2.4	36.0	0.15	6.03	3.20	2.28
UGC 1230	LSB	54.1	0.37	4.7	37.1	0.65	0.67	1.82	0.97
NGC 3198	HSB	14.1	3.24	4.0	38.6	1.06	3.64	1.12	2.09
NGC 5371	HSB	35.3	7.59	4.4	41.0	0.89	8.52	1.44	3.98
NGC 2998	HSB	59.3	5.19	4.8	41.1	1.78	7.16	1.75	3.43
NGC 5055	HSB	9.2	3.62	2.9	44.4	0.76	6.04	1.87	2.36
NGC 5033	HSB	15.3	3.06	7.5	45.6	1.07	0.27	3.28	3.16
NGC 0801	HSB	63.0	4.75	9.5	46.7	1.39	6.93	2.37	3.59
NGC 5907	HSB	16.5	5.40	5.5	48.0	1.90	2.49	1.89	3.44
NGC 3992	HSB	25.6	8.46	5.7	49.6	1.94	13.94	1.65	4.08
NGC 2841	HSB	14.1	4.74	3.5	51.6	0.86	19.55	4.12	5.83
UGC 0128	LSB	64.6	0.60	6.9	54.8	0.73	2.75	4.60	1.03
NGC 5533	HSB	42.0	3.17	7.4	56.0	1.39	2.00	4.14	3.31
NGC 6674	HSB	42.0	4.94	7.1	59.1	2.18	2.00	2.52	3.57
UGC 6614	LSB	86.2	2.11	8.2	62.7	2.07	9.70	4.60	2.39
UGC 2885	HSB	80.4	23.96	13.3	74.1	3.98	8.47	0.72	4.31



Fitting to the rotational velocities (in km $\sec^{-1}$) of the 20 large galaxy sample. No dark matter is assumed.

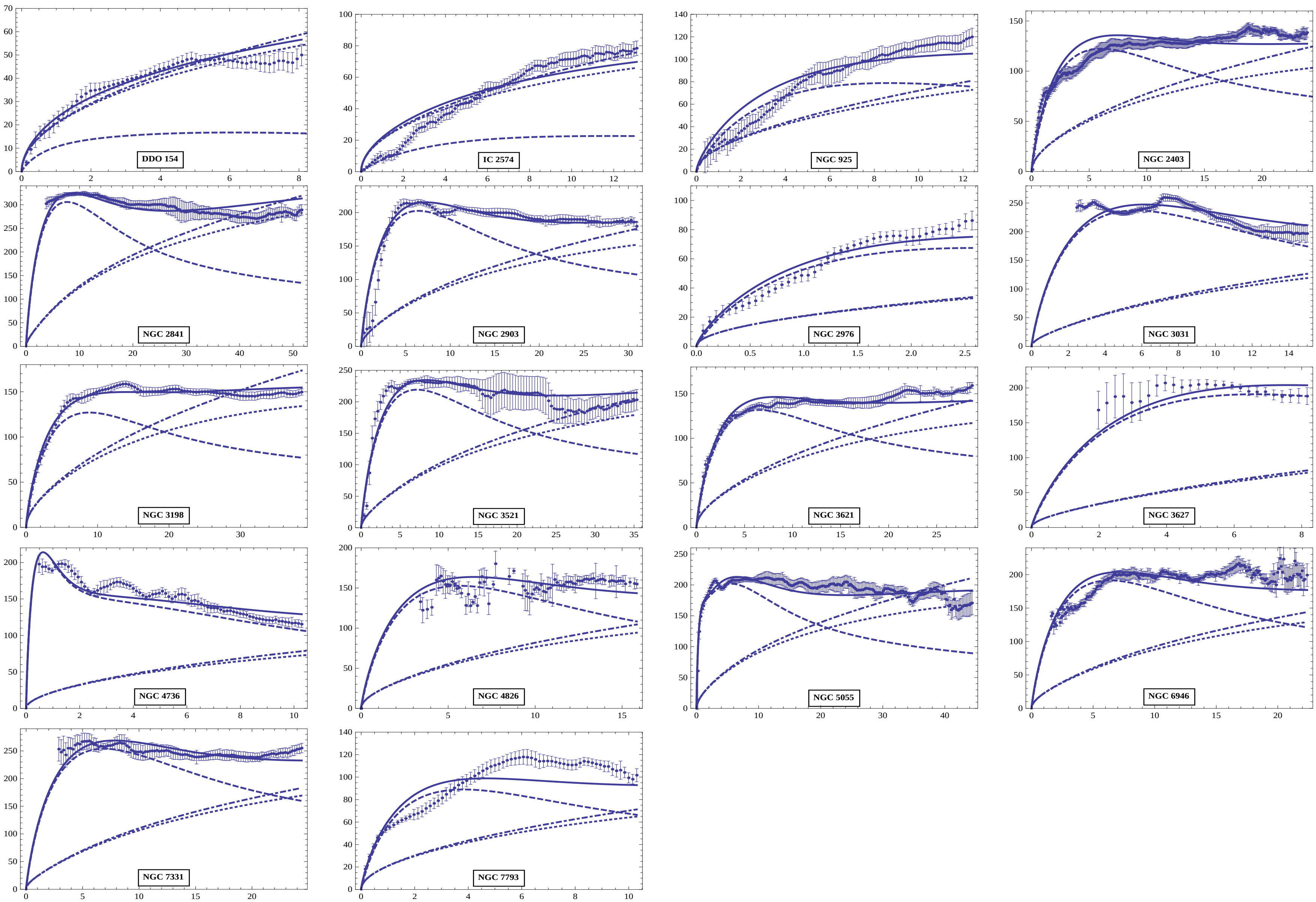


FIG. 1: Fitting to the rotational velocities (in km sec^{-1}) of the THINGS 18 galaxy sample.

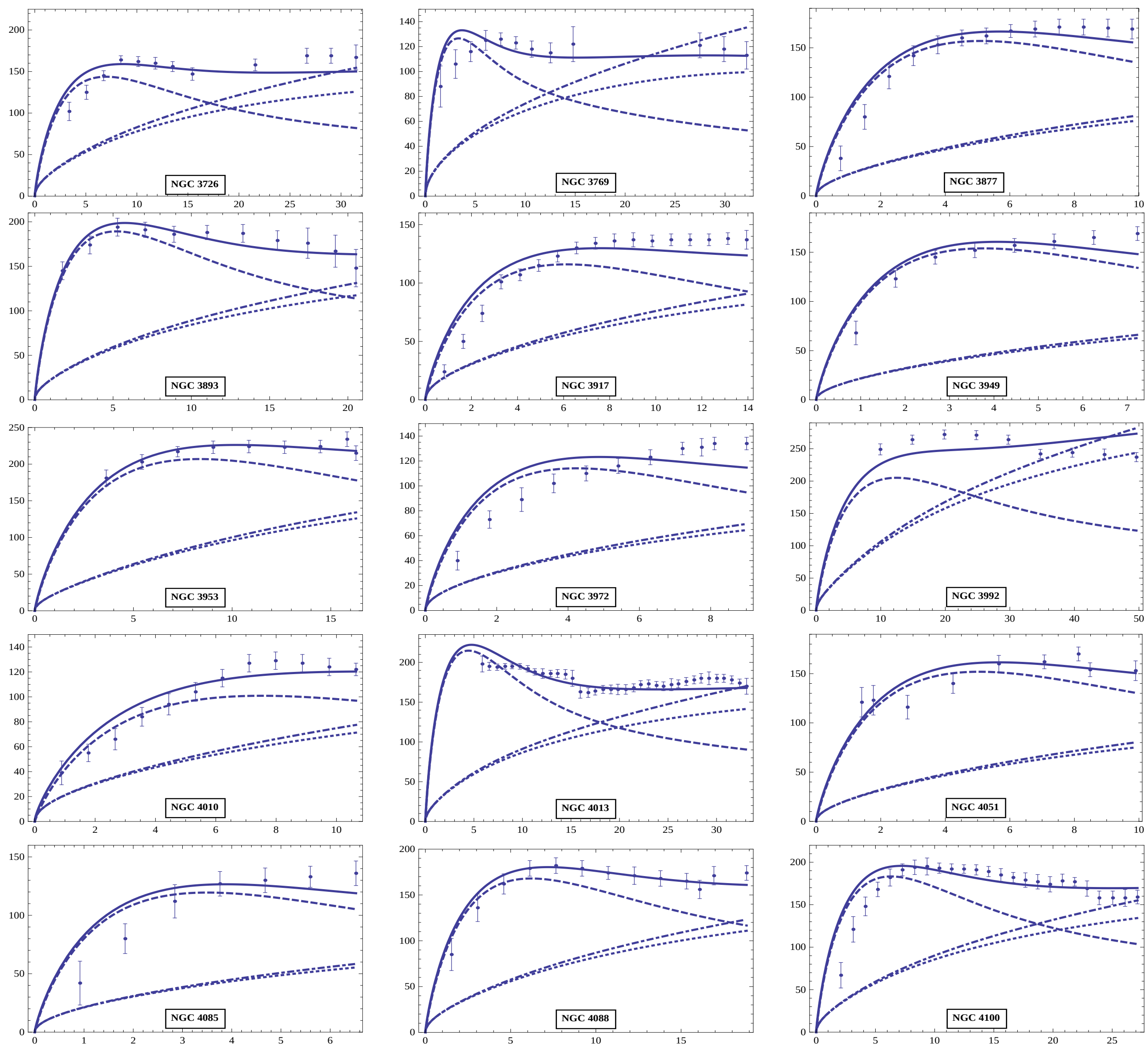


FIG. 2: Fitting to the rotational velocities of the Ursa Major 30 galaxy sample – Part 1

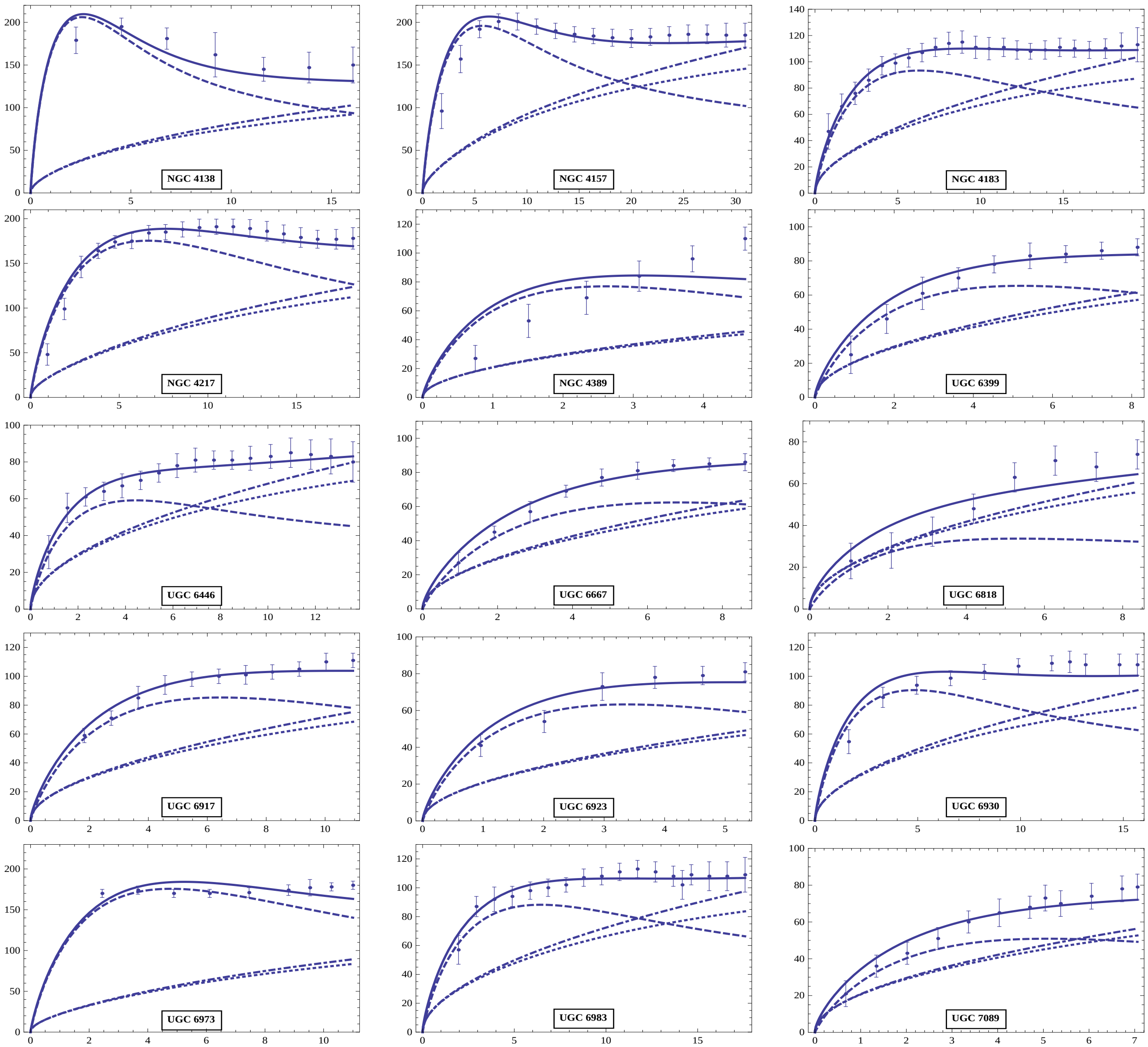


FIG. 2: Fitting to the rotational velocities of the Ursa Major 30 galaxy sample – Part 2

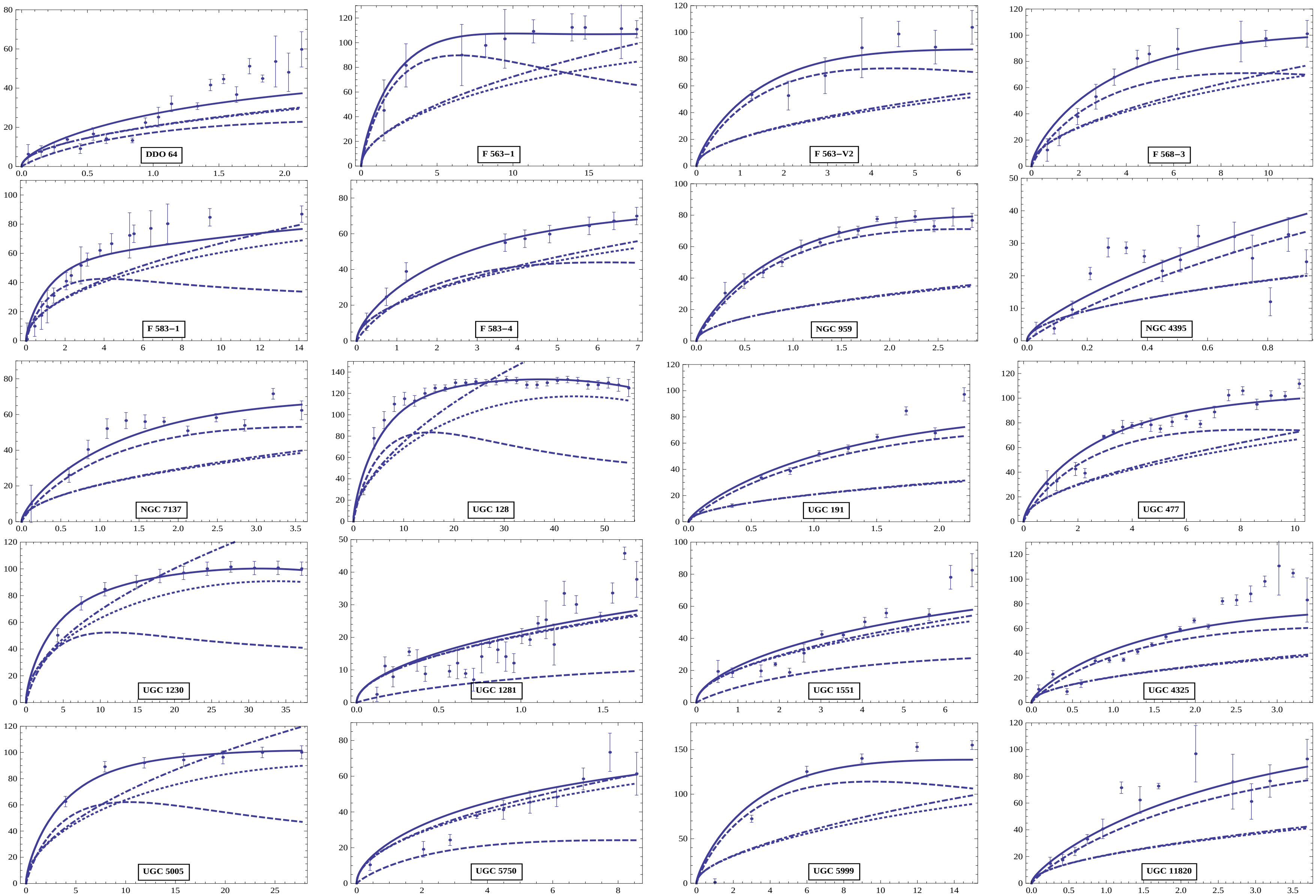


FIG. 3: Fitting to the rotational velocities of the LSB 20 galaxy sample

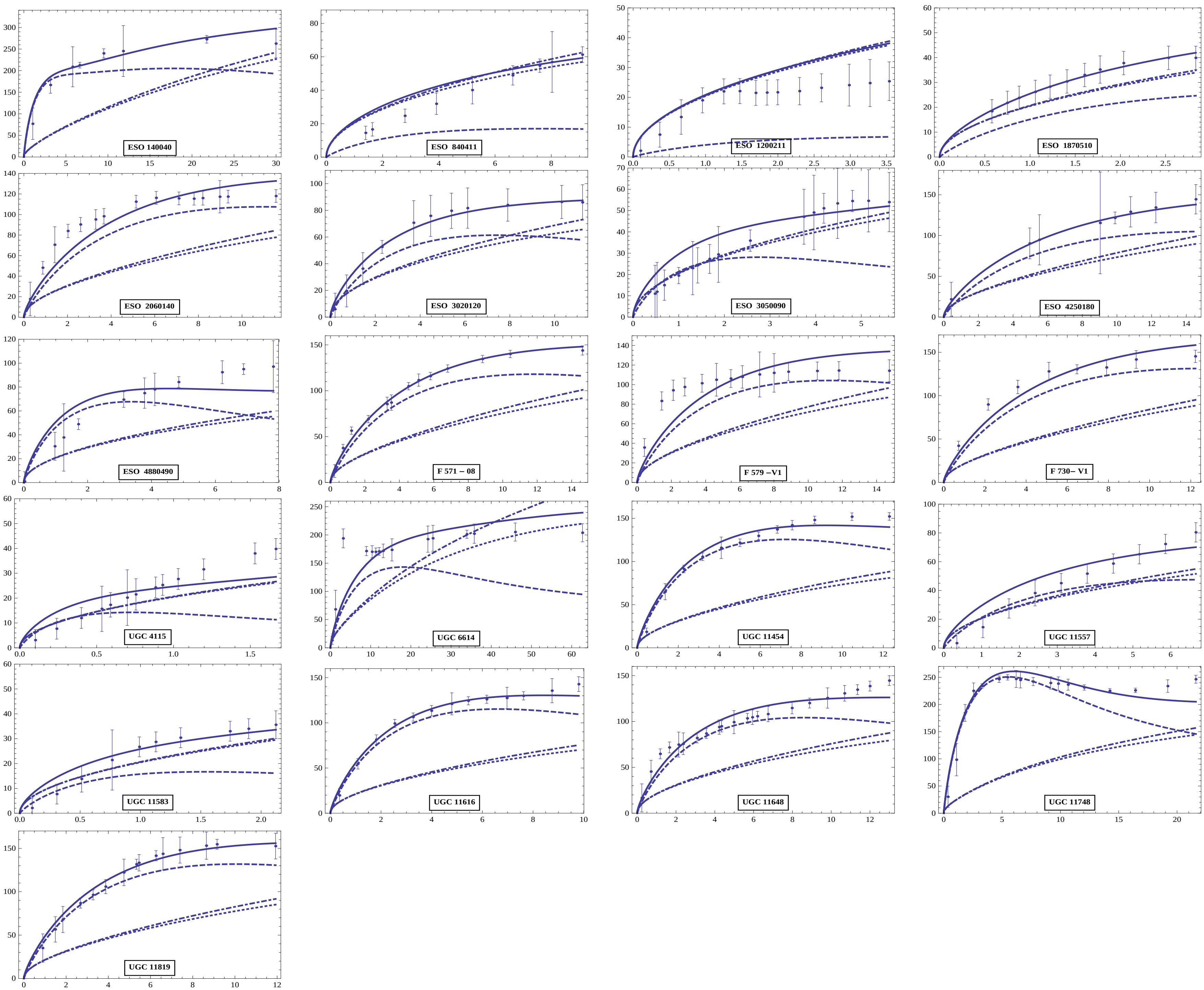


FIG. 4: Fitting to the rotational velocities of the LSB 21 galaxy sample

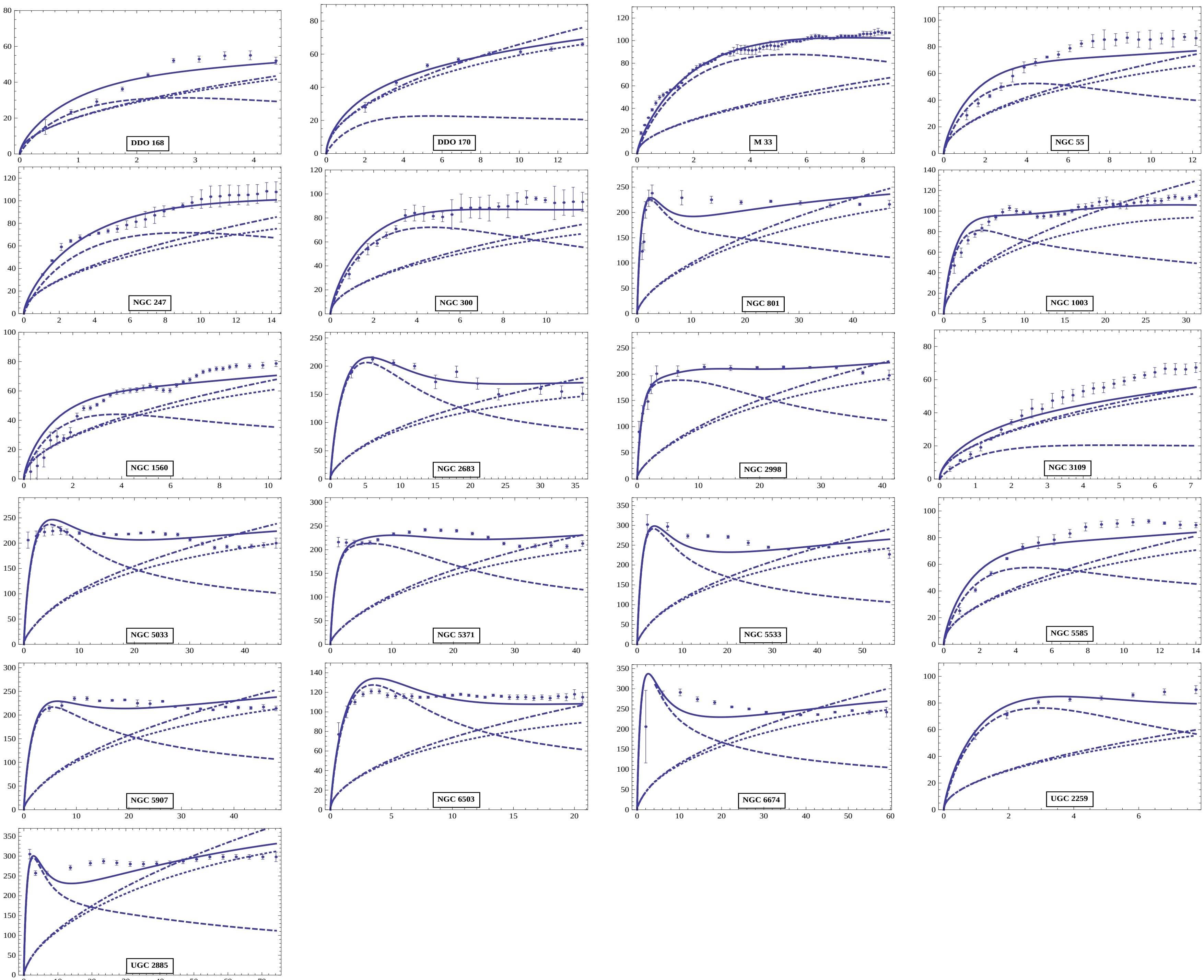


FIG. 5: Fitting to the rotational velocities of the Miscellaneous 21 galaxy sample

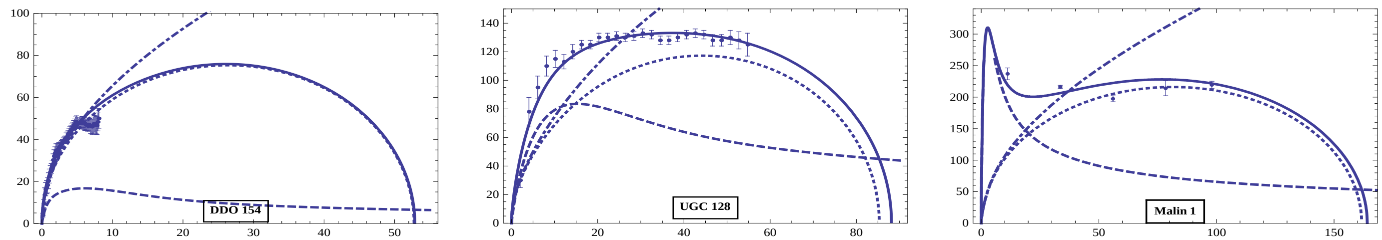


FIG. 6: Extended distance predictions for DDO 154, UGC 128, and Malin 1.

Table 2: Properties of the THINGS 18 Galaxy Sample

Galaxy	Type	Distance (Mpc)	$L_{\rm B}$ ($10^{10}{\rm L}_\odot$)	R_0 (kpc)	$R_{\rm last}$ (kpc)	$M_{\rm HI}$ ($10^{10}M_\odot$)	$M_{\rm disk}$ ($10^{10}M_\odot$)	$(M/L)_{\rm stars}$ ($M_\odot/L_\odot$)	$(v^2/c^2R)_{\rm last}$ ($10^{-30}{\rm cm}^{-1}$)
DDO 0154	LSB	4.2	0.007	0.8	8.1	0.03	0.003	0.45	1.12
IC 2574	LSB	4.5	0.345	4.2	13.1	0.19	0.098	0.28	1.69
NGC 0925	LSB	8.7	1.444	3.9	12.4	0.41	1.372	0.95	4.17
NGC 2403	HSB	4.3	1.647	2.7	23.9	0.46	2.370	1.44	2.89
NGC 2841	HSB	14.1	4.742	3.5	51.6	0.86	19.552	4.12	5.83
NGC 2903	HSB	9.4	4.088	3.0	30.9	0.49	7.155	1.75	3.75
NGC 2976	LSB	3.6	0.201	1.2	2.6	0.01	0.322	1.60	10.43
NGC 3031	HSB	3.7	3.187	2.6	15.0	0.38	8.662	2.72	9.31
NGC 3198	HSB	14.1	3.241	4.0	38.6	1.06	3.644	1.12	2.09
NGC 3521	HSB	12.2	4.769	3.3	35.3	1.03	9.245	1.94	4.21
NGC 3621	HSB	7.4	2.048	2.9	28.7	0.89	2.891	1.41	3.18
NGC 3627	HSB	10.2	3.700	3.1	8.2	0.10	6.622	1.79	15.64
NGC 4736	HSB	5.0	1.460	2.1	10.3	0.05	1.630	1.60	4.66
NGC 4826	HSB	5.4	1.441	2.6	15.8	0.03	3.640	2.53	5.46
NGC 5055	HSB	9.2	3.622	2.9	44.4	0.76	6.035	1.87	2.36
NGC 6946	HSB	6.9	3.732	2.9	22.4	0.57	6.272	1.68	6.39
NGC 7331	HSB	14.2	6.773	3.2	24.4	0.85	12.086	1.78	9.61
NGC 7793	HSB	5.2	0.910	1.7	10.3	0.16	0.793	0.87	3.61

Table 3: Properties of the Ursa Major 30 Galaxy Sample

Galaxy	Type	Distance (Mpc)	$L_{\rm B}$ ($10^{10}{\rm L}_\odot$)	R_0 (kpc)	$R_{\rm last}$ (kpc)	$M_{\rm HI}$ ($10^{10}M_\odot$)	$M_{\rm disk}$ ($10^{10}M_\odot$)	$(M/L)_{\rm stars}$ ($M_\odot/L_\odot$)	$(v^2/c^2R)_{\rm last}$ ($10^{-30}\,\mathtt{cm}^{-1}$)	v
NGC 3726	HSB	17.4	3.340	3.2	31.5	0.60	3.82	1.15	3.19	
NGC 3769	HSB	15.5	0.684	1.5	32.2	0.41	1.36	1.99	1.43	
NGC 3877	HSB	15.5	1.948	2.4	9.8	0.11	3.44	1.76	10.51	
NGC 3893	HSB	18.1	2.928	2.4	20.5	0.59	5.00	1.71	3.85	
NGC 3917	LSB	16.9	1.334	2.8	13.9	0.17	2.23	1.67	4.85	
NGC 3949	HSB	18.4	2.327	1.7	7.2	0.35	2.37	1.02	14.23	
NGC 3953	HSB	18.7	4.236	3.9	16.3	0.31	9.79	2.31	10.20	
NGC 3972	HSB	18.6	0.978	2.0	9.0	0.13	1.49	1.53	7.18	
NGC 3992	HSB	25.6	8.456	5.7	49.6	1.94	13.94	1.65	4.08	
NGC 4010	LSB	18.4	0.883	3.4	10.6	0.29	2.03	2.30	5.03	
NGC 4013	HSB	18.6	2.088	2.1	33.1	0.32	5.58	2.67	3.14	
NGC 4051	HSB	14.6	2.281	2.3	9.9	0.18	3.17	1.39	8.52	
NGC 4085	HSB	19.0	1.212	1.6	6.5	0.15	1.34	1.11	10.21	
NGC 4088	HSB	15.8	2.957	2.8	18.8	0.64	4.67	1.58	5.79	
NGC 4100	HSB	21.4	3.388	2.9	27.1	0.44	5.74	1.69	3.35	
NGC 4138	LSB	15.6	0.827	1.2	16.1	0.11	2.97	3.59	5.04	
NGC 4157	HSB	18.7	2.901	2.6	30.9	0.88	5.83	2.01	3.99	
NGC 4183	HSB	16.7	1.042	2.9	19.5	0.30	1.43	1.38	2.36	
NGC 4217	HSB	19.6	3.031	3.1	18.2	0.30	5.53	1.83	6.28	
NGC 4389	HSB	15.5	0.610	1.2	4.6	0.04	0.42	0.68	9.49	
UGC 6399	LSB	18.7	0.291	2.4	8.1	0.07	0.59	2.04	3.42	
UGC 6446	LSB	15.9	0.263	1.9	13.6	0.24	0.36	1.36	1.70	
UGC 6667	LSB	19.8	0.422	3.1	8.6	0.10	0.71	1.67	3.09	
UGC 6818	LSB	21.7	0.352	2.1	8.4	0.16	0.11	0.33	2.35	
UGC 6917	LSB	18.9	0.563	2.9	10.9	0.22	1.24	2.20	4.05	
UGC 6923	LSB	18.0	0.297	1.5	5.3	0.08	0.35	1.18	4.43	
UGC 6930	LSB	17.0	0.601	2.2	15.7	0.29	1.02	1.69	2.68	
UGC 6973	HSB	25.3	1.647	2.2	11.0	0.35	3.99	2.42	10.58	
UGC 6983	LSB	20.2	0.577	2.9	17.6	0.37	1.28	2.22	2.43	
UGC 7089	LSB	13.9	0.352	2.3	7.1	0.07	0.35	0.98	3.18	

Table 4: Properties of the LSB 20 Galaxy Sample

Galaxy	Type	Distance (Mpc)	$L_{\rm B}$ (10^{10}L$_\odot$)	R_0 (kpc)	$R_{\rm last}$ (kpc)	$M_{\rm HI}$ ($10^{10}M_\odot$)	$M_{\rm disk}$ ($10^{10}M_\odot$)	$(M/L)_{\rm stars}$ ($M_\odot/L_\odot$)	$(v^2/c^2R)_{\rm last}$ ($10^{-30}\texttt{cm}^{-1}$)
DDO 0064	LSB	6.8	0.015	1.3	2.1	0.02	0.04	2.87	6.05
F563-1	LSB	46.8	0.140	2.9	18.2	0.29	1.35	9.65	2.44
F563-V2	LSB	57.8	0.266	2.0	6.3	0.20	0.60	2.26	6.15
F568-3	LSB	80.0	0.351	4.2	11.6	0.30	1.20	3.43	3.16
F583-1	LSB	32.4	0.064	1.6	14.1	0.18	0.15	2.32	1.92
F583-4	LSB	50.8	0.096	2.8	7.0	0.06	0.31	3.25	2.52
NGC 0959	LSB	13.5	0.333	1.3	2.9	0.05	0.37	1.11	7.43
NGC 4395	LSB	4.1	0.374	2.7	0.9	0.13	0.83	2.21	2.29
NGC 7137	LSB	25.0	0.959	1.7	3.6	0.10	0.27	0.28	3.91
UGC 0128	LSB	64.6	0.597	6.9	54.8	0.73	2.75	4.60	1.03
UGC 0191	LSB	15.9	0.129	1.7	2.2	0.26	0.49	3.81	15.48
UGC 0477	LSB	35.8	0.871	3.5	10.2	1.02	1.00	1.14	4.42
UGC 1230	LSB	54.1	0.366	4.7	37.1	0.65	0.67	1.82	0.97
UGC 1281	LSB	5.1	0.017	1.6	1.7	0.03	0.01	0.53	3.02
UGC 1551	LSB	35.6	0.780	4.2	6.6	0.44	0.16	0.20	3.69
UGC 4325	LSB	11.9	0.373	1.9	3.4	0.10	0.40	1.08	7.39
UGC 5005	LSB	51.4	0.200	4.6	27.7	0.28	1.02	5.11	1.30
UGC 5750	LSB	56.1	0.472	3.3	8.6	0.10	0.10	0.21	1.58
UGC 5999	LSB	44.9	0.170	4.4	15.0	0.18	3.36	19.81	5.79
UGC 11820	LSB	17.1	0.169	3.6	3.7	0.40	1.68	9.95	8.44

Table 5: Properties of the LSB 21 Galaxy Sample

Galaxy	Type	Distance (Mpc)	$L_{\rm B}$ ($10^{10}{\rm L}_\odot$)	R_0 (kpc)	$R_{\rm last}$ (kpc)	$M_{\rm HI}$ ($10^{10}M_\odot$)	$M_{\rm disk}$ ($10^{10}M_\odot$)	$(M/L)_{\rm stars}$ ($M_\odot/L_\odot$)	$(v^2/c^2R)_{\rm last}$ ($10^{-30}{\rm cm}^{-1}$)
ESO 0140040	LSB	217.8	7.169	10.1	30.0		20.70	3.38	8.29
ESO 0840411	LSB	82.4	0.287	3.5	9.1		0.06	0.21	1.49
ESO 1200211	LSB	15.2	0.028	2.0	3.5		0.01	0.20	0.66
ESO 1870510	LSB	16.8	0.054	2.1	2.8		0.09	1.62	2.02
ESO 2060140	LSB	59.6	0.735	5.1	11.6		3.51	4.78	4.34
ESO 3020120	LSB	70.9	0.717	3.4	11.2		0.77	1.07	2.37
ESO 3050090	LSB	13.2	0.186	1.3	5.6		0.06	0.32	1.87
ESO 4250180	LSB	88.3	2.600	7.3	14.6		4.79	1.84	5.17
ESO 4880490	LSB	28.7	0.139	1.6	7.8		0.43	3.07	4.34
F571-8	LSB	50.3	0.191	5.4	14.6	0.16	4.48	23.49	5.10
F579-V1	LSB	86.9	0.557	5.2	14.7	0.21	3.33	5.98	3.18
F730-V1	LSB	148.3	0.756	5.8	12.2		5.95	7.87	6.22
UGC 04115	LSB	5.5	0.004	0.3	1.7		0.01	0.97	3.42
UGC 06614	LSB	86.2	2.109	8.2	62.7	2.07	9.70	4.60	2.39
UGC 11454	LSB	93.9	0.456	3.4	12.3		3.15	6.90	6.79
UGC 11557	LSB	23.7	1.806	3.0	6.7	0.25	0.37	0.20	3.49
UGC 11583	LSB	7.1	0.012	0.7	2.1		0.01	0.96	2.15
UGC 11616	LSB	74.9	2.159	3.1	9.8		2.43	1.13	7.49
UGC 11648	LSB	49.0	4.073	4.0	13.0		2.57	0.63	5.79
UGC 11748	LSB	75.3	23.930	2.6	21.6		9.67	0.40	1.01
UGC 11819	LSB	61.5	2.155	4.7	11.9		4.83	2.24	7.03

Table 6: Properties of the Miscellaneous 21 Galaxy Sample

Galaxy	Type	Distance (Mpc)	$L_{\rm B}$ ($10^{10}{\rm L}_\odot$)	R_0 (kpc)	$R_{\rm last}$ (kpc)	$M_{\rm HI}$ ($10^{10}M_\odot$)	$M_{\rm disk}$ ($10^{10}M_\odot$)	$(M/L)_{\rm stars}$ ($M_\odot/L_\odot$)	$(v^2/c^2R)_{\rm last}$ ($10^{-30}{\tt cm}^{-1}$)
DDO 0168	LSB	4.5	0.032	1.2	4.4	0.03	0.06	2.03	2.22
DDO 0170	LSB	16.6	0.023	1.9	13.3	0.09	0.05	1.97	1.18
M 0033	HSB	0.9	0.850	2.5	8.9	0.11	1.13	1.33	4.62
NGC 0055	LSB	1.9	0.588	1.9	12.2	0.13	0.30	0.50	2.22
NGC 0247	LSB	3.6	0.512	4.2	14.3	0.16	1.25	2.43	2.94
NGC 0300	LSB	2.0	0.271	2.1	11.7	0.08	0.65	2.41	2.69
NGC 0801	HSB	63.0	4.746	9.5	46.7	1.39	6.93	2.37	3.59
NGC 1003	LSB	11.8	1.480	1.9	31.2	0.63	0.66	0.45	1.53
NGC 1560	LSB	3.7	0.053	1.6	10.3	0.12	0.17	3.16	2.16
NGC 2683	HSB	10.2	1.882	2.4	36.0	0.15	6.03	3.20	2.28
NGC 2998	HSB	59.3	5.186	4.8	41.1	1.78	7.16	1.75	3.43
NGC 3109	LSB	1.5	0.064	1.3	7.1	0.06	0.02	0.35	2.29
NGC 5033	HSB	15.3	3.058	7.5	45.6	1.07	0.27	3.28	3.16
NGC 5371	HSB	35.3	7.593	4.4	41.0	0.89	8.52	1.44	3.98
NGC 5533	HSB	42.0	3.173	7.4	56.0	1.39	2.00	4.14	3.31
NGC 5585	HSB	9.0	0.333	2.0	14.0	0.28	0.36	1.09	2.06
NGC 5907	HSB	16.5	5.400	5.5	48.0	1.90	2.49	1.89	3.44
NGC 6503	HSB	5.5	0.417	1.6	20.7	0.14	1.53	3.66	2.30
NGC 6674	HSB	42.0	4.935	7.1	59.1	2.18	2.00	2.52	3.57
UGC 2259	LSB	10.0	0.110	1.4	7.8	0.04	0.47	4.23	3.76
UGC 2885	HSB	80.4	23.955	13.3	74.1	3.98	8.47	0.72	4.31