Exceptional Points in Microwave Billiards: Eigenvalues and Eigenfunctions

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Dresden 2011

- Microwave billiards and quantum billiards
- Microwave billiards as a scattering system
- Eigenvalues and eigenfunctions of a dissipative system near an exceptional point
- Properties of exceptional points in the time domain
- Induced violation of time-reversal invariance
  $\rightarrow$ determination of $\hat{\mathrm{H}}_{\mathrm{eff}}$ at the EP

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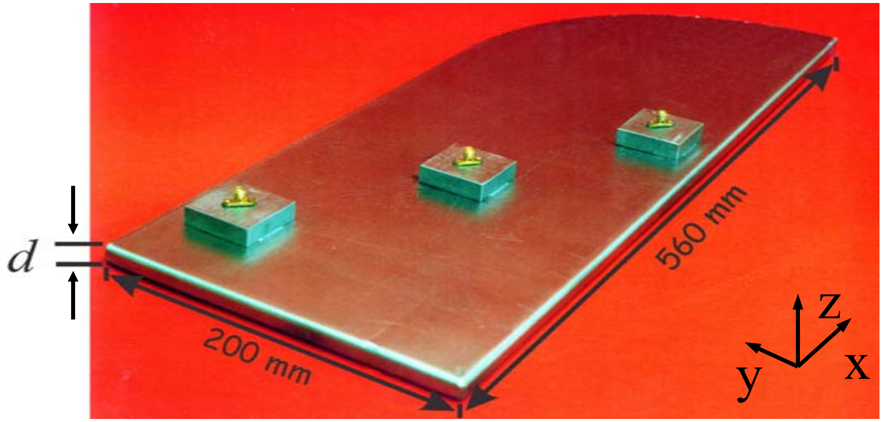
Cylindrical Microwave Billiards



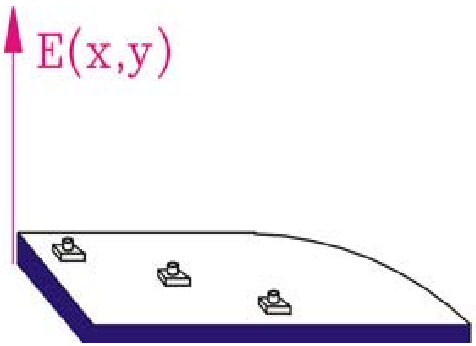
vectorial Helmholtz equation $\left(\Delta + k_\mu^2\right)\vec{E}_\mu(\vec{r}) = 0, \quad \vec{n} \times \vec{E}_\mu(\vec{r})\big|_{\partial G} = 0, \quad k_\mu = \frac{2\pi f_\mu}{c}$

cylindrical resonators $f \leq f_{max} = \frac{c}{2d} \quad \Rightarrow \quad \vec{E}_\mu(\vec{r}) = E_\mu(x,y)\vec{e}_z$

scalar Helmholtz equation $\left(\Delta + k_\mu^2\right)E_\mu(x,y) = 0, \quad E_\mu(x,y)\big|_{\partial G} = 0$

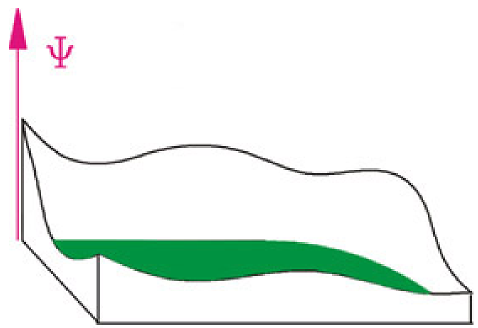
Microwave and Quantum Billiards

microwave billiard



$$(\Delta + k^2)E_z(x,y) = 0, E_z(x,y)|_{\partial G} = 0$$

quantum billiard



$$(\Delta + k^2)\Psi(x,y) = 0, \Psi(x,y)|_{\partial G} = 0$$

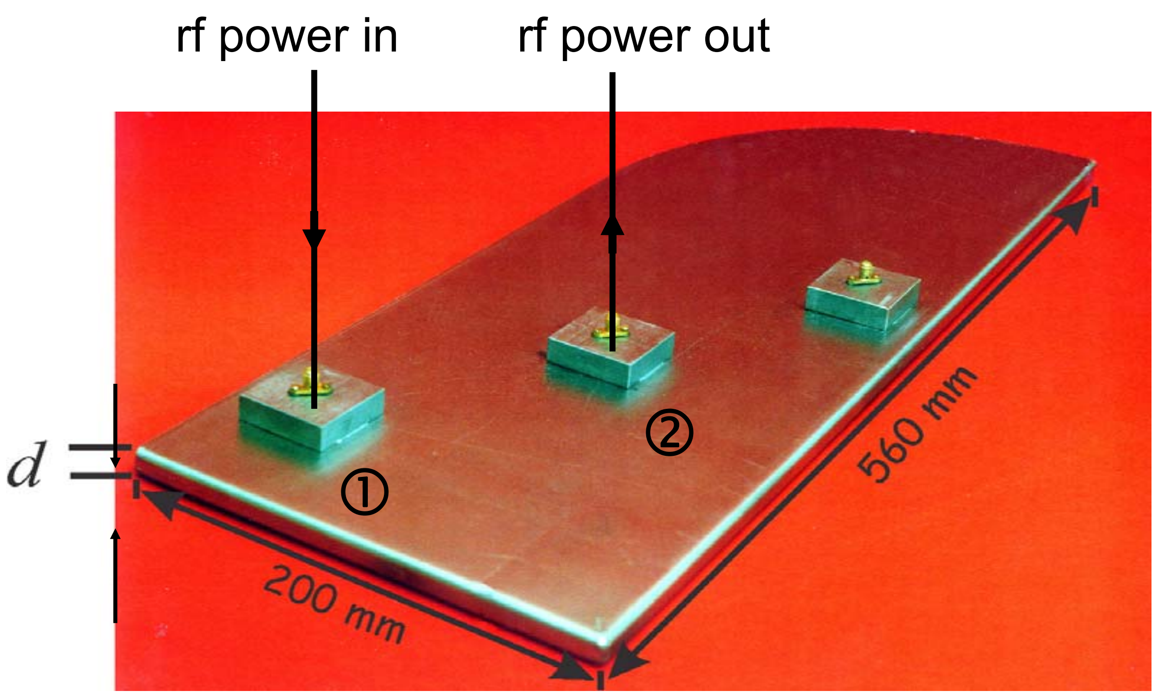
resonance frequencies ↔ eigenvalues

electric field strengths ↔ eigenfunctions

normal conducting resonators → ~700 eigenfunctions

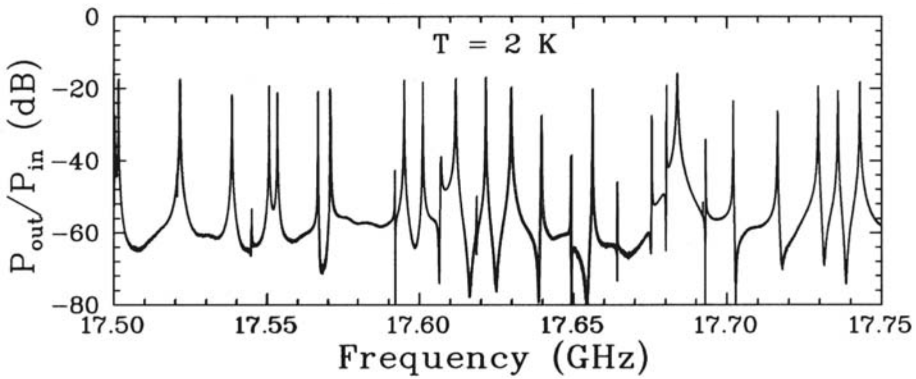
superconducting resonators → ~1000 eigenvalues

Microwave Resonator as a Scattering System



- Microwave power is **emitted** into the resonator by antenna ① and the output signal is **received** by antenna ② → **Open scattering system**
- The antennas act as **single scattering channels**

Transmission Spectrum



- **Transmission measurements**: relative power transmitted from a to b

$$P_{out,b} / P_{in,a} \propto |S_{ba}|^2$$

- Scattering matrix $\hat{S} = \hat{I} - 2\pi i \hat{W}^T (E - \hat{H} + i\pi \hat{W}\hat{W}^T)^{-1} \hat{W}$
- $\hat{H}$: resonator Hamiltonian
- $\hat{W}$: coupling of resonator states to antenna states and to the walls

Resonance Parameters

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- Use eigenrepresentation of

$$\hat{H}_{eff} = \hat{H} - i\pi\hat{W}\hat{W}^T$$

and obtain for a scattering system with isolated resonances

a → resonator → b

$$S_{ba} = \delta_{ba} - i\sum_{\mu}\frac{\sqrt{\Gamma_{\mu a}\Gamma_{\mu b}}}{f - f_{\mu} + (i/2)\Gamma_{\mu}}$$

- Here: f_{μ} = real part, Γ_{μ} = imaginary part } of eigenvalues e_{μ} of $\hat{H}_{eff}$

- Partial widths $\Gamma_{\mu,a}$, $\Gamma_{\mu,a}$ and total width Γ_{μ}

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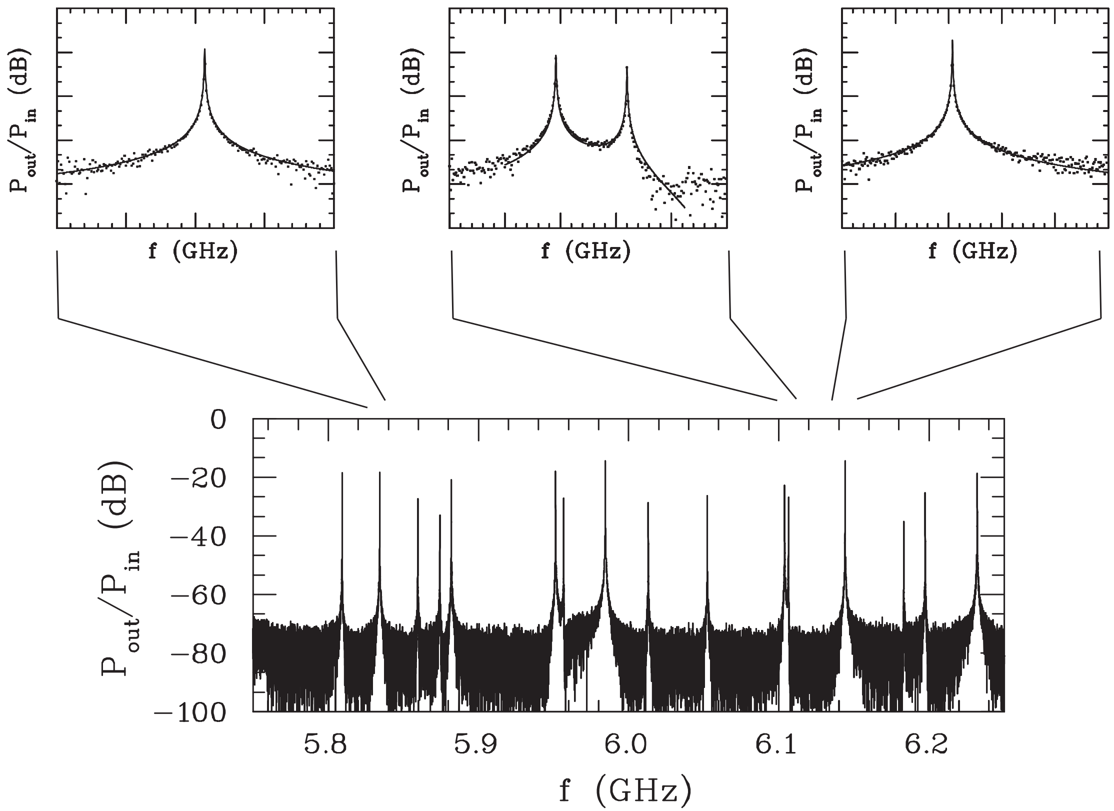
Typical Transmission Spectrum

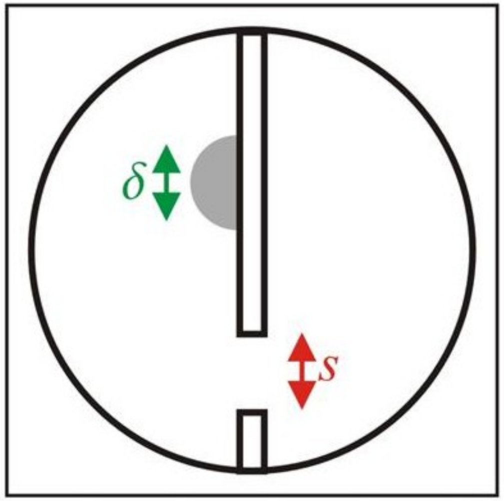


- Transmission measurements: relative power from antenna a → b

$$|S_{ba}|^2 = P_{out,b} / P_{in,a}$$

Eigenvalues and Eigenfunctions of a Dissipative System near an EP

- At an **exceptional point (EP)** two (or more) complex eigenvalues *and* the corresponding eigenfunctions of a dissipative system coalesce
- The crossing of two eigenvalues is accomplished by the **variation of two parameters**
- Sketch of the experimental setup:



- Divide a circular microwave billiard into two approximately equal parts
- The opening s controls the coupling of the eigenmodes of the two billiard parts
- The position δ of the Teflon disc determines the resonance frequencies of the left part

Two-state Matrix Model

(C. Dembowski et al., Phys. Rev. E **69**, 056216 (2004))

- Isolated EP
  → in its vicinity the dynamics is determined by the two eigenstates
  → model system with a **2d non-Hermitian symmetric matrix**

$$\hat{H}_{eff}(s,\delta) = \begin{pmatrix} E_1 & H_{12}^S \\ H_{12}^S & E_2 \end{pmatrix}$$

- All entries are functions of δ and s

- Eigenvalues:

$$e_{1,2} = \left(\frac{E_1 + E_2}{2}\right) \pm \Re$$

$$\Re = H_{12}^S \sqrt{Z^2 + 1}; \ Z = \frac{E_1 - E_2}{2H_{12}^S}$$

- EPs:

$$\Re = 0 : Z = \pm i \leftrightarrow \delta = \delta_{EP}, \ s = s_{EP}$$

Encircling the EP in the Parameter Space

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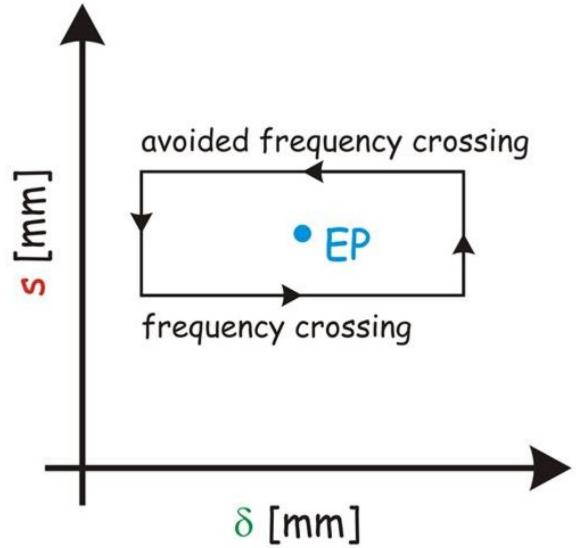
- **Encircling the EP** located at the parameter values s^{EP} and δ^{EP}

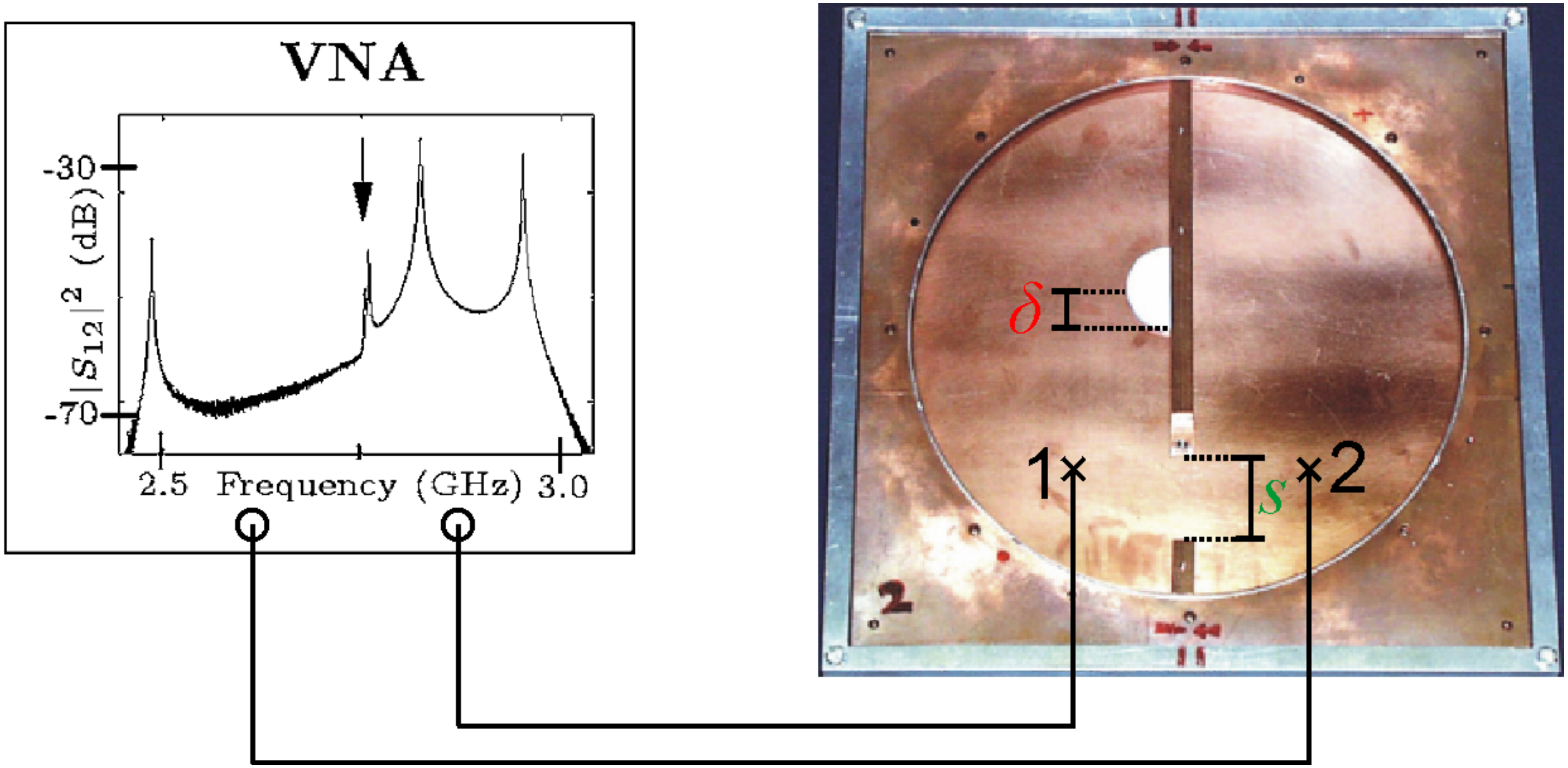


- The **eigenvalues are interchanged** and one of the eigenfunctions in addition picks up a **topological phase** π

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \rightarrow \begin{pmatrix} e_2 \\ e_1 \end{pmatrix} \qquad \begin{pmatrix} |\Psi_1\rangle \\ |\Psi_2\rangle \end{pmatrix} \rightarrow \begin{pmatrix} |\Psi_2\rangle \\ -|\Psi_1\rangle \end{pmatrix}$$

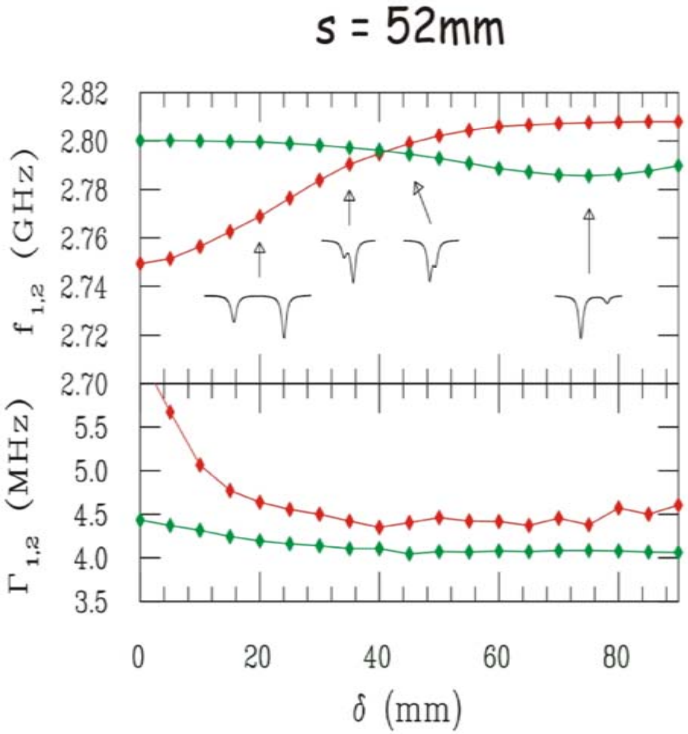
Experimental Setup

(C. Dembowski et al., Phys. Rev. Lett. **86**, 787 (2001))

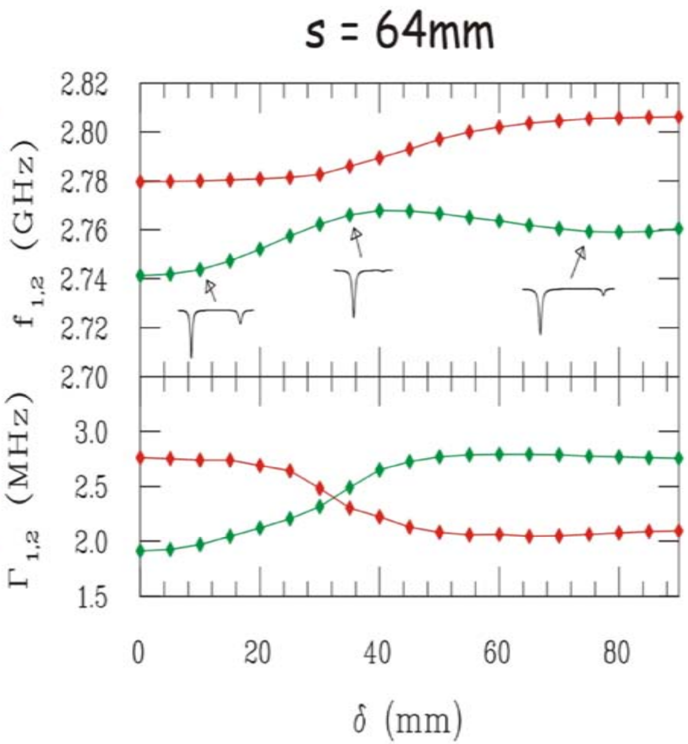


Frequencies and Widths

s < s^{EP}

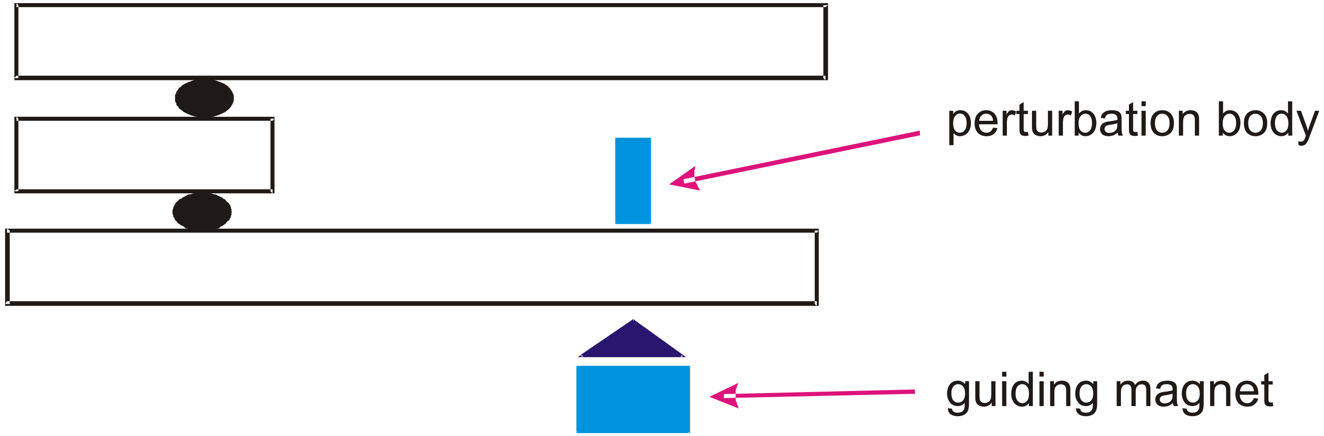


s > s^{EP}



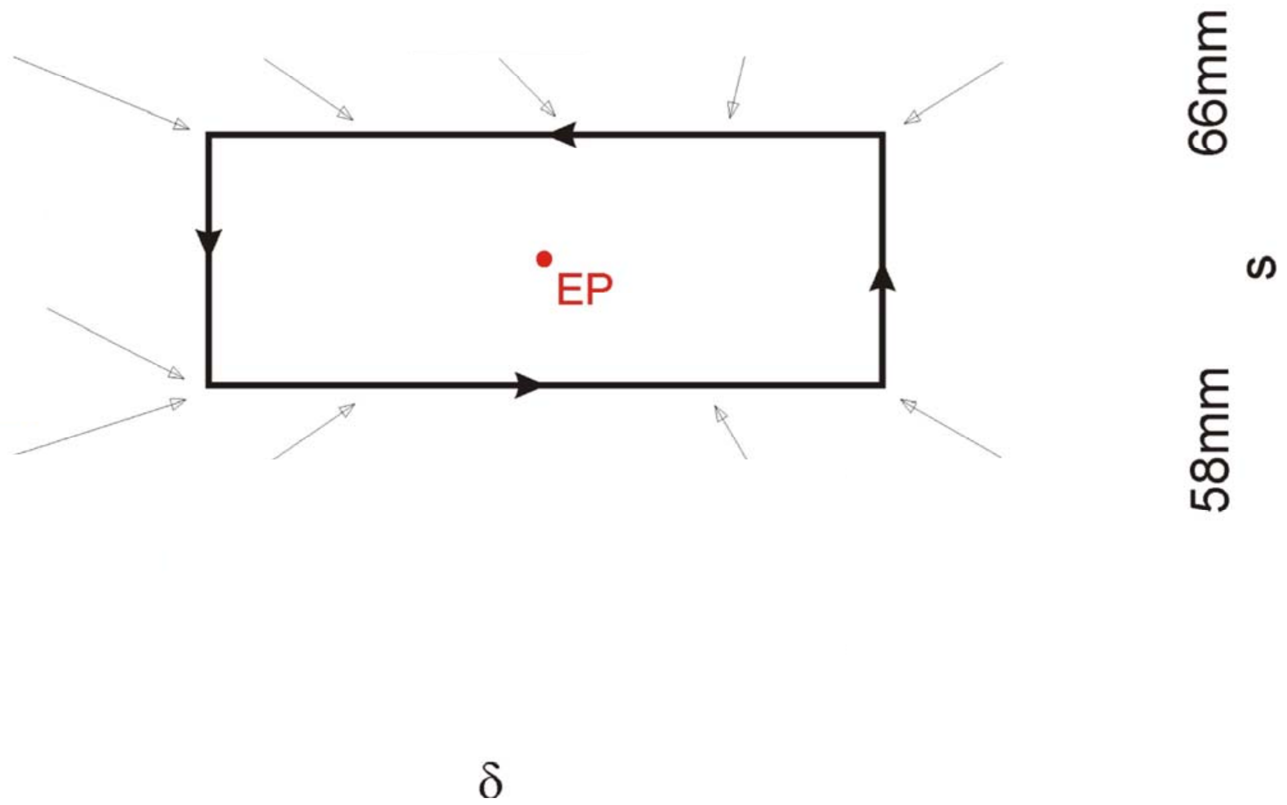
- Change of the real (resonance frequency) and the imaginary (resonance width) part of the eigenvalue $e_{1,2} = f_{1,2} + i \cdot \Gamma_{1,2}$ for varying δ and fixed s

Measuring Field Distributions



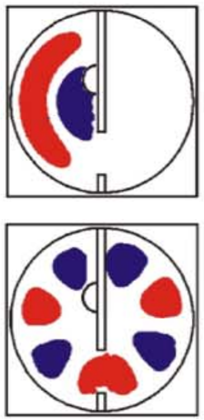
- Use dielectric perturbation body, e.g. **magnetic rubber**
- **Maier-Slater theorem**: $\delta f(x,y) = c_1 \cdot E^2(x,y)$
- Reconstruct the eigenfunctions from the pattern of nodal lines, where E(x,y)=0 → extraction of $\Psi(x, y)$

Evolution of Wave Functions



Chirality of Eigenfunctions

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

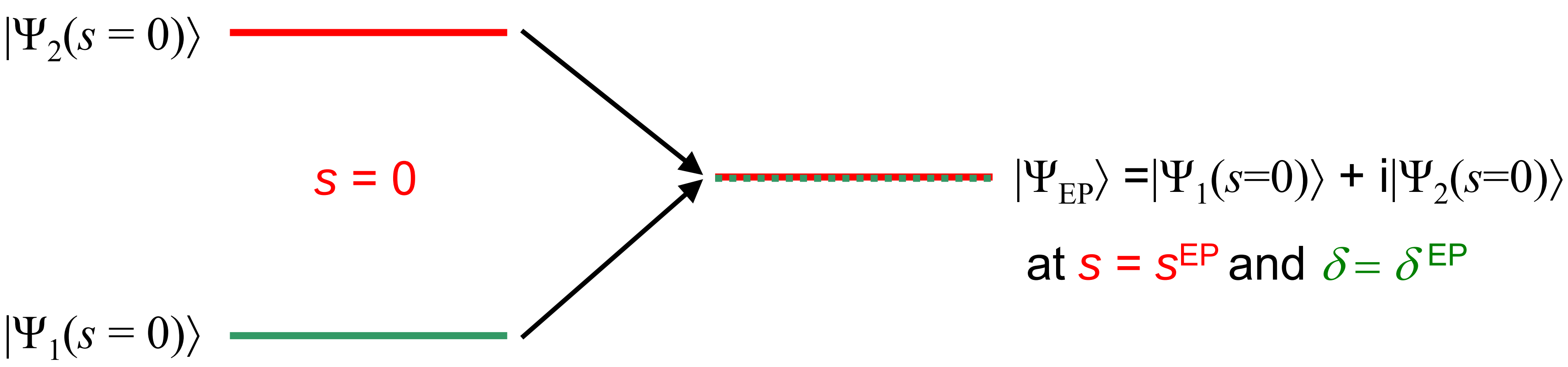


- Eigenfunctions are interchanged and one eigenfunction picks up a **topological phase** of π
- Surrounding the EP four times restores the original situation

What happens at the EP?

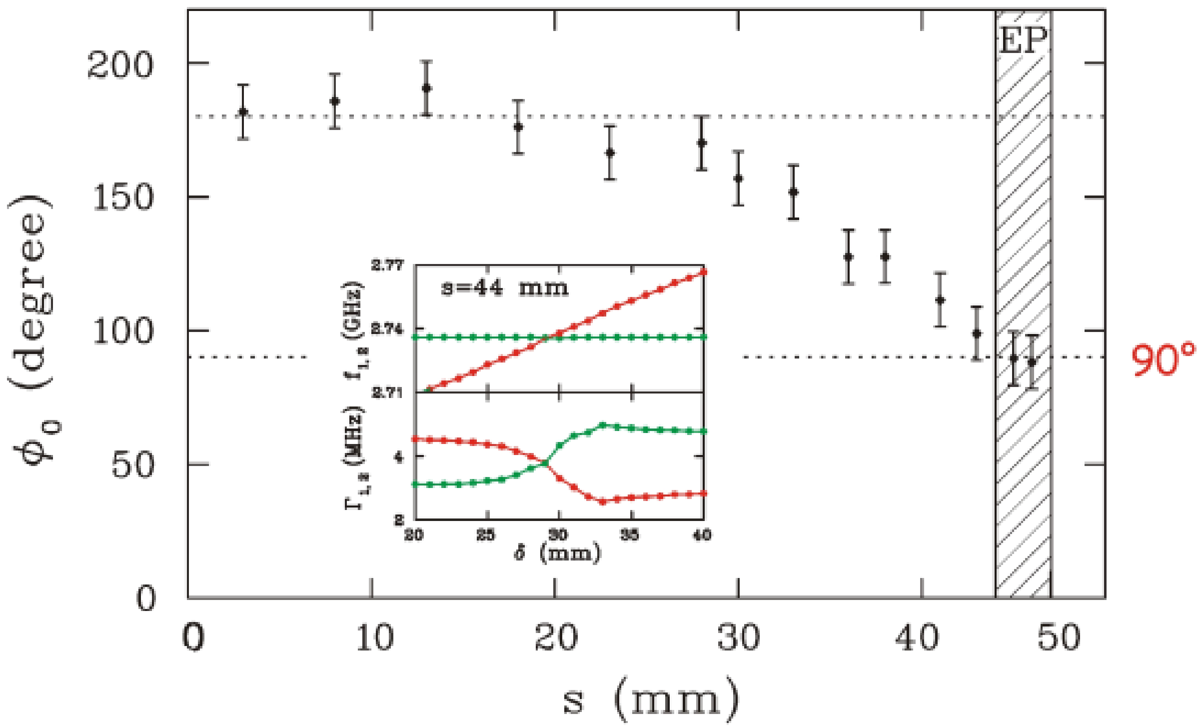
(C. Dembowski et al., Phys. Rev. Lett. **90**, 034101 (2003))

- At the exceptional point the two eigenmodes coalesce with a phase difference $\pi/2$



- Can this phase difference be measured?
- Choose two eigenmodes which are each localized in one of the semicircular parts of the cavity for $s < s^{EP}$ and $\delta = \delta^{EP}$ (i.e. $f_1 = f_2$)
  $\rightarrow$ these can be excited separately

Measured Phase Difference



- Modes Ψ_1 and Ψ_2 are excited separately by antennas 1 and 2
- Measurement of the **phase difference ϕ_0** between the oscillating fields $|\Psi_1\rangle$ and $|\Psi_2\rangle$ at the positions of the antennas

Time Decay of the Resonances near and at the EP

(Dietz et al., Phys. Rev. E **75**, 027201 (2007))

- In the vicinity of an EP the two eigenmodes can be described as a pair of coupled damped oscillators

- Near the EP the time spectrum exhibits besides the decay of the resonances oscillations → **Rabi oscillations** with frequency $\Omega = 2\pi f$

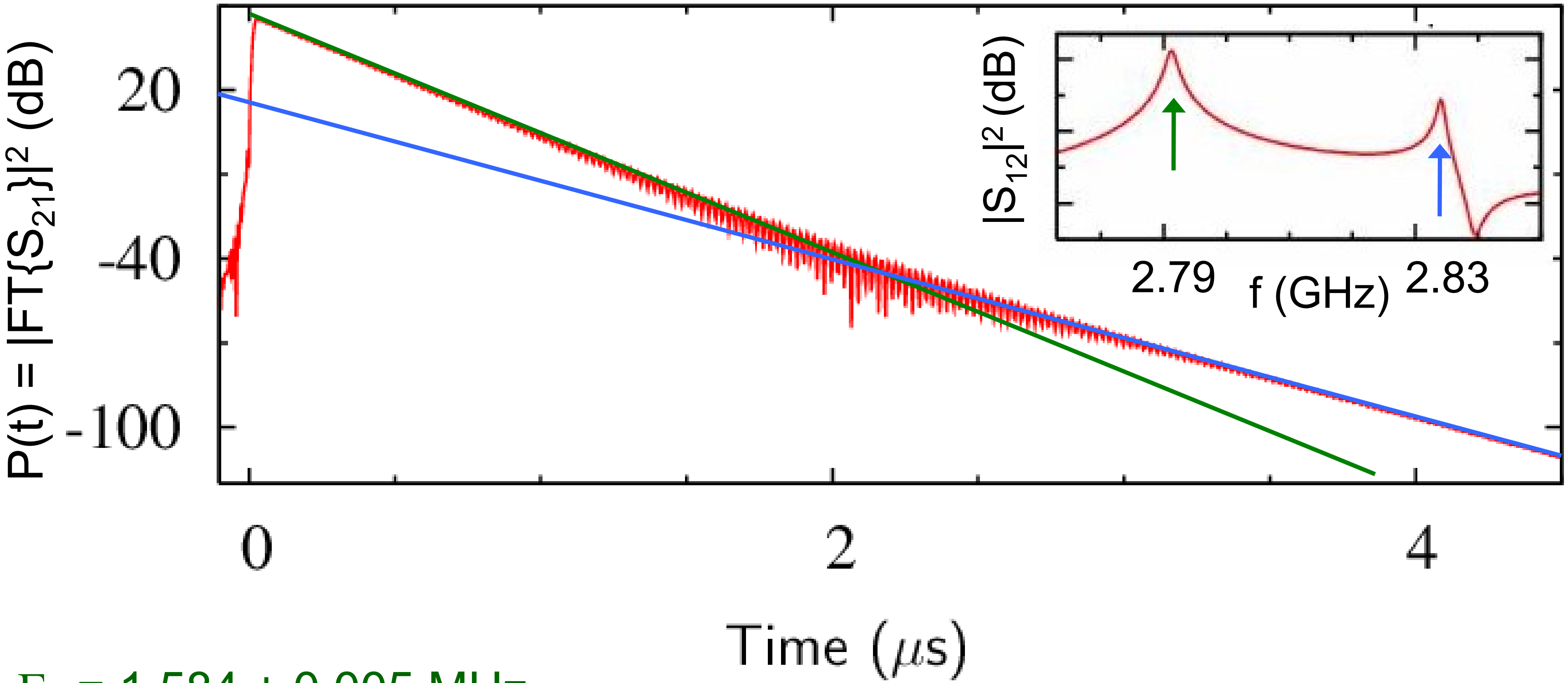
$$P(t) \propto \exp(-\Gamma t)\,\frac{\sin^2(\Omega t)}{\Omega^2};\ \Omega \propto \Re$$

- At the EP $\Re$ and Ω vanish → no oscillations

$$P(t) \propto t^2 \exp(-\Gamma t)$$

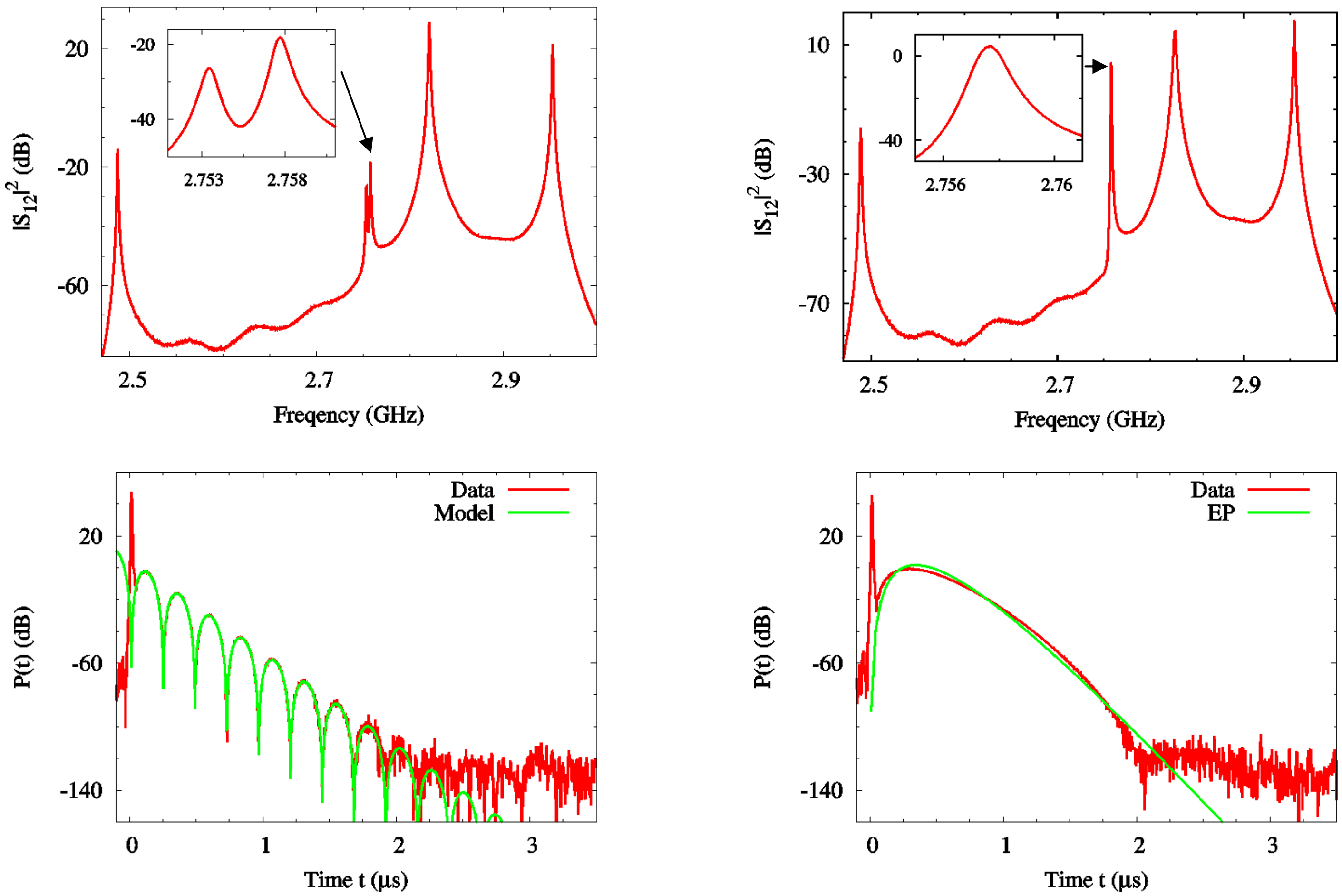
- In distinction, an isolated resonance decays simply exponentially → line-shape at EP is not of a Breit-Wigner form

Time Decay of the Resonances "far" from the EP



- Γ_1 = 1.584 ± 0.005 MHz
- Γ_2 = 1.022 ± 0.002 MHz
- Region of interaction at about 2 μs → Rabi oscillations

Time Decay of the Resonances near and at the EP



Detailed Balance in Nuclear Reactions

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Volume 56B, number 2 PHYSICS LETTERS 14 April 1975

TIME-REVERSIBILITY VIOLATION AND ISOLATED NUCLEAR RESONANCES*

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Received 21 January 1975

It is pointed out that measurements of differential cross-sections in nuclear reactions proceeding via an isolated resonance can provide in principle a test for time reversibility.

- Search for Time-reversal (T-) Invariance Violation (TIV) in nuclear reactions

Detailed Balance in Nuclear Reactions

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2.A.1: 2.C

Nuclear Physics **A317** (1979) 300–312; *North-Holland Publishing Co., Amsterdam*

TEST OF DETAILED BALANCE AT ISOLATED RESONANCES IN THE REACTIONS $^{27}Al+p \rightleftarrows {}^{24}Mg+\alpha$ AND TIME REVERSIBILITY †

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Received 15 September 1978

Abstract: The principle of detailed balance has been tested in the reactions $^{27}Al(p, \alpha_0)^{24}Mg$ (Q = 1.600 MeV) and $^{24}Mg(\alpha, p_0)^{27}Al$ (Q = −1.600 MeV) at bombarding energies E_p^{lab} = 1.35–1.46 MeV and E_α^{lab} = 3.38–3.52 MeV, respectively. Protons and α-particles were detected at $\theta_{c.m.}$ = 177.7°. The relative strengths of two resonances at E_x = 12.901 MeV ($J^\pi = 2^+$) and E_x = 12.974 MeV ($J^\pi = 1^-$) in ^{28}Si excited in the forward and backward reaction agree within the experimental uncertainty $\delta = 0.0025 \pm 0.0192$. This experimental result is converted into a difference of phase angles for reduced widths amplitudes, $\Delta\xi = (0.3 \pm 3)°$, which is consistent with time reversibility.

- Search for TIV in nuclear reactions → upper limits

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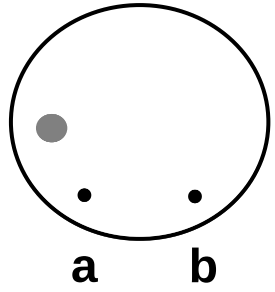
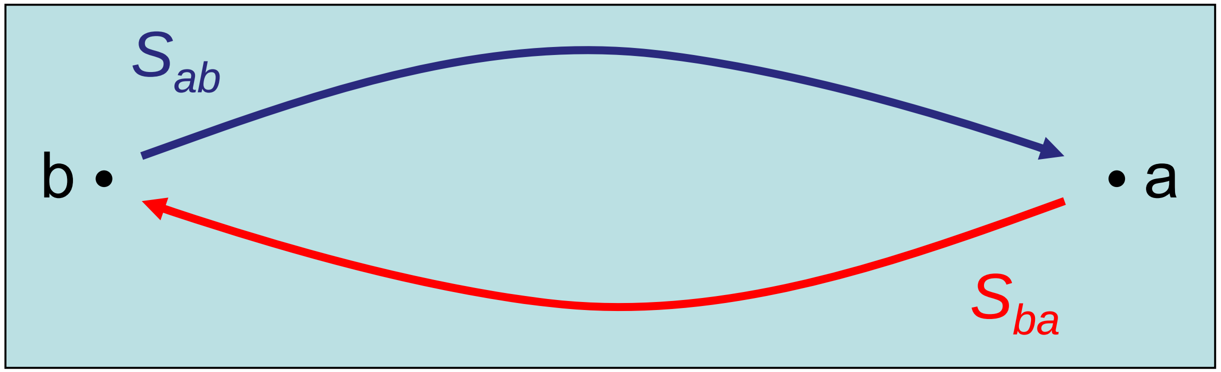
Induced *T*-Invariance Violation in Microwave Billiards

(B. Dietz et al., Phys. Rev. Lett. **98**, 074103 (2007))

- *T*-invariance violation caused by a magnetized ferrite
- Ferrite features Ferromagnetic Resonance (FMR)
- Coupling of microwaves to the ferrite depends on the direction a ⇆ b





- Principle of detailed balance: $|S_{ab}|^2 = |S_{ba}|^2$
- Principle of reciprocity: $S_{ab} = S_{ba}$

Scattering Matrix Description

Remember: Scattering matrix formalism

$$\hat{S}(f) = I - 2\pi i \hat{W}^T \left(fI - \hat{H} + i\pi \hat{W}\hat{W}^T \right)^{-1} \hat{W}$$

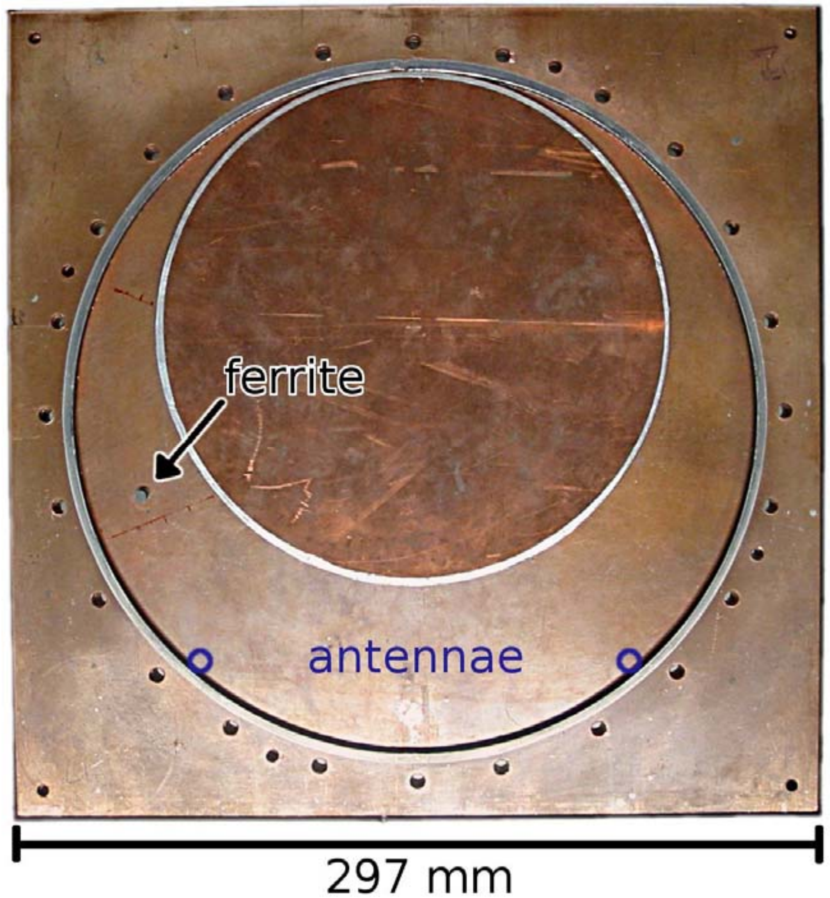
- Investigation of T-invariant / T-noninvariant systems

→ replace $\hat{H}$ by a real symmetric / Hermitian matrix

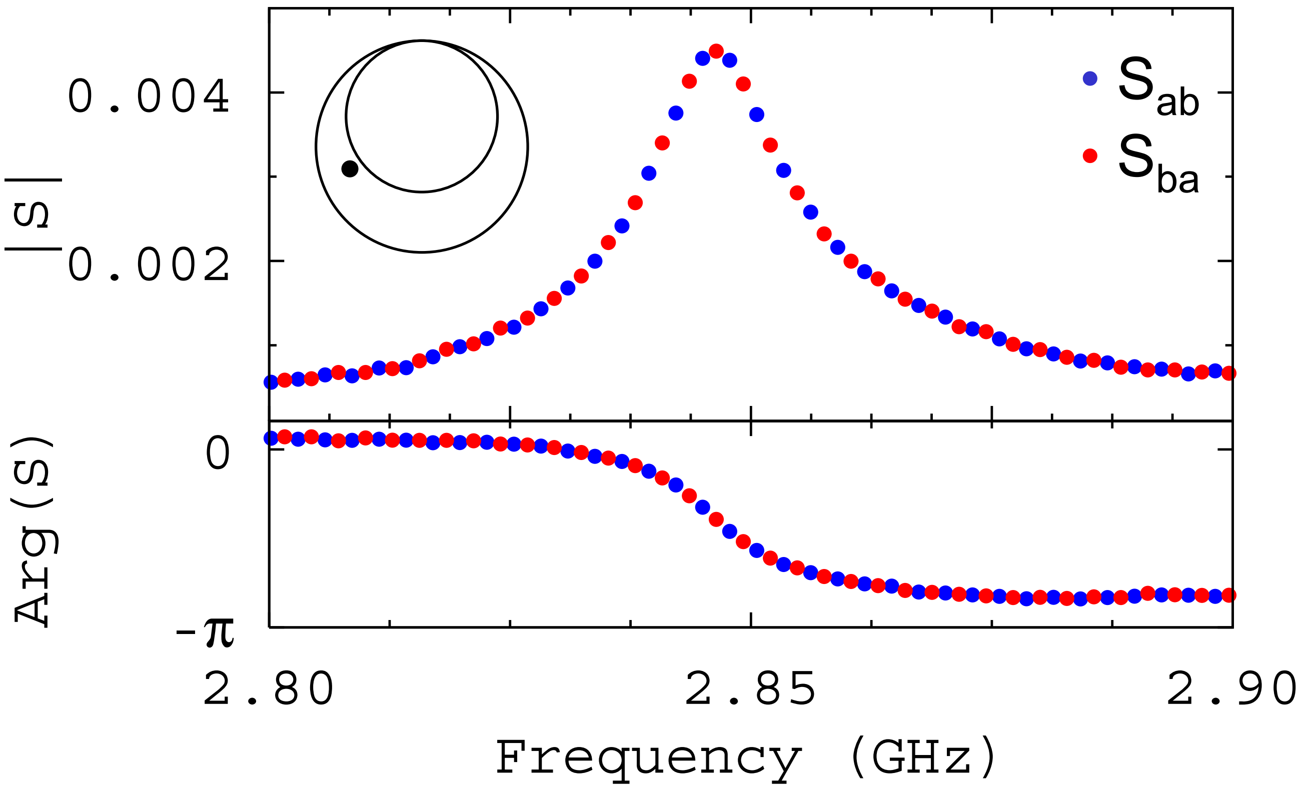
- Note: $\hat{H}_{eff} = \hat{H} - i\pi\hat{W}\hat{W}^T$ is non-Hermitian in both cases
- Isolated resonances: singlets and doublets → $\hat{H}$ is 1D or 2D
- Overlapping resonances ($\Gamma > D$) $\hat{H}$ = GOE / GUE in RMT

(B. Dietz et al., Phys. Rev. Lett. **103**, 064101 (2009))

Isolated Resonances - Setup

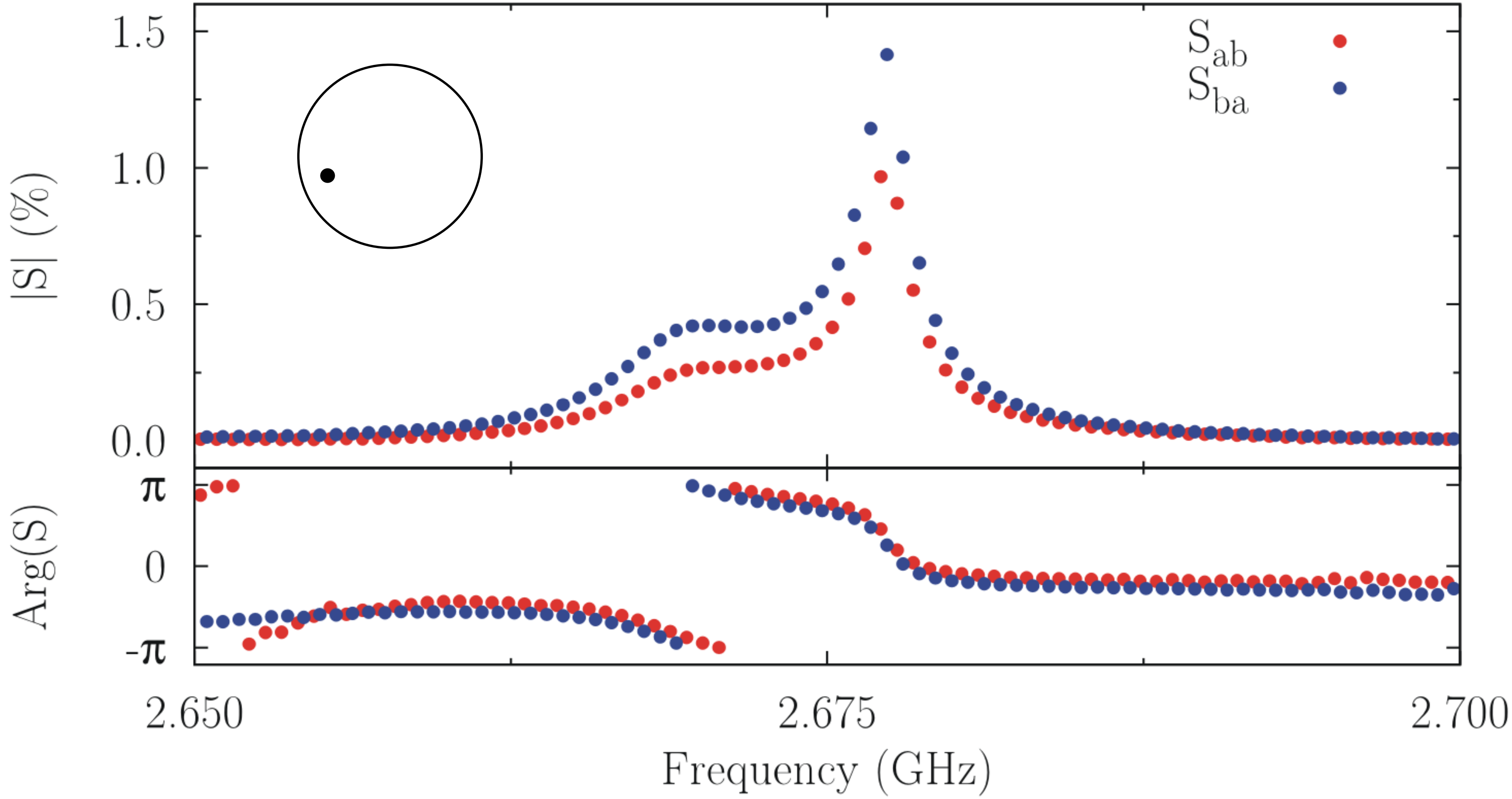


Singlet with T-Violation



- Reciprocity holds → T-violation cannot be detected this way

Doublet with T-Violation



- Violation of reciprocity due to interference of two resonances

Scattering Matrix and *T*-violation

- Scattering matrix element ($\omega = 2\pi f$)

$$S_{ab}(\omega) = \delta_{ab} - 2\pi i \langle a | \hat{W}^{+} (\omega - \hat{H}_{eff})^{-1} \hat{W} | b \rangle$$

- Decomposition of effective Hamiltonian

$$\hat{H}_{eff} = i\hat{H}^{a} + \hat{H}^{s}$$

$$\downarrow$$

$$\begin{pmatrix} 0 & iH^{a}_{12} \\ -iH^{a}_{12} & 0 \end{pmatrix}$$

- Ansatz for *T*-violation incorporating the ferromagnetic resonance and its selective coupling to the microwaves
- Note: $\hat{H}^{s}$ conserves *T*-invariance and $\hat{H}^{a}$ violates it

T-Violating Matrix Element

$$H_{12}^{a}(B) = \frac{\pi}{2} \cdot \lambda \cdot B \cdot T^{r} \cdot \frac{\omega_M^2}{\omega_0(B) - \overline{\omega} - i/T^{r}}$$

- λ: coupling strength
- B: external field
- T^{r}: spin relaxation time
- $\frac{\omega_M^2}{\omega_0(B) - \overline{\omega} - i/T^{r}}$: magnetic susceptibility

- Fit parameters: λ and $\overline{\omega}$

Measured T-Violating Matrix Element

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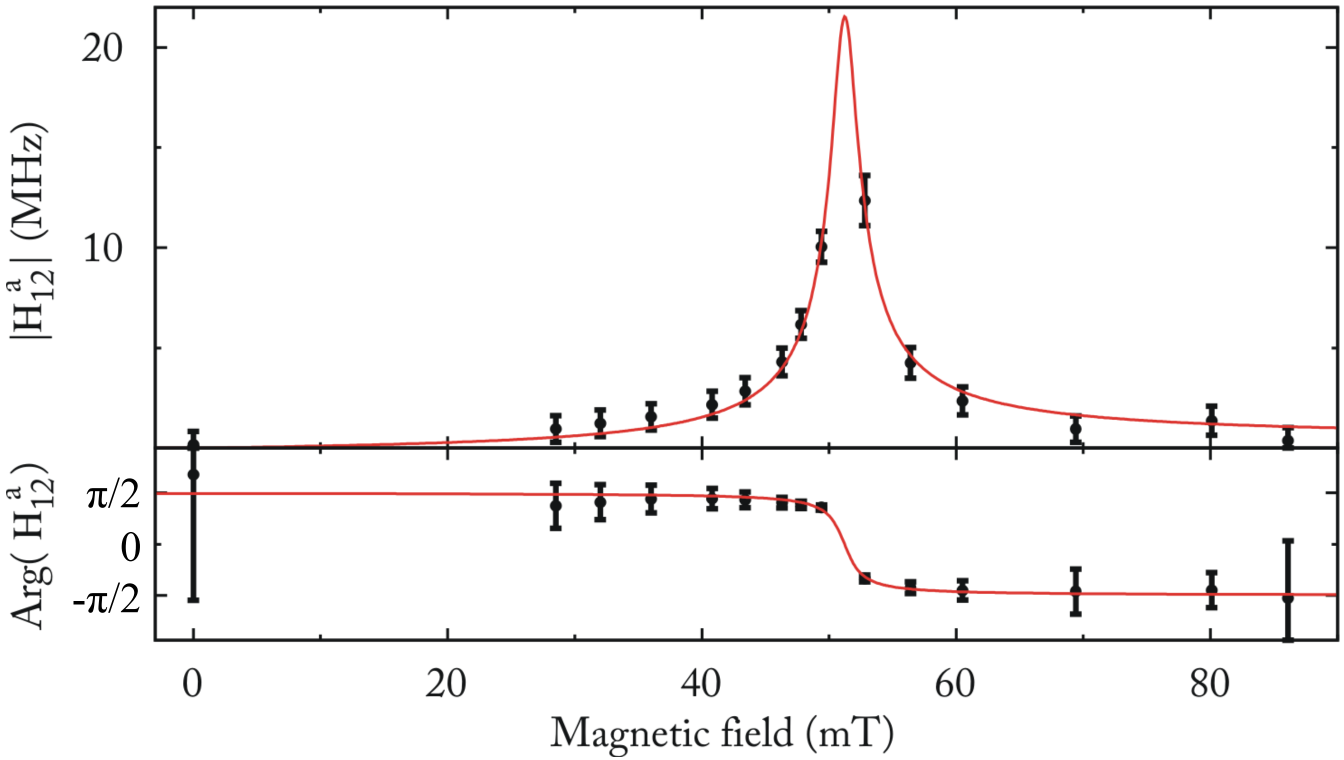
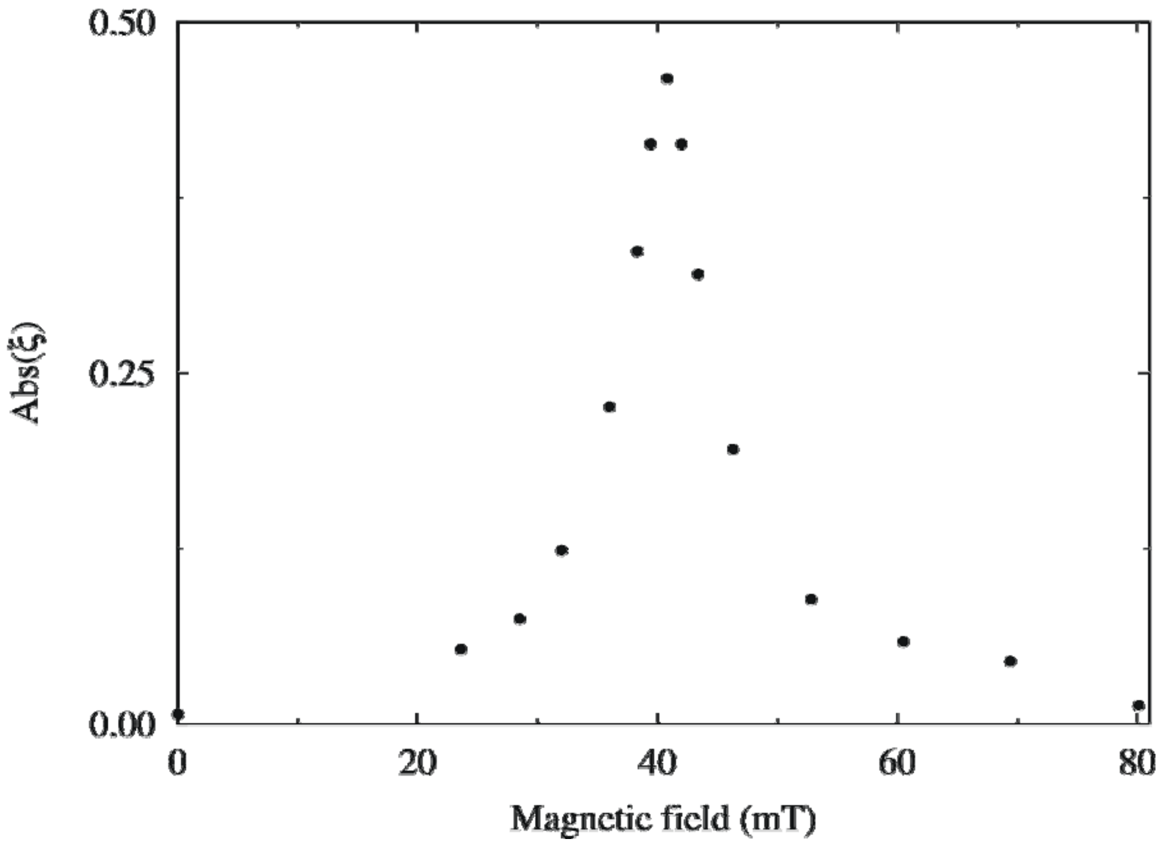


- T-violating matrix element shows resonance like structure
- Successful description of dependence on magnetic field
  $\rightarrow \hat{H}_{eff}$ was determined and will be used to describe TIV at the EP

Relative Strength of T-Violation

- Compare: TIV matrix element H_{12}^a to the energy difference of two eigenvalues of the T-invariant system

$$\xi = \left| \frac{2H_{12}^a}{E_1^s - E_2^s} \right|$$



Summary and Outlook

- Microwave billiards are precisely controlled, parameter-dependent dissipative systems
- Complex eigenvalues and eigenfunctions can be determined from the billiard scattering matrix
  → Observation of EPs in microwave billiards
- Behavior of eigenvalues close to and at an EP investigated experimentally
- Also time-behavior close to and at an EP was studied
  → disappearance of echoes at the EP → t^2-behavior
- T-invariance violation induced in microwave billiards and observed for doublets of resonances → determination of complete $\hat{H}_{eff}$ possible at an exceptional point
- Second talk: Measurement and interpretation of T-violation