Bound states in the continuum in quantum dot pairs

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Bound states in the continuum in quantum dot pairs

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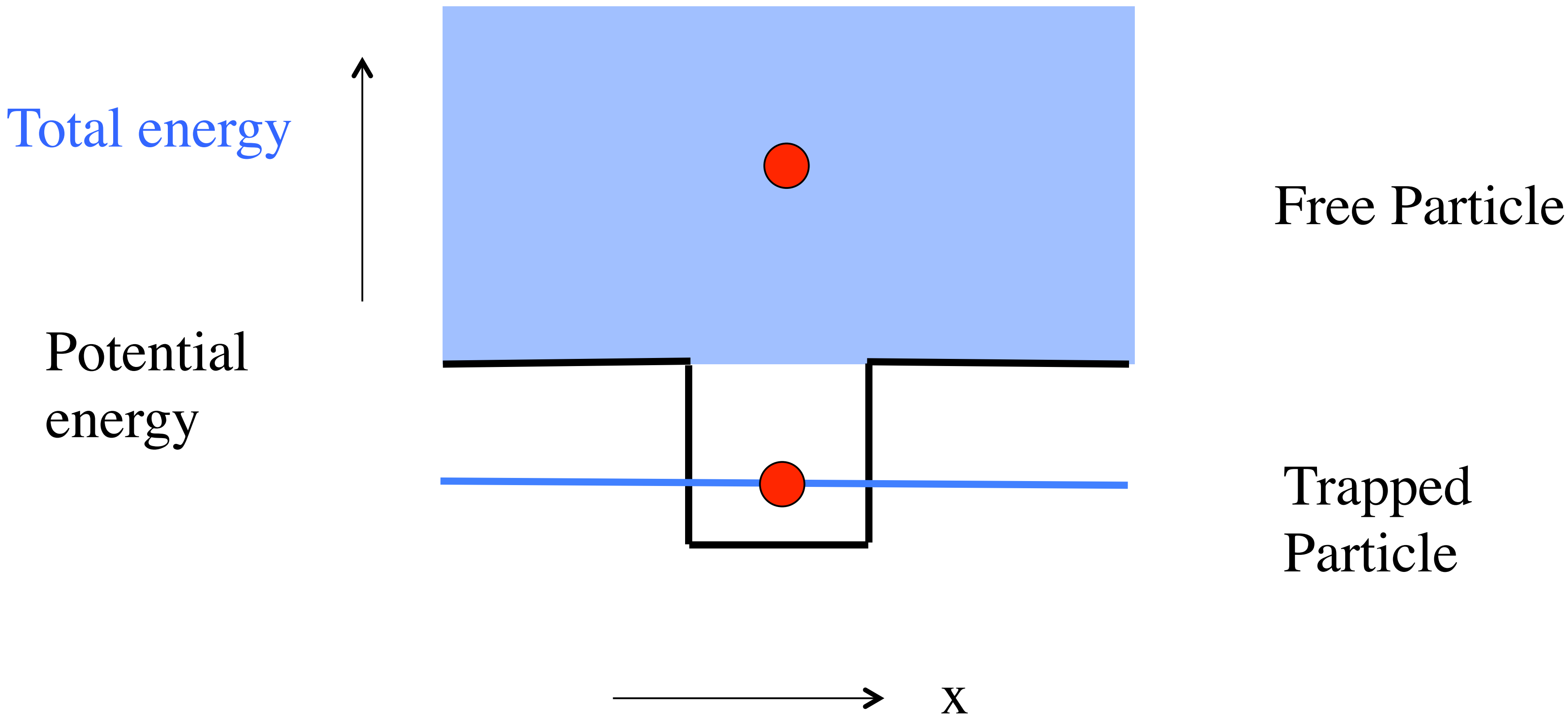
I. Prigogine

Time symmetry breaking from total Hermitian Hamiltonian

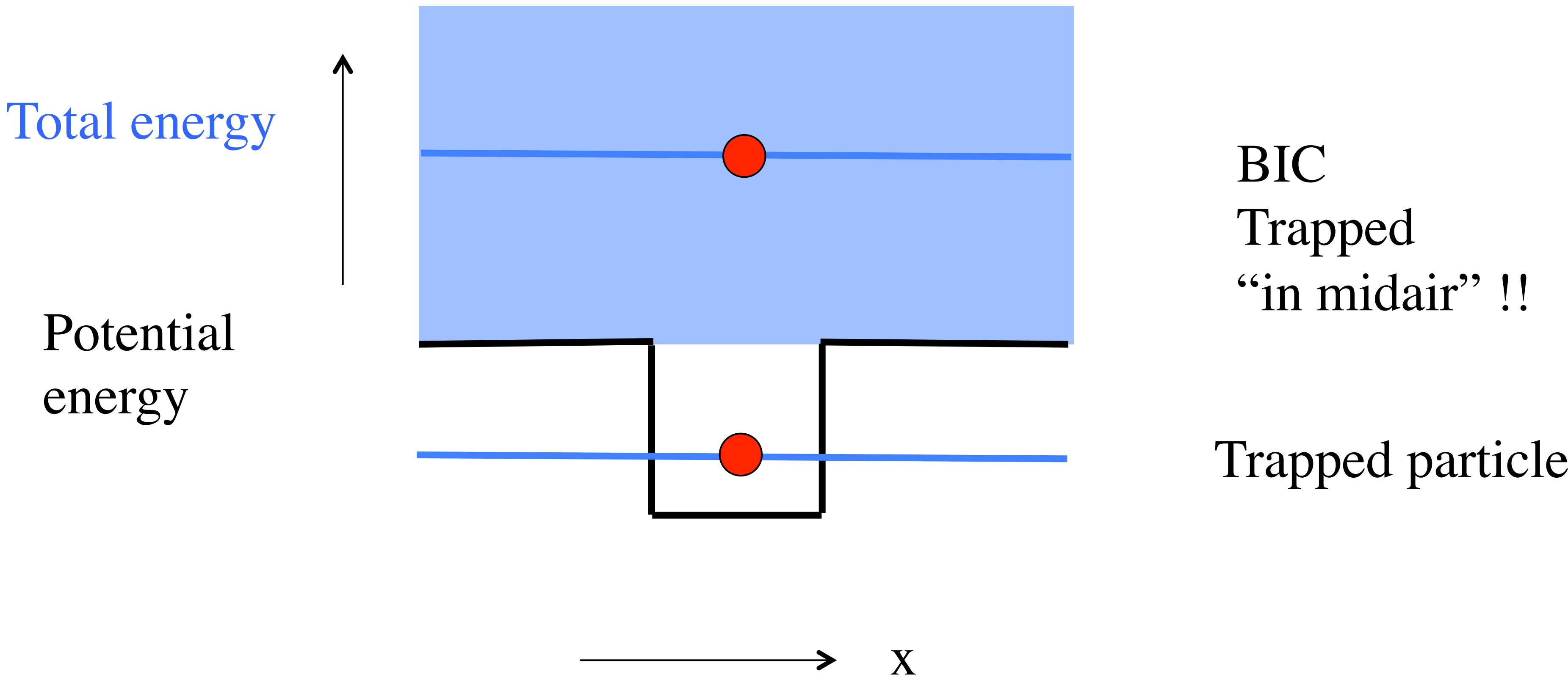
Complex eigenvalues of total Hamiltonian exist
= complex eigenvalues of effective Hamiltonian

Price to pay: Generalized wave functions

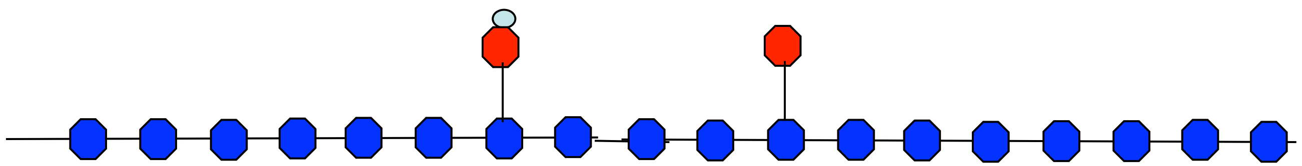
Bound states and continuum states



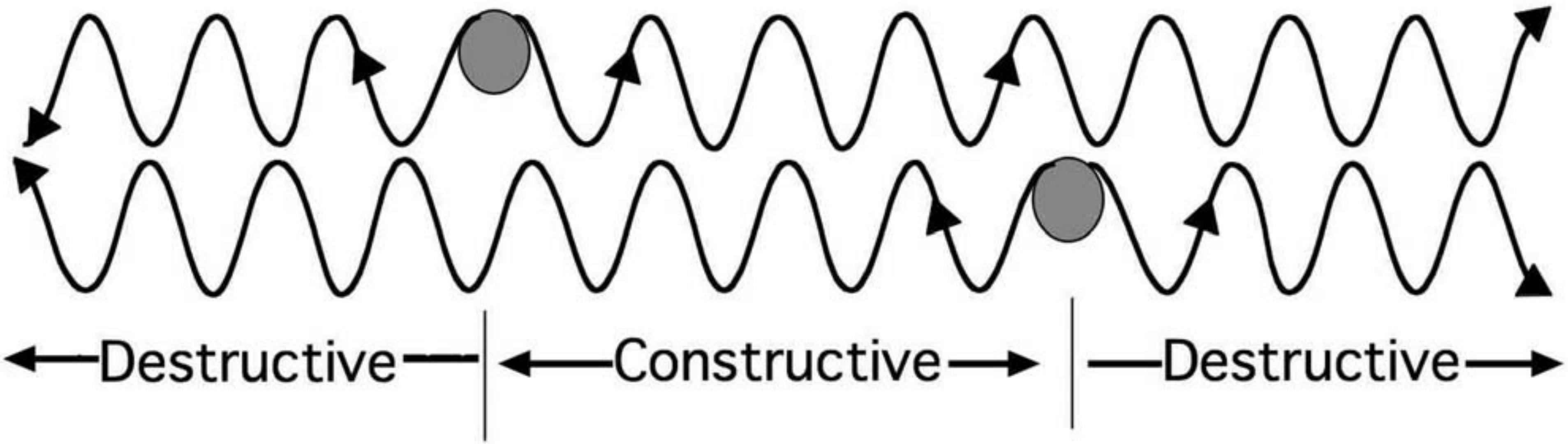
Bound states in the continuum
BIC



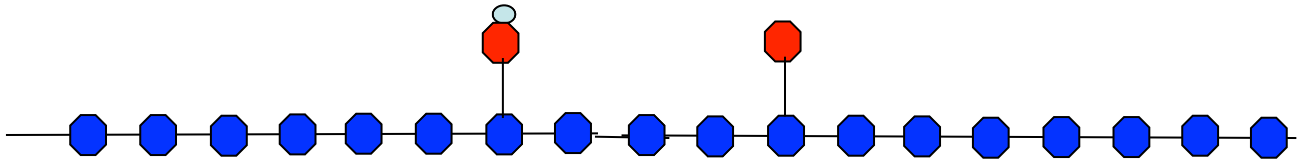
Physical realization of BIC: electron in an infinite quantum wire



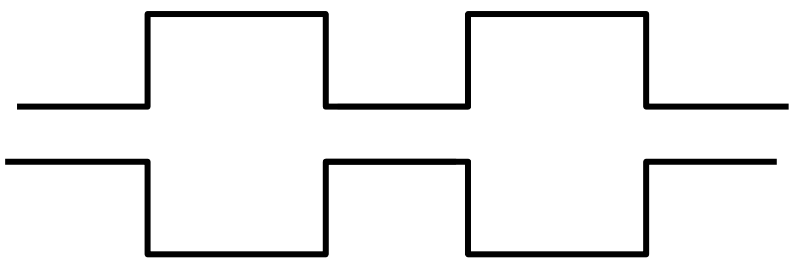
How does it work?
Wave interference



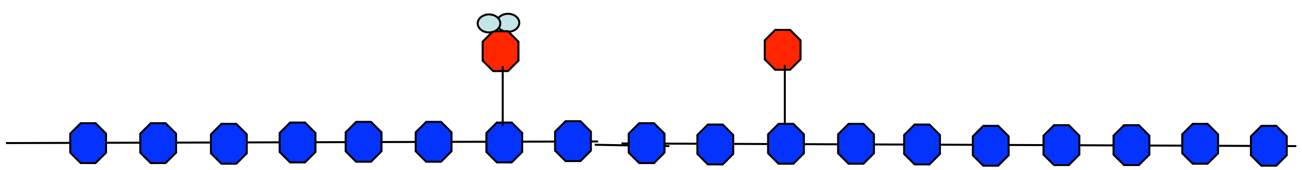
Photonic crystal



Waveguides



Question
Can <u>two</u> electrons exist in a BIC?

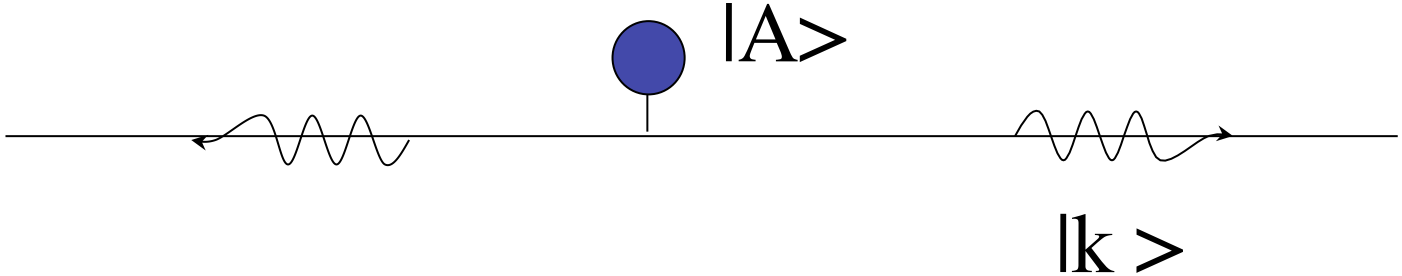


Two-Electron Bound States in a Continuum in Quantum Dots
A. F. Sadreev and T. V. Babushkina
JETP Letters, 2008, Vol. 88, No. 5, pp. 312–317

Outline

- One photon, one atom
- One photons, two atoms
- Quantum wire

One photon and one atom



$$H = \omega_A |A\rangle\langle A| + \int dk\ \omega_k |k\rangle\langle k| + g\int dk\ v_k \left[|k\rangle\langle A| + |A\rangle\langle k|\right]$$

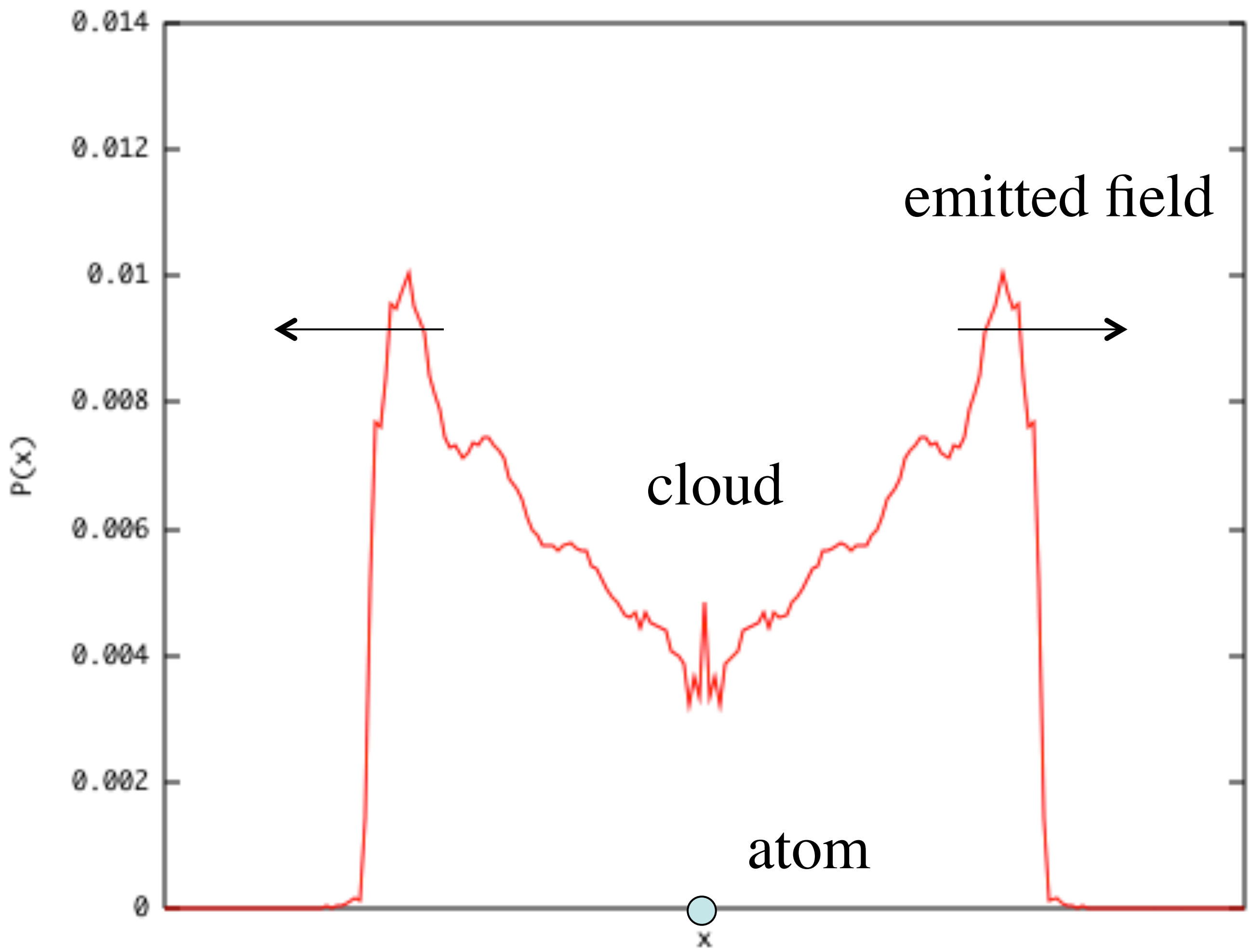
$\omega_k \rightarrow |k|$

Resonant state: Excited atom

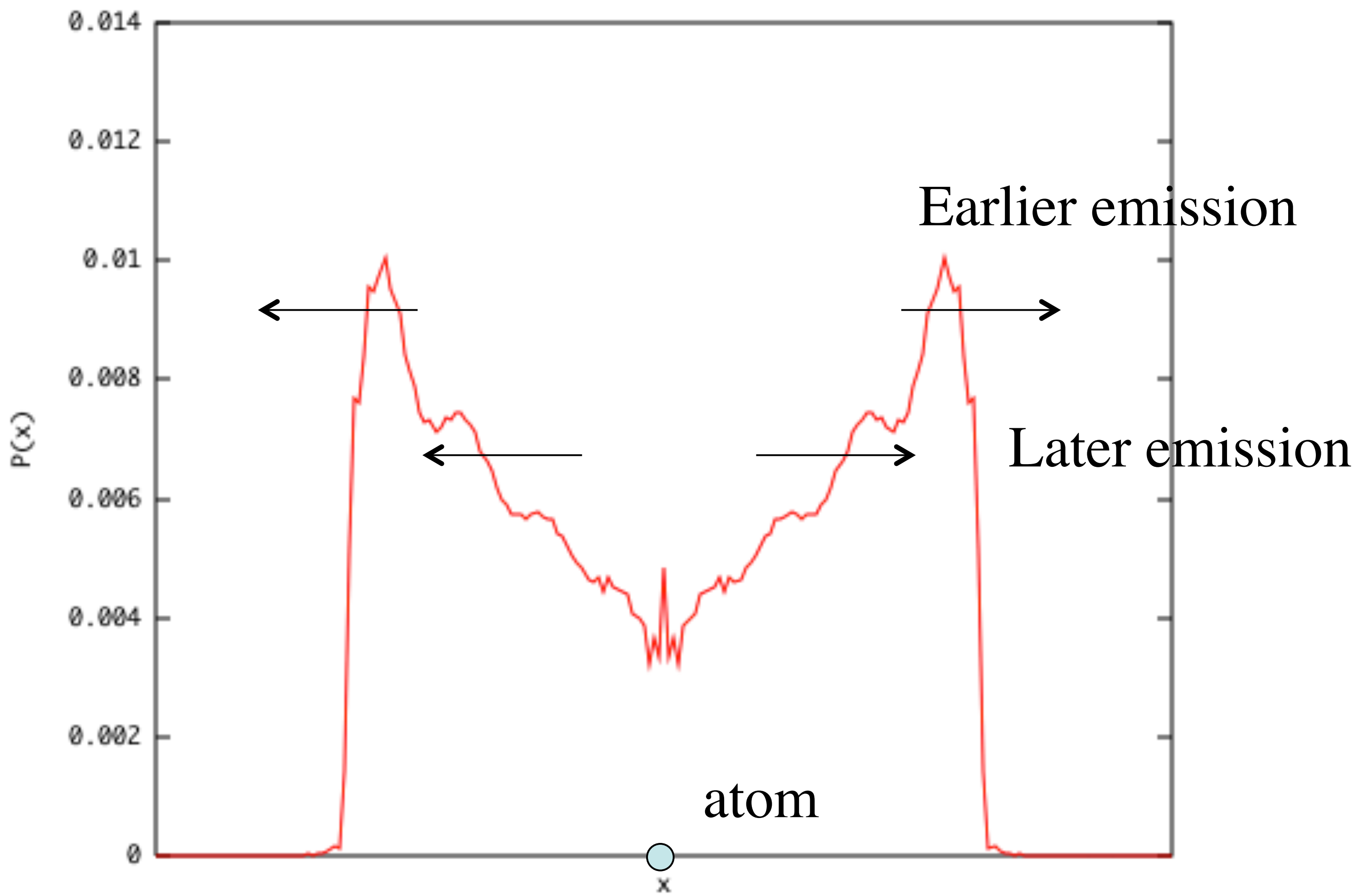
Finite lifetime

Exponential decay and emission

Emitted photon



Emitted photon



Resonant state

$$H\left|\psi^{RES}\right\rangle = z\left|\psi^{RES}\right\rangle$$

Resonant state

$$H\left|\psi^{RES}\right\rangle = z\left|\psi^{RES}\right\rangle$$

$$\left\langle\psi^{RES}\middle|\psi^{RES}\right\rangle = 0$$

Anti-resonant state

$$H\left|\psi^{ANTI-RES}\right\rangle = z^{*}\left|\psi^{ANTI-RES}\right\rangle$$

Anti-resonant state

$$H\left|\psi^{ANTI-RES}\right\rangle = z^{*}\left|\psi^{ANTI-RES}\right\rangle$$

$$\left\langle\psi^{ANTI-RES}\middle|\psi^{ANTI-RES}\right\rangle = 0$$

Duality

$$\left\langle \psi^{ANTI-RES} \middle| \psi^{RES} \right\rangle = 1$$

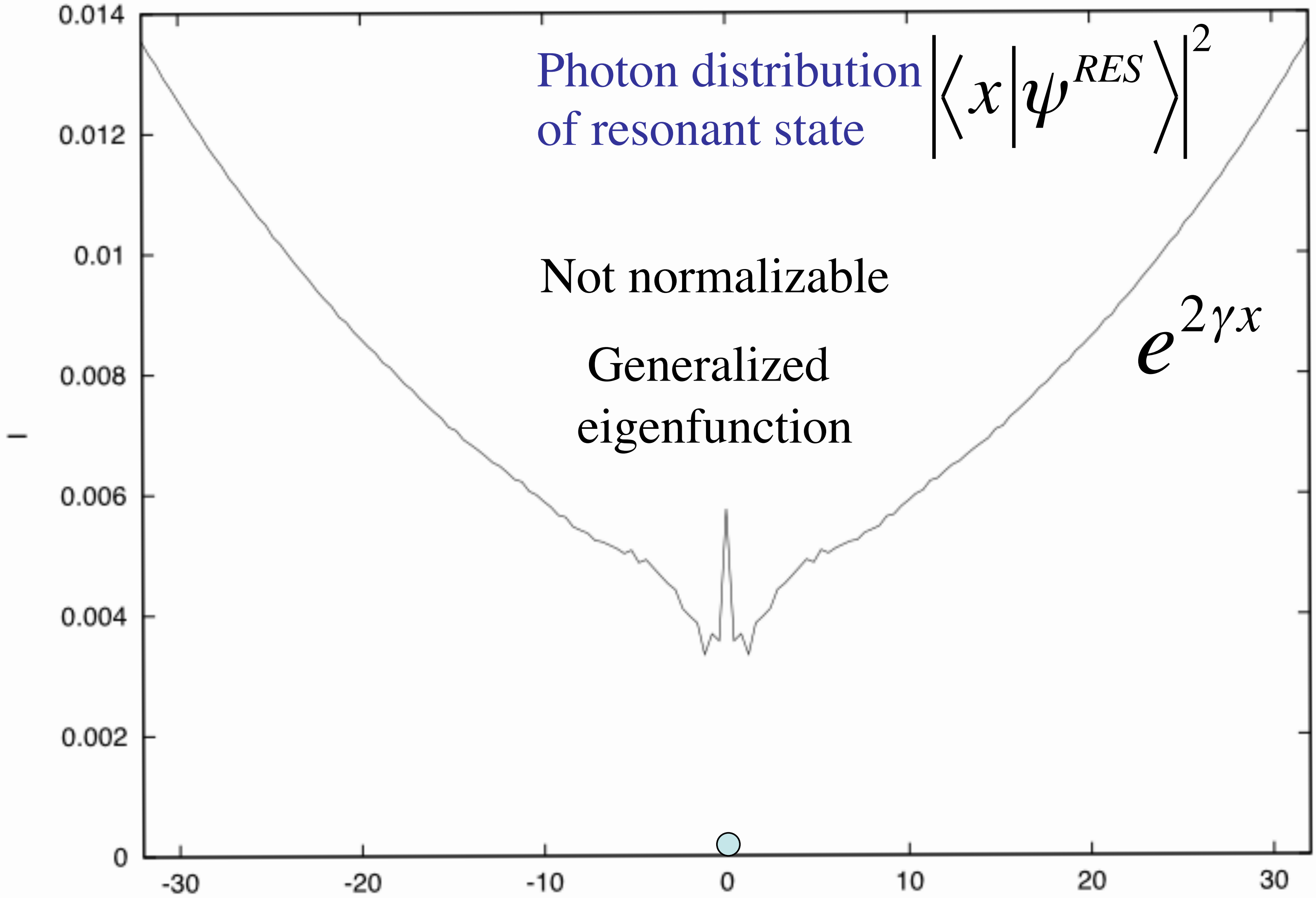
Resonant state decays for t > 0

$$z = E - i\gamma$$

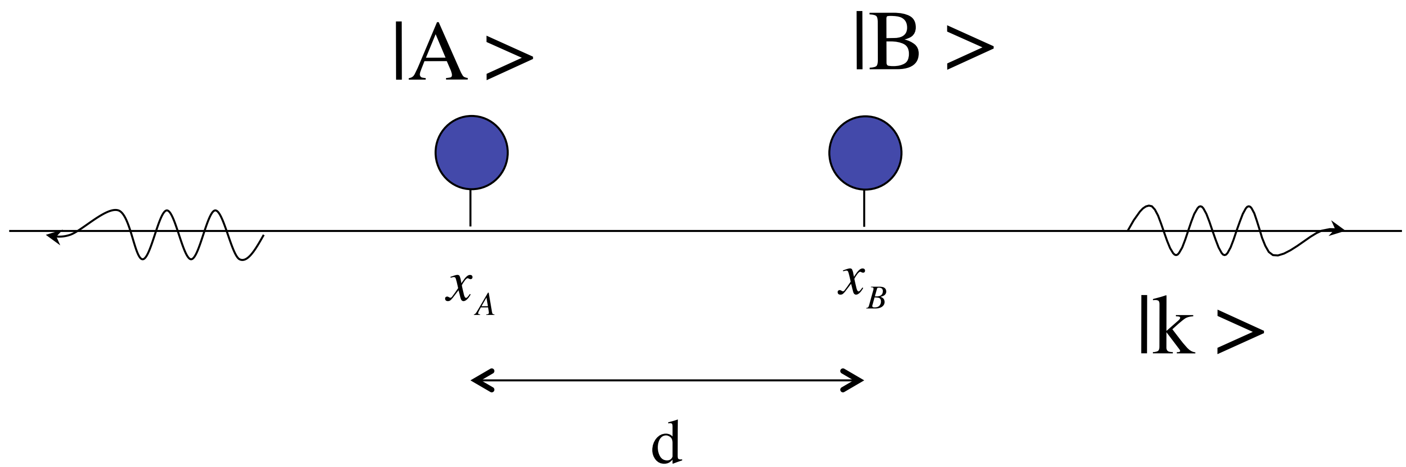
$$e^{-iHt}\left|\psi^{RES}\right\rangle = e^{-iEt}e^{-\gamma t}\left|\psi^{RES}\right\rangle$$

exponential decay

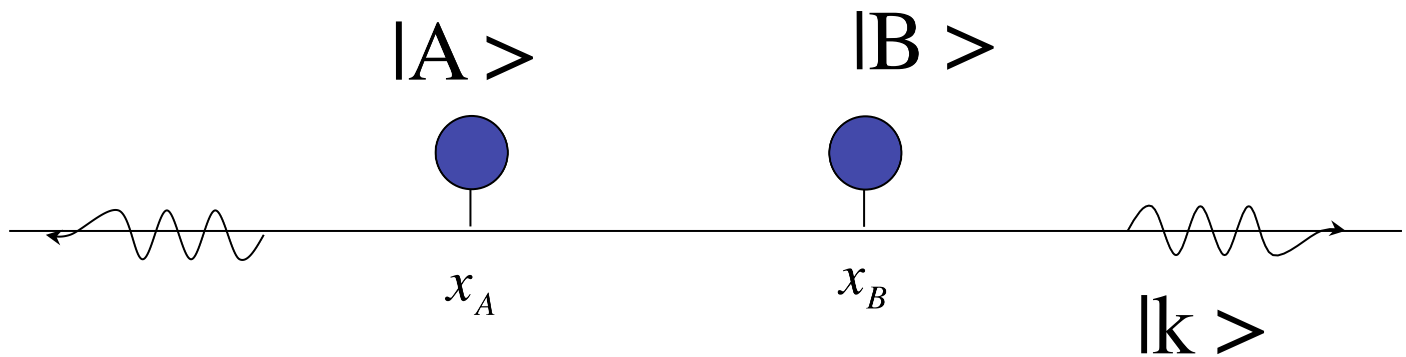
But diverges in space



One photon and two atoms



Photon and two atoms



$$H = \omega_A |A\rangle\langle A| + \omega_B |B\rangle\langle B| + \int dk\ \omega_k |k\rangle\langle k|$$

$$+\ g\int dk\ v_k \left[e^{ikx_A} |k\rangle\langle A| + e^{-ikx_A} |A\rangle\langle k| \right]$$

$$+ g\int dk\ v_k \left[e^{ikx_B} |k\rangle\langle B| + e^{-ikx_B} |B\rangle\langle k| \right]$$

Identical atoms

$$\omega_A = \omega_B = \omega$$

$$|s\rangle = \frac{|A\rangle + |B\rangle}{\sqrt{2}}$$ Symmetric

$$|a\rangle = \frac{|A\rangle - |B\rangle}{\sqrt{2}}$$ Antisymmetric

Resonant and anti-resonant states Symmetric

$$H\left|\psi_s^{RES}\right\rangle = z_s\left|\psi_s^{RES}\right\rangle$$

$$H\left|\psi_s^{ANTI-RES}\right\rangle = z_s^*\left|\psi_s^{ANTI\_RES}\right\rangle$$

Resonant and anti-resonant states Antisymmetric

$$H\left|\psi_a^{RES}\right\rangle = z_a\left|\psi_a^{RES}\right\rangle$$

$$H\left|\psi_a^{ANTI-RES}\right\rangle = z_a^*\left|\psi_a^{ANTI-RES}\right\rangle$$

Effective Hamiltonian

$$\begin{pmatrix} \omega + \Xi_s(E) & 0 \\ 0 & \omega + \Xi_a(E) \end{pmatrix}$$

Complex eigenvalue of the Hamiltonian

$$z = \omega + \Xi(z)$$

Self energies

$$\Xi_s(E) = 2\int_0^\infty dk\ \ g^2 v_k^2 \frac{1}{E-k}\left(1+\cos\left(kd\right)\right)$$

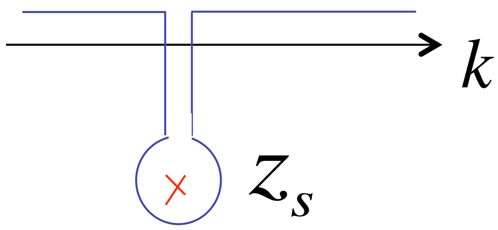
$$\Xi_a(E) = 2\int_0^\infty dk\ \ g^2 v_k^2 \frac{1}{E-k}\left(1-\cos\left(kd\right)\right)$$

Distance between atoms

Eigenvalue equation

$$z_s = \omega + \Xi_s(z_s)$$

$$z_s = \omega + 2\int_0^\infty dk\ \lambda^2 v_k^2 \frac{1}{z_s - k}\left(1 + \cos(kd)\right)$$



$$e^{\gamma_s d}$$

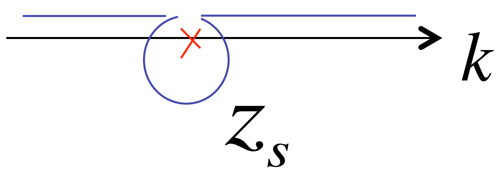
$$z_s = E_s - i\gamma_s$$

Exponential growth!

Eigenvalue equation

$$z_s = \omega + \Xi_s(z_s)$$

$$z_s = \omega + 2\int_0^\infty dk\ \lambda^2 v_k^2 \frac{1}{z_s - k}\left(1 + \cos(kd)\right)$$



-1 interference

n odd

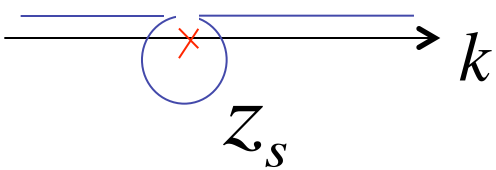
BIC

$$z_s = E_s = \frac{n\pi}{d}$$

Eigenvalue equation

$$z_a = \omega + \Xi_a(z_a)$$

$$z_a = \omega + 2\int_0^\infty dk \;\; \lambda^2 v_k^2 \frac{1}{z_a - k}\left(1 - \cos\left(kd\right)\right)$$



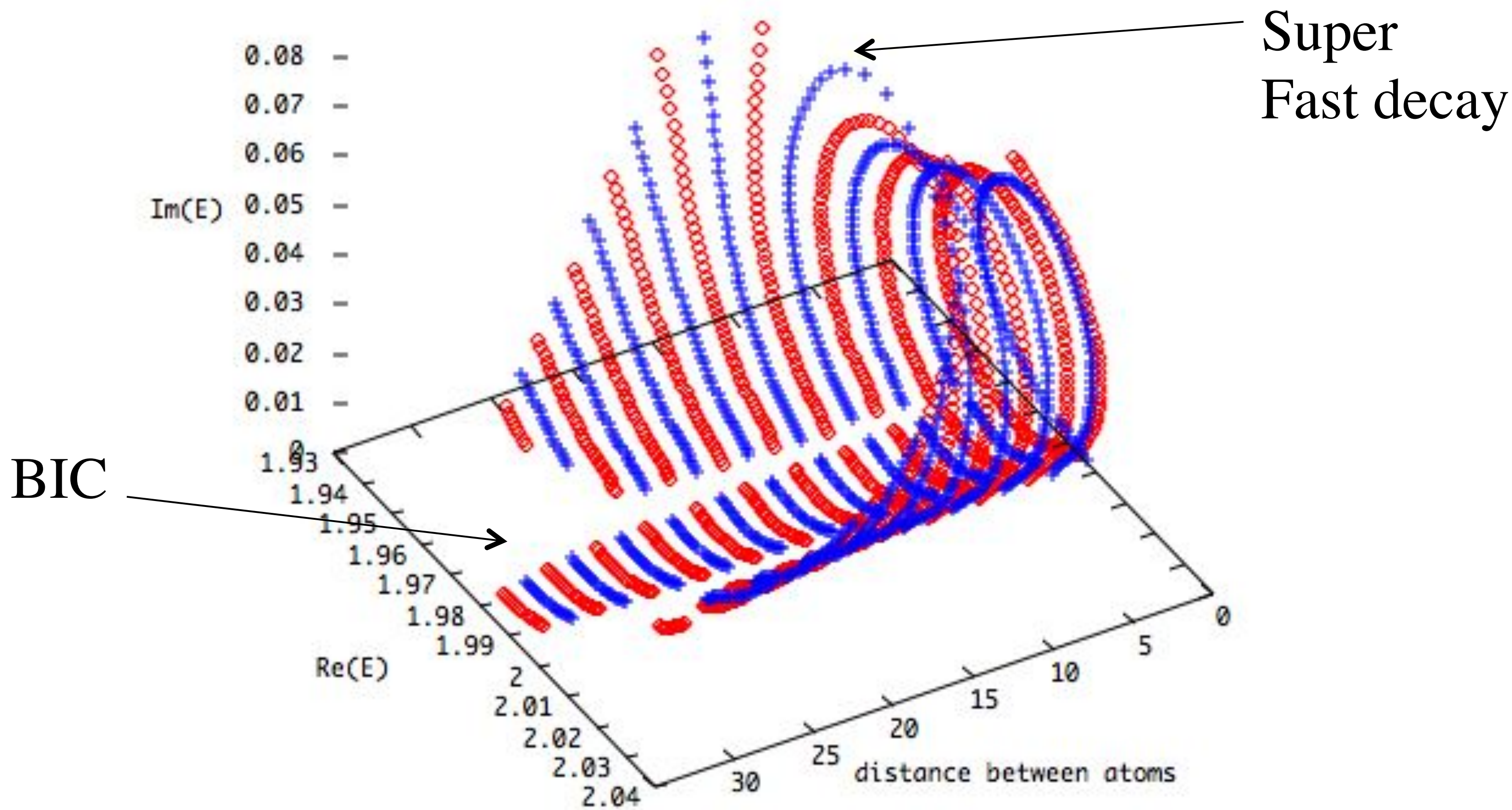
+1 interference

even

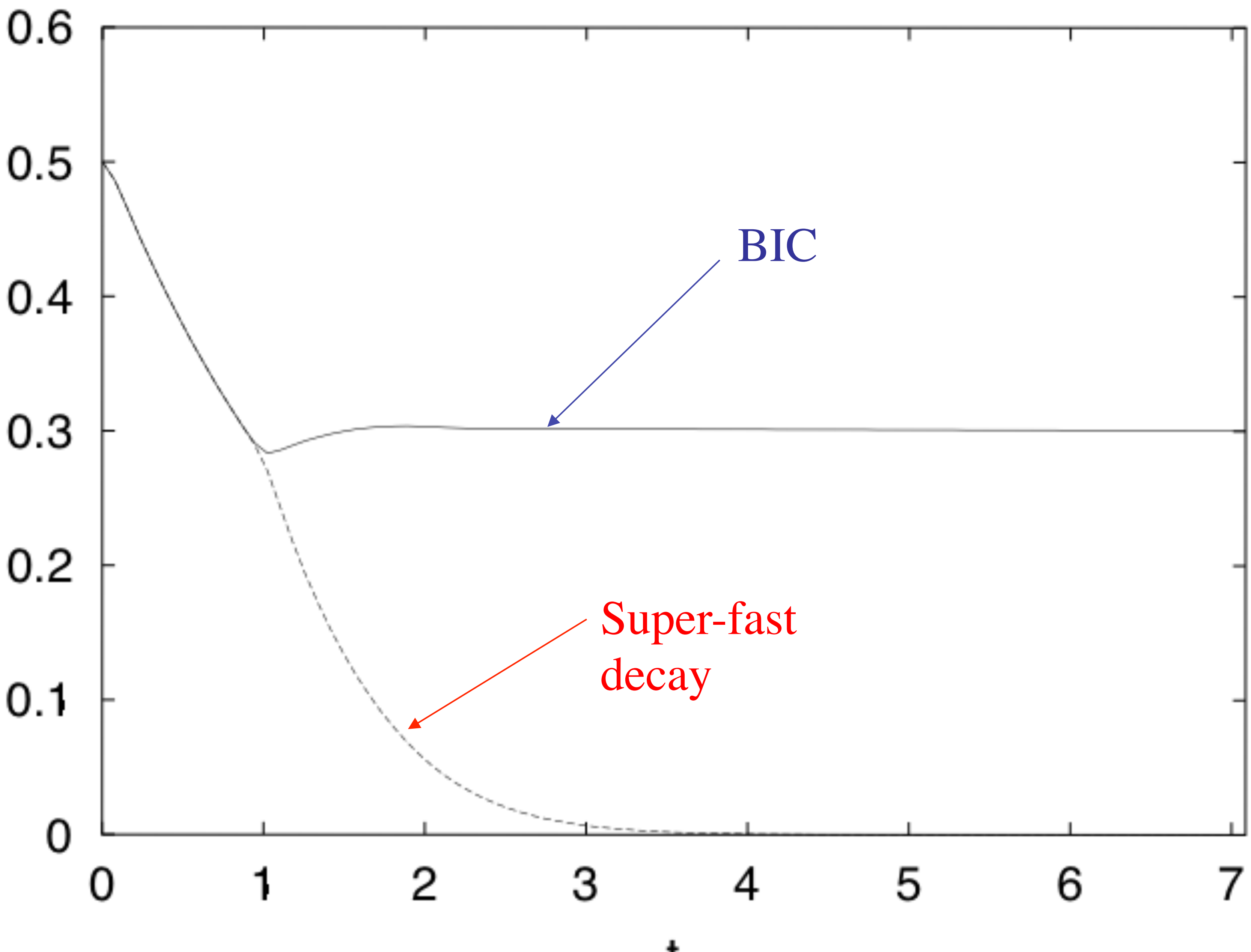
BIC

$$z_a = E_a = \frac{n\pi}{d}$$

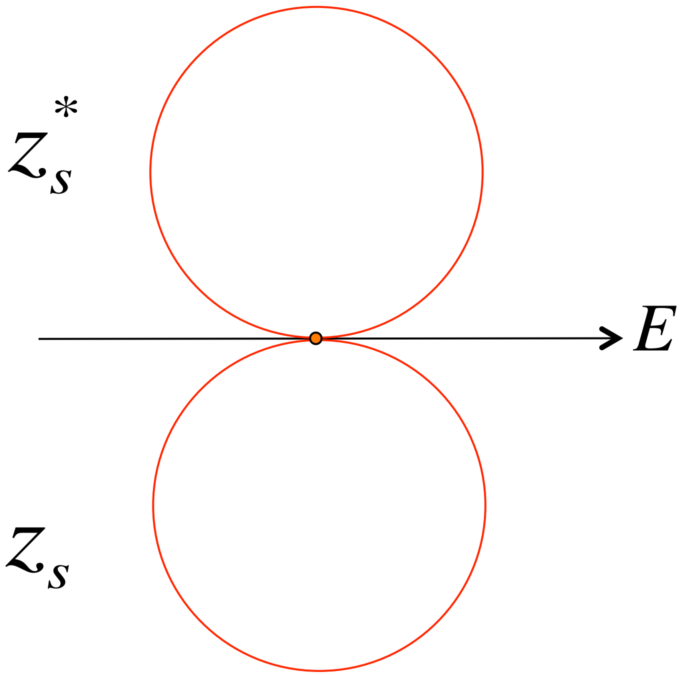
Poles vs. distance



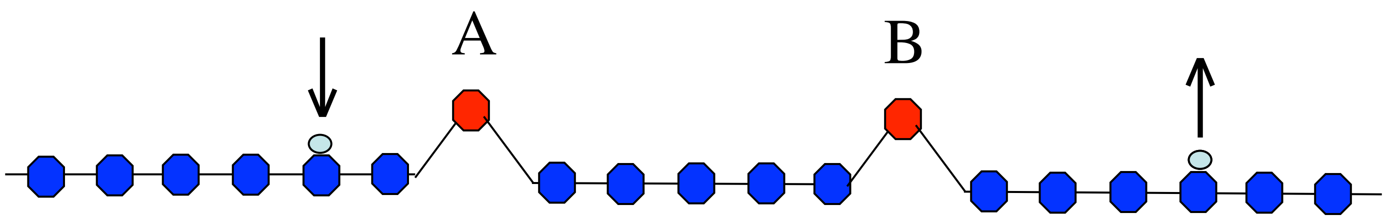
Survival probability at atom A



BIC = exceptional point?

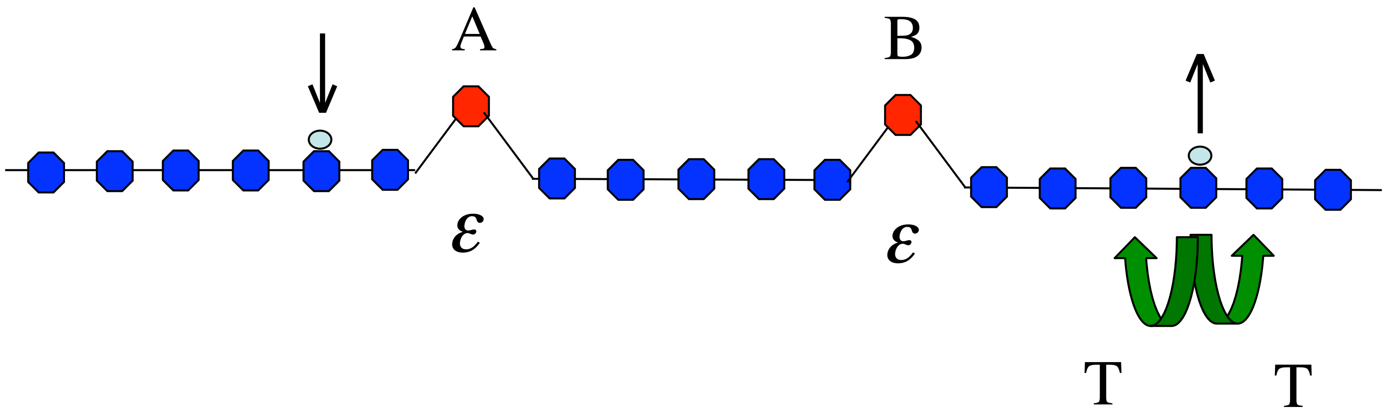


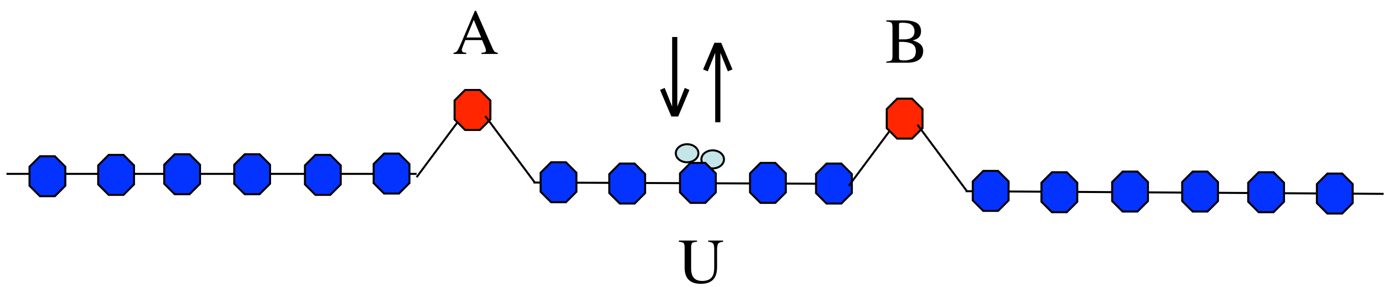
Quantum wire with two adatoms and two electrons



Infinite wire →continuous spectrum of energy inside an energy band

Quantum wire with two adatoms and two electrons





BIC for U=0

$$E = \varepsilon + \Xi(E)$$

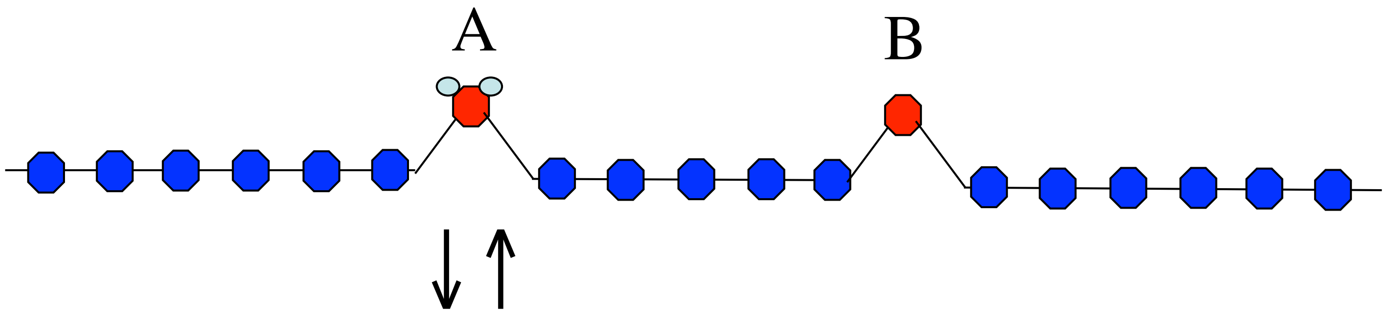
$$\Xi(E) = -\frac{2\lambda^2 T^2}{\pi} \int_{-\pi}^{\pi} dk \; \frac{1}{E + 2T\cos(k)} \left(1 \pm \cos(kd)\right)$$

Condition for BIC

$$E = -2T\cos k \qquad k = \frac{\pi n}{d}$$

$$\Xi(E) = 0 \qquad E = \varepsilon = -2T\cos\frac{\pi n}{d}$$

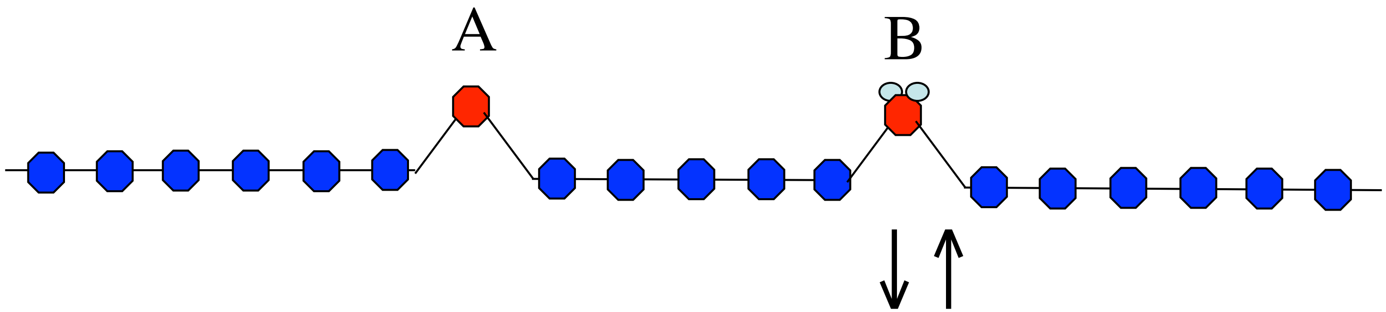
Numerical test
Initial condition



Two electrons

Quantum superposition at A and B

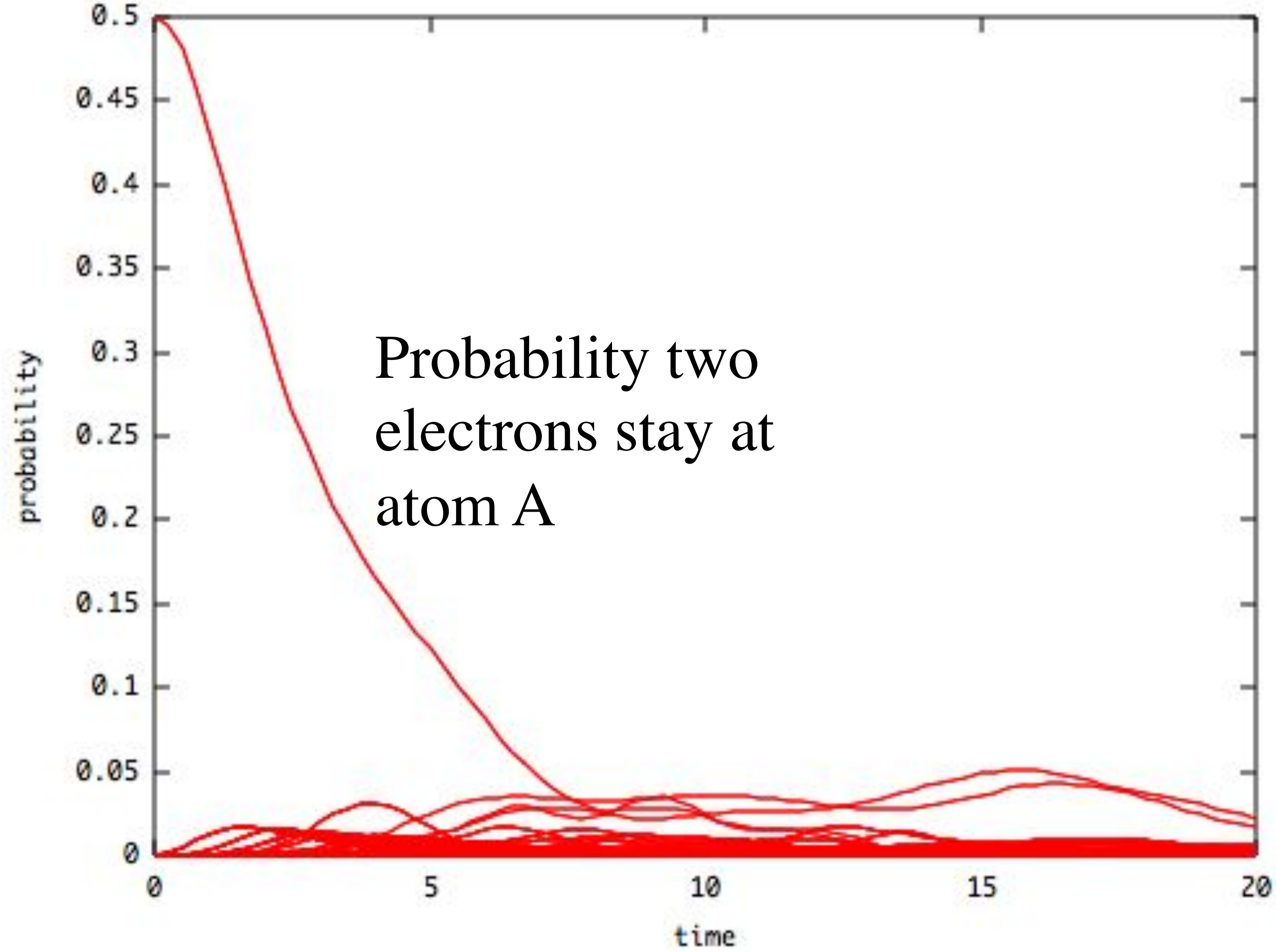
Numerical test
Initial condition



Two electrons
Quantum superposition at A and B

No BIC (U=0)

$$\varepsilon \neq -2T\cos\frac{\pi n}{d}$$



$\varepsilon = 0.5$

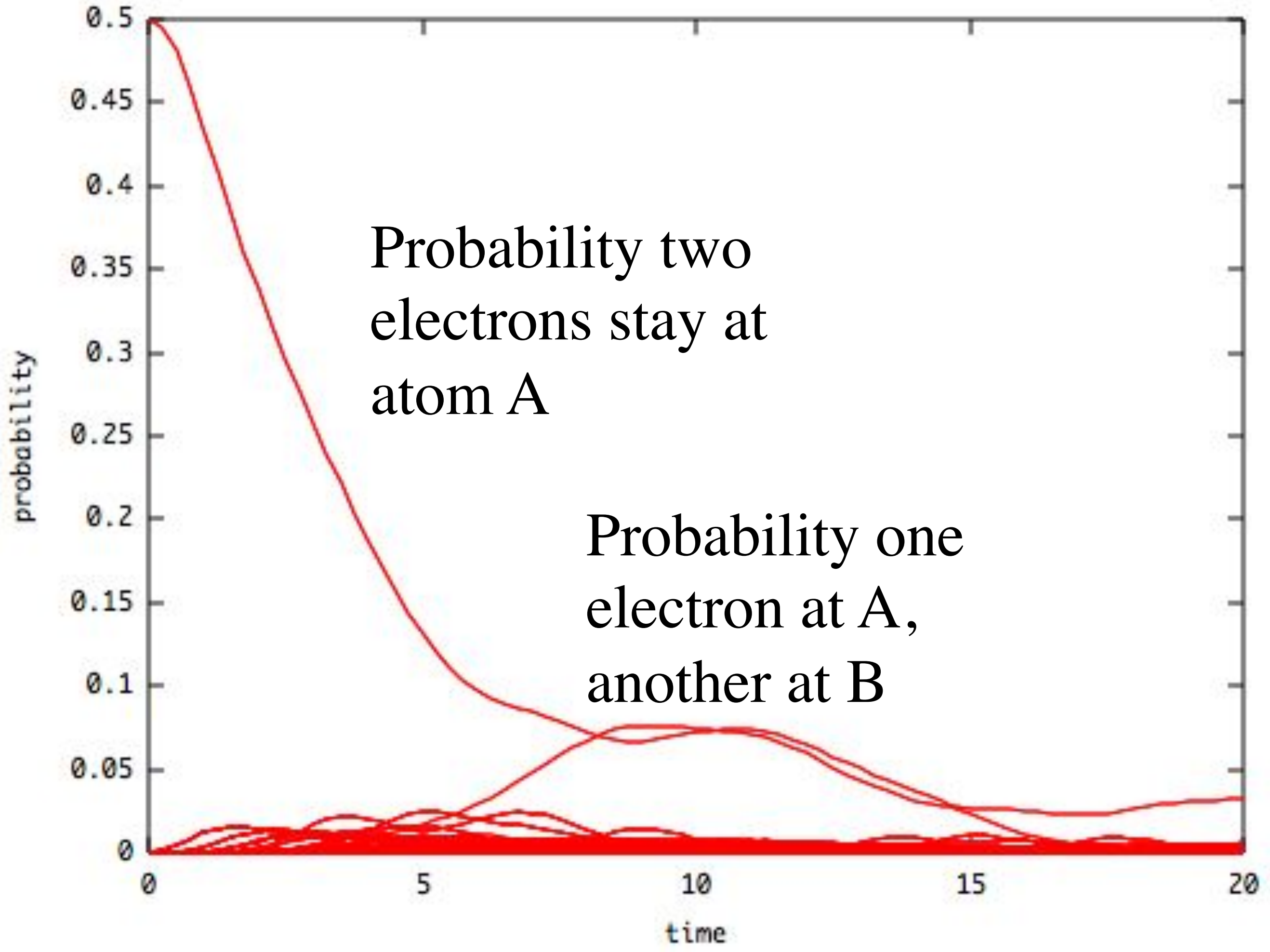
$d = 3$

$n = 1$

$T = -1$

BIC (U=0)

$$\varepsilon = -2T\cos\frac{\pi n}{d}$$



$\varepsilon = -1$

$d = 3$

$n = 1$

$T = -1$

What will happen if U is not zero?

What will happen if U is not zero?

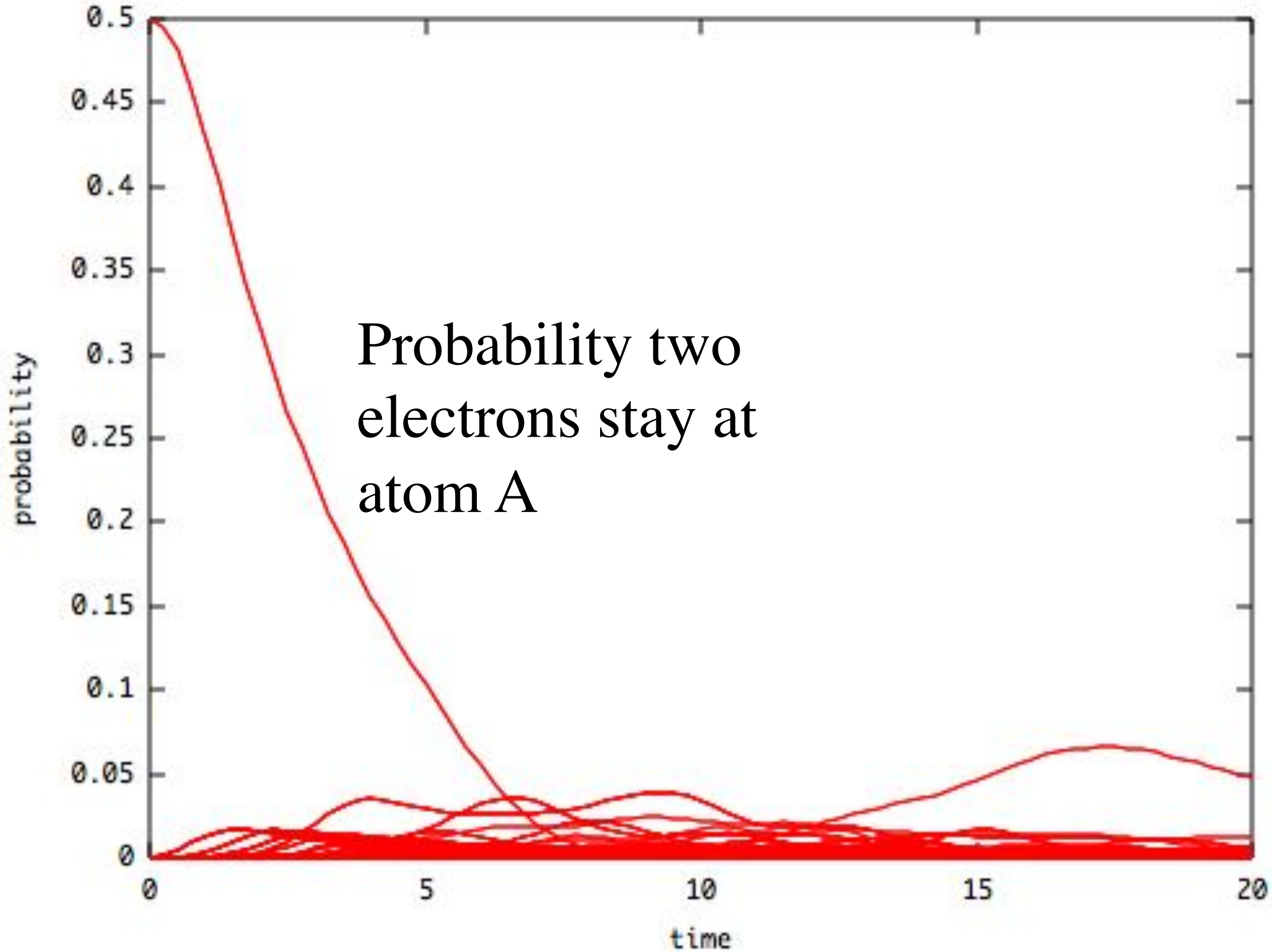
Numerical result:

Condition for BIC becomes

$$\varepsilon + U = -2T\cos\frac{\pi n}{d}$$

No BIC

$$\varepsilon + U \neq -2T\cos\frac{\pi n}{d}$$



$\varepsilon = -1$

$d = 3$

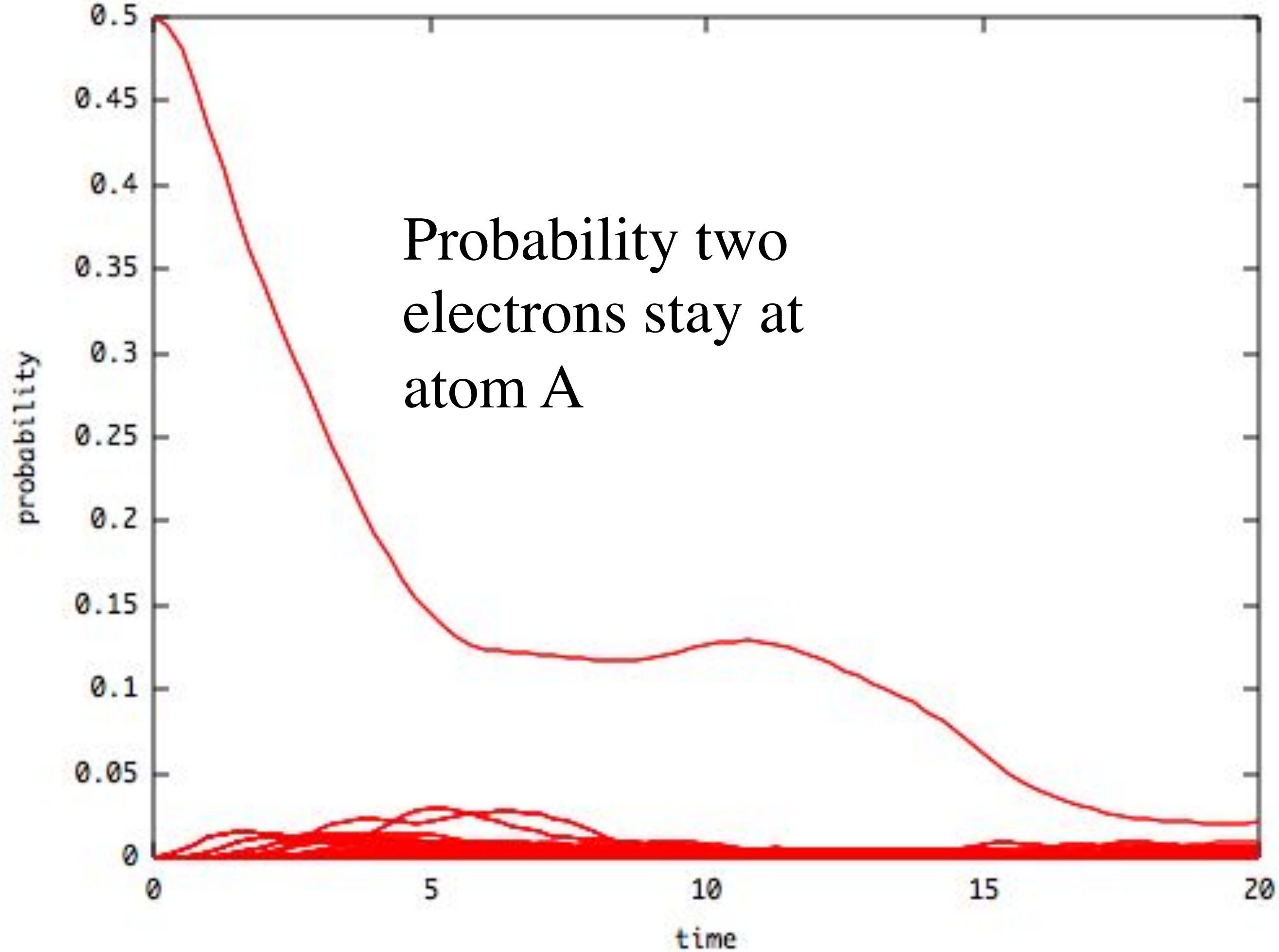
$n = 1$

$T = -1$

$U = 1.5$

BIC

$$\varepsilon + U = -2T\cos\frac{\pi n}{d}$$



$\varepsilon = -1$

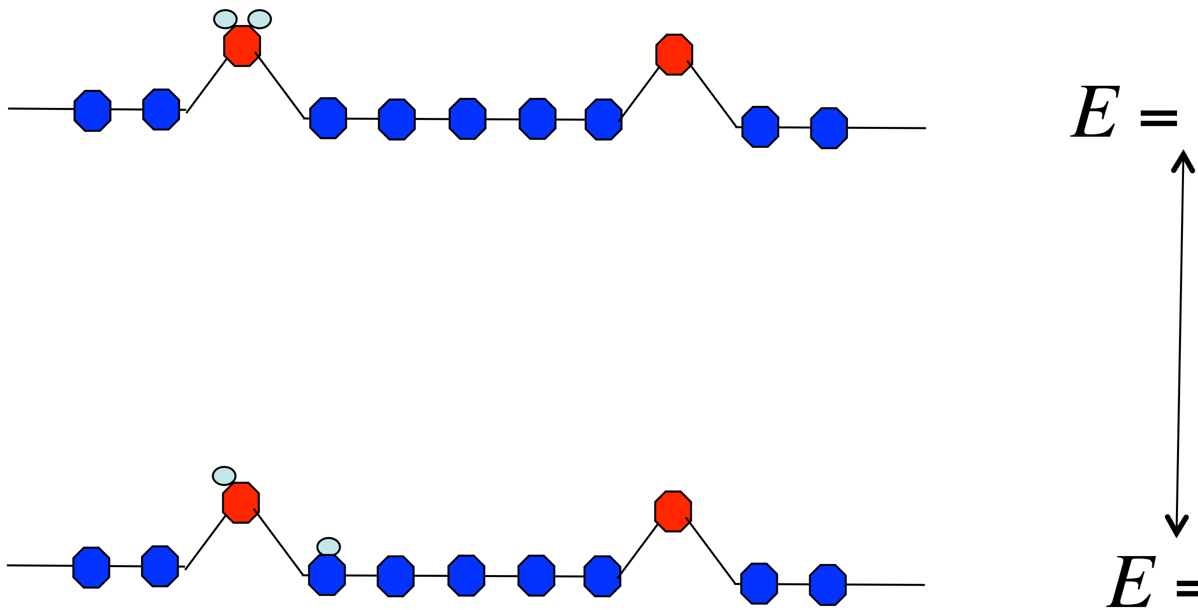
$d = 3$

$n = 1$

$T = -1$

$U = 2$

Why $\varepsilon + U$

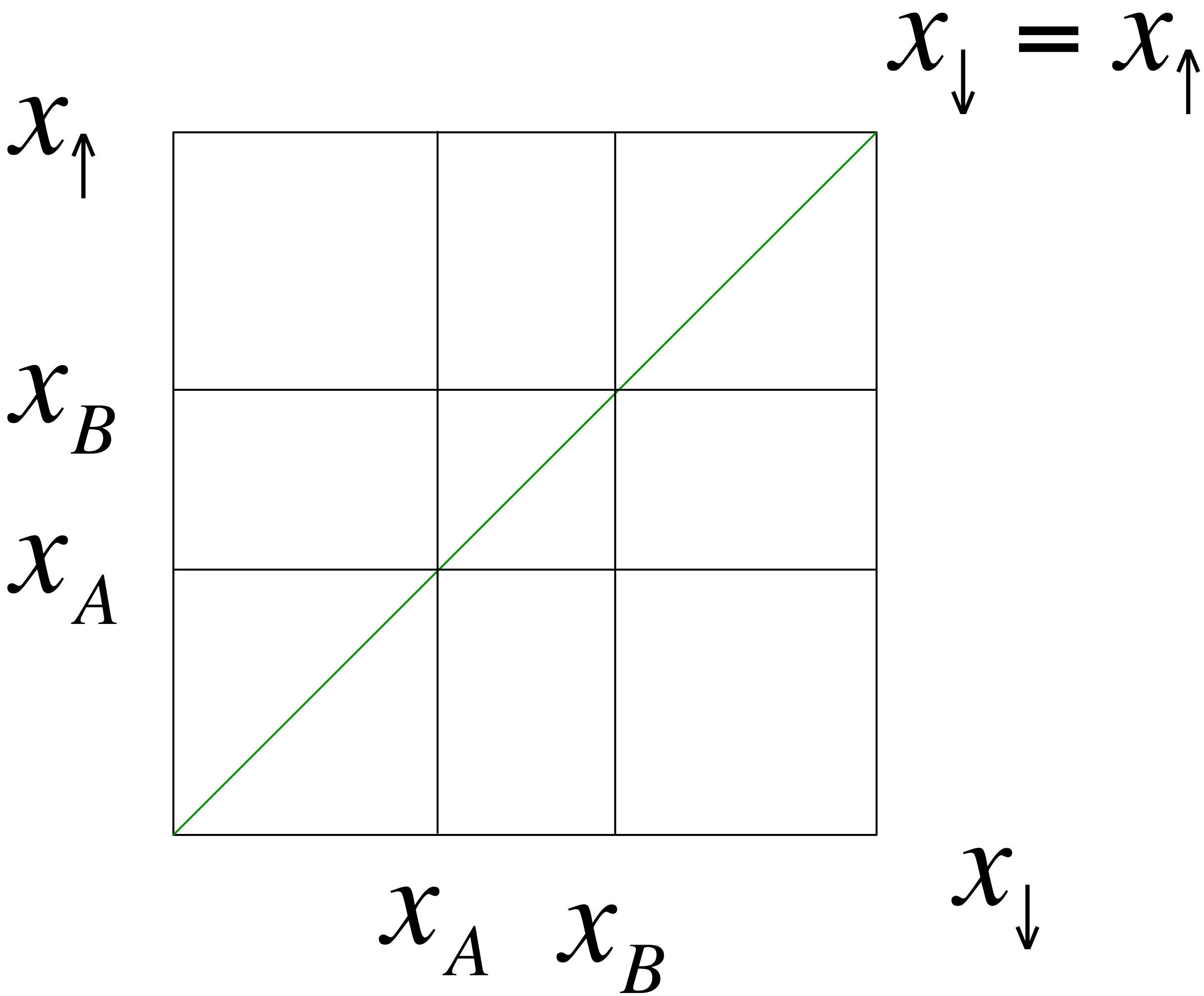


$$E = 2\varepsilon + U$$

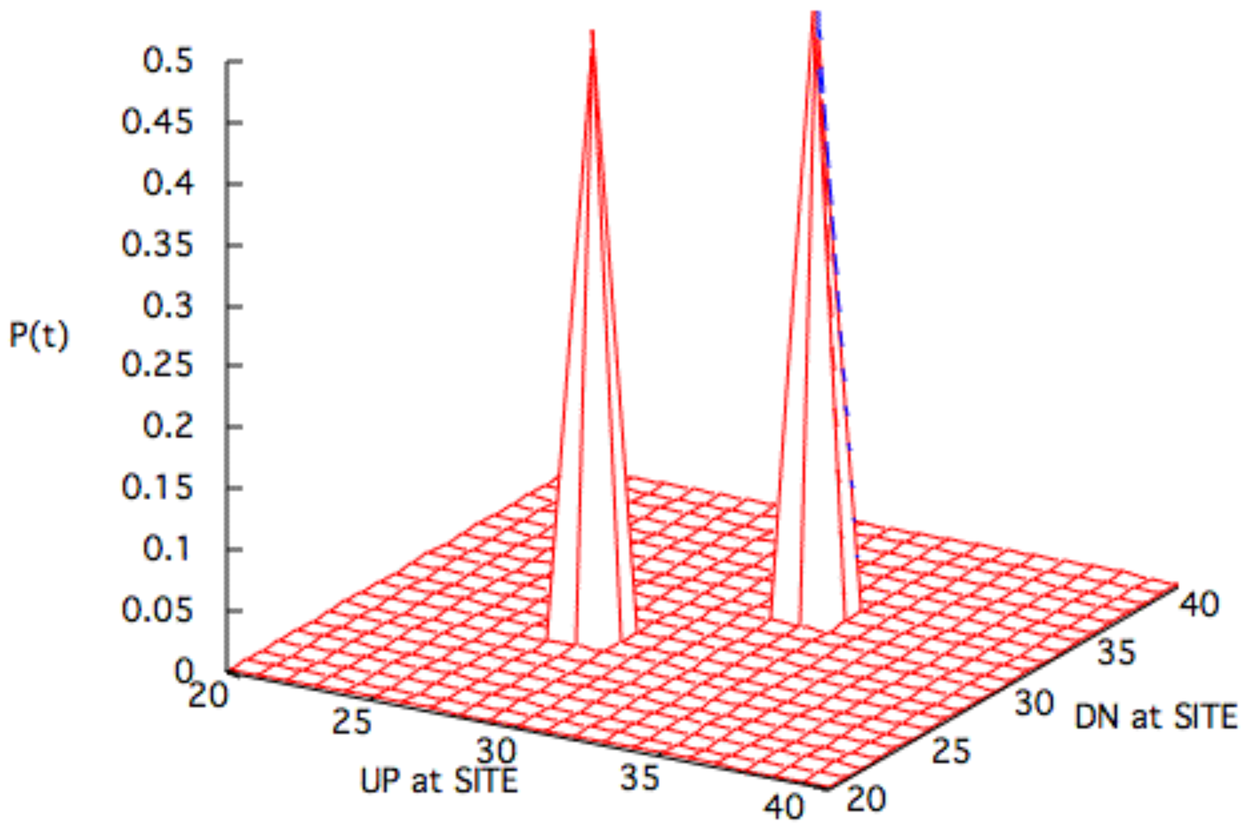
$$\Delta E = \varepsilon + U$$

$$E = \varepsilon$$

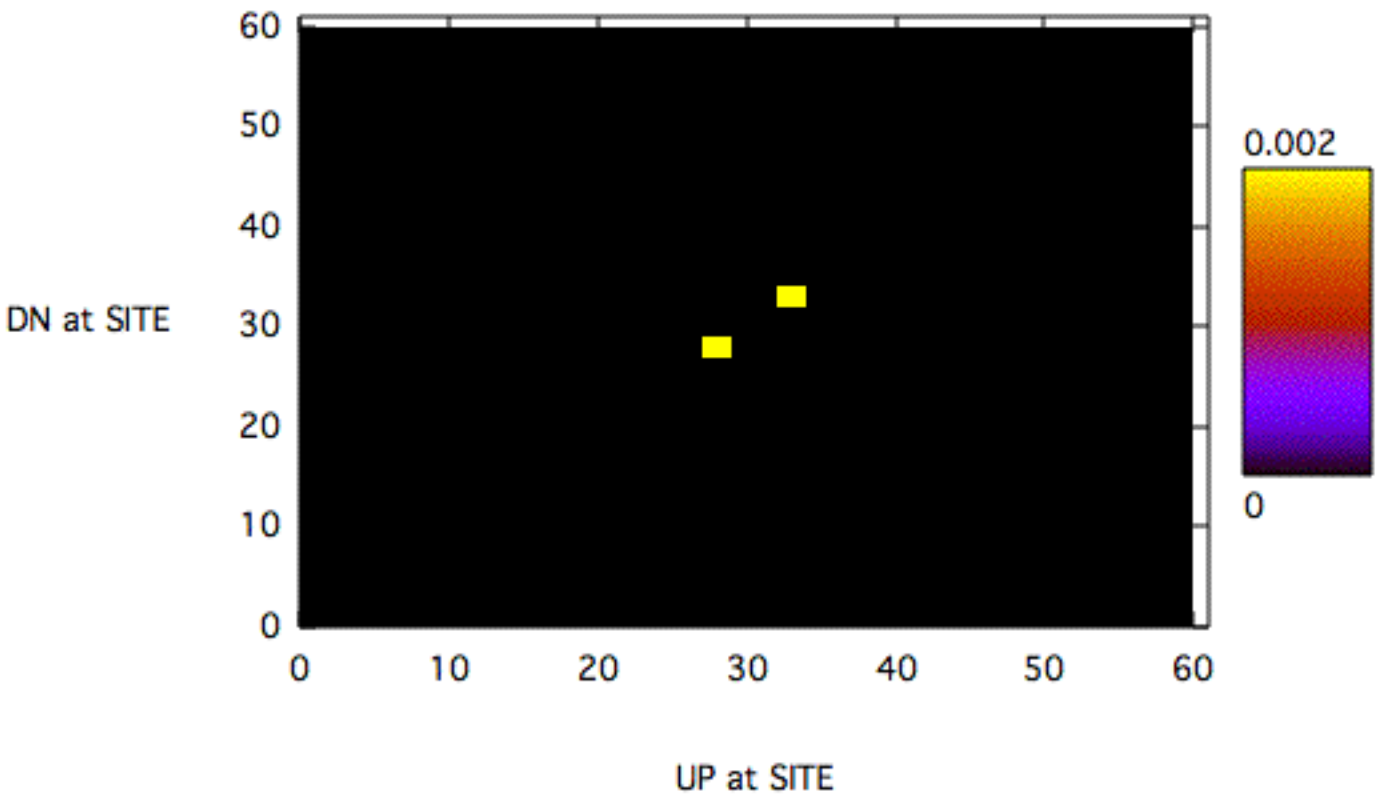
Analytical solution



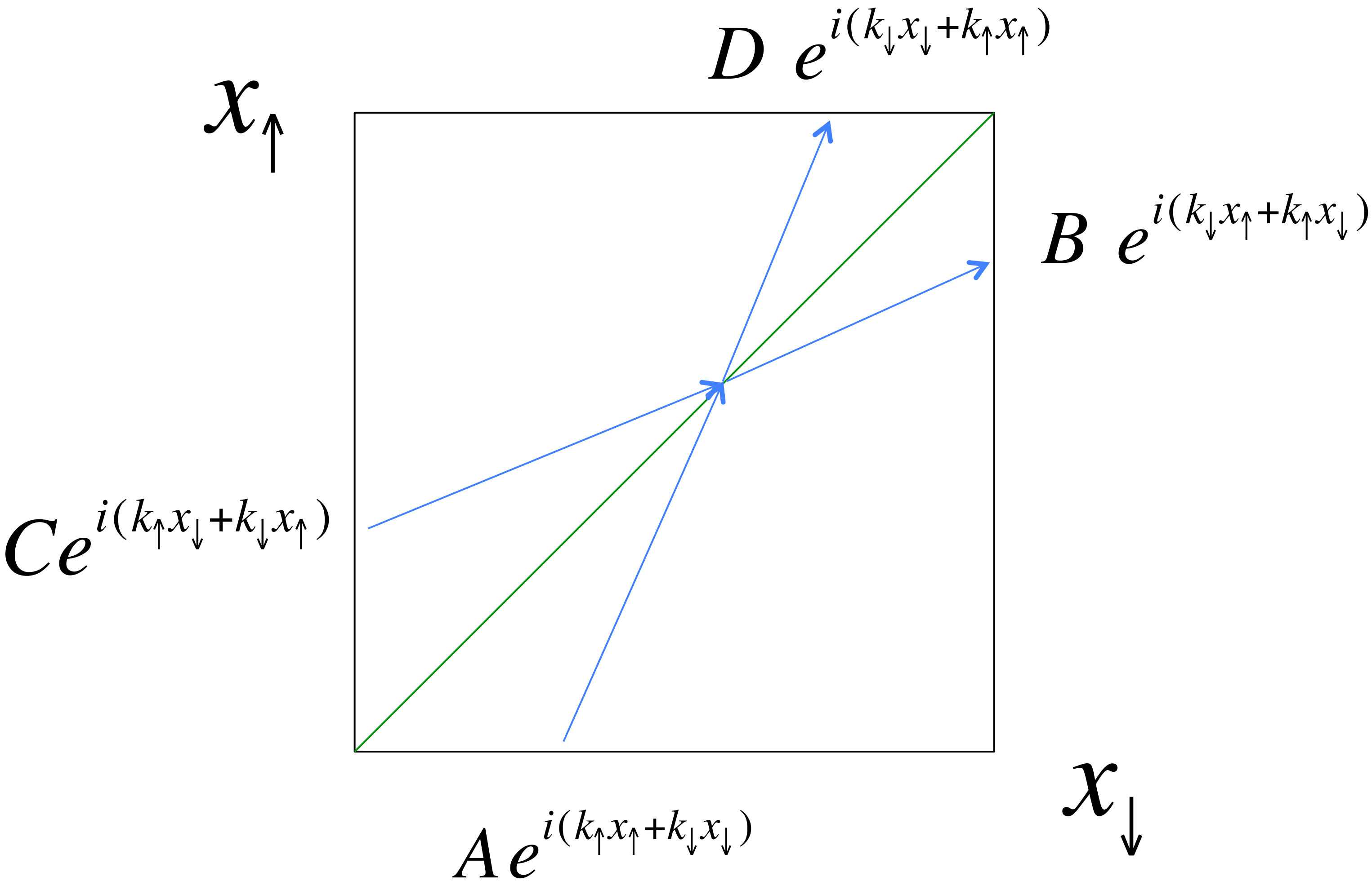
Time evolution (BIC)



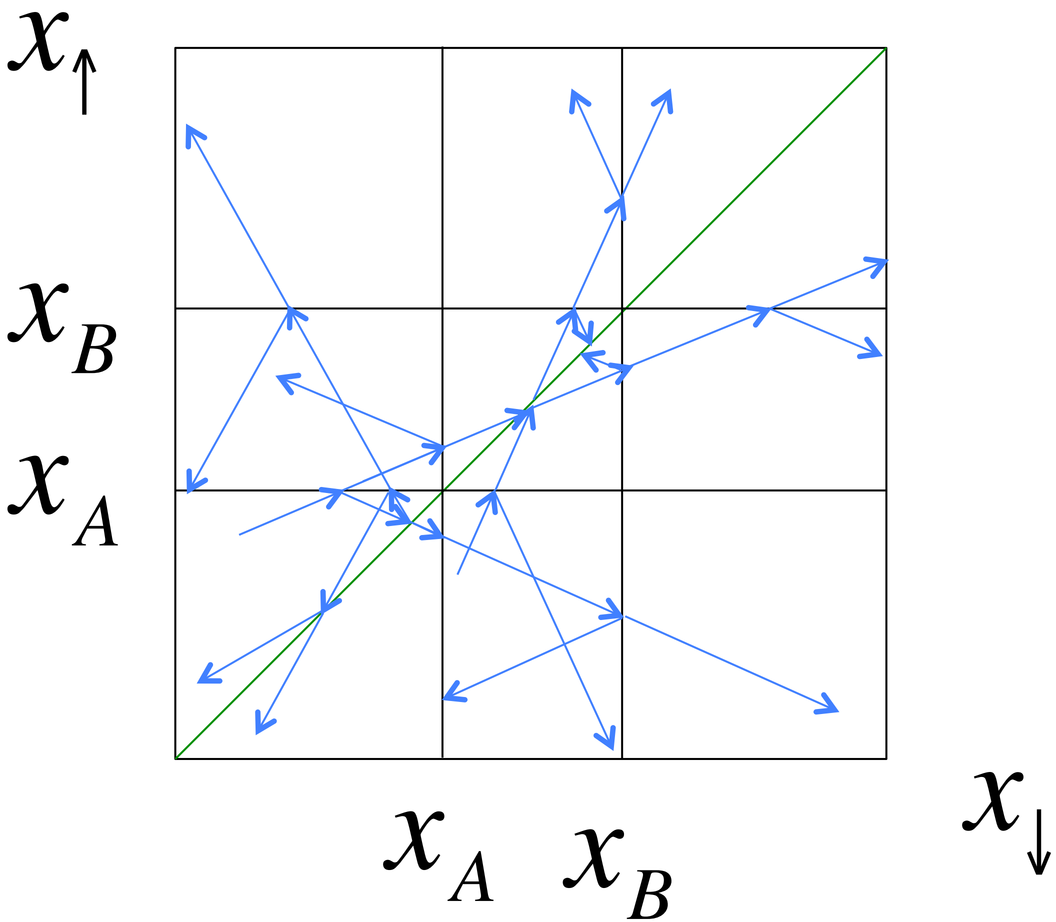
Time evolution (BIC)



Bethe Ansatz



Extended Bethe Ansatz



Conclusions

- BIC for 1 electron in quantum wire model
- Delocalized state.
- Numerical evidence of 2-electron BIC in quantum wire model, even with Coulomb repulsion
- Future work: analytical solution