Interplay between system and environment in quantum mechanics

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System and environment

- the system is
 - a many-particle system with many levels
 - described by standard quantum mechanics
 - localized in space
- the (natural) environment is
 - the continuum of scattering wavefunctions
 - described by standard quantum mechanics
 - extended in space
- the states of the system have, generally, a finite lifetime due to embedding the system into the environment
- the coupling between system and environment has to be taken into account nonperturbatively

\longrightarrow the system is open

Solution of the problem by means of the Feshbach projection operator technique

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Schrödinger equation in the whole function space with discrete and scattering states

 $(\mathbf{H}^{\mathrm{full}} - \mathbf{E}) \Psi_{\mathbf{F}}^{\mathrm{c}} = 0$ $(H_{B} - E_{\lambda}^{B}) \Phi_{\lambda}^{B} = 0$ $\label{eq:Q} \begin{array}{rcl} &\longrightarrow \mathbf{Q} = \sum_{\lambda} |\Phi^{\mathrm{B}}_{\lambda}\rangle \langle \Phi^{\mathrm{B}}_{\lambda}| \\ (\mathrm{H_c} &- \mathrm{E})\,\xi^{\mathrm{E}(+)}_{\mathrm{c}} = \mathbf{0} \end{array} .$ \longrightarrow P = $\sum_{c} \int_{\epsilon_{c}}^{\epsilon_{c}'} dE |\xi_{c}^{E(+)}\rangle\langle\xi_{c}^{E(+)}|$ $\gamma_{\lambda a}^{0} = \sqrt{2\pi} \langle \Phi_{\lambda}^{\mathsf{B}} | \mathsf{QHP} | \xi_{a}^{\mathsf{E}(+)} \rangle$ $H^{\text{full}} = H_{\Omega\Omega} + H_{\OmegaP} + H_{PQ} + H_{PP}$ Q + P = 1 $(\Psi_{c}^{\mathsf{E}} \text{ contains everything})$

H^{full} is Hermitian

Solution (called 'formal' by Feshbach)

$$\begin{split} \Psi_{c}^{E} &= \xi_{c}^{E} + \sum_{\lambda,\lambda'=1}^{N} (\Phi_{\lambda}^{B} + \omega_{\lambda}^{0}) \langle \Phi_{\lambda}^{B} | \frac{1}{E - H_{eff}} | \Phi_{\lambda'}^{B} \rangle \langle \Phi_{\lambda'}^{B} | H_{QP} | \xi_{c}^{E} \rangle \\ H_{eff} &= H_{QQ} + H_{QP} G_{P}^{(+)} H_{PQ} \quad (\text{non-Hermitian}) \\ \omega_{\lambda}^{0(+)} &= G_{P}^{(+)} H_{PQ} \cdot \Phi_{\lambda}^{B} \\ G_{P}^{(+)} &= P(E - H_{PP})^{-1} P \\ (H_{eff} - z_{\lambda}) \Phi_{\lambda} &= 0 \end{split}$$
$$\begin{aligned} \Psi_{c}^{E} &= \xi_{c}^{E} \quad + \sum_{\lambda=1}^{N} \Omega_{\lambda} \cdot \frac{\langle \Phi_{\lambda}^{*} | H_{QP} | \xi_{c}^{E} \rangle}{E - z_{\lambda}} \\ \Omega_{\lambda} &= (1 + G_{P}^{(+)} H_{PQ}) \Phi_{\lambda} \\ &= \Phi_{\lambda} + \omega_{\lambda} \end{split}$$

Solution is exact within P + Q = 1

- The system with N states is localized (Q subsystem)
- In a certain energy window, it is embedded in the extended continuum of scattering wave functions ξ^E_C (P subsystem)
- The states of the Q subspace can interact via the states of the P subspace

- System and environment are well defined
 - \rightarrow P + Q = 1
- The system is open

The effective non-Hermitian Hamiltonian

- ▶ H_{eff} is non-Hermitian ↔ states of the (localized) system (Q subspace) interact via a common (extended) environment (P subspace)
- Eigenfunctions Φ_{λ} of H_{eff} are biorthogonal

 $\langle \Phi_{\mathsf{k}}^* | \Phi_{\mathsf{l}} \rangle = \delta_{\mathsf{k},\mathsf{l}}$

such that

$$\langle \Phi_k | \Phi_k
angle \equiv A_k \geq 1$$

Effective Hamiltonian consists of a first-order and a second-order interaction term

$$\begin{split} H_{\rm eff} &= H_{\rm B} + V_{\rm BC} G_{\rm C}^{(+)} V_{\rm CB} \\ {\rm Re} \left\{ \langle \Phi_i^{\rm B} | H_{\rm eff} | \Phi_j^{\rm B} \rangle \right\} &= \langle \Phi_i^{\rm B} | H_{\rm B} | \Phi_j^{\rm B} \rangle + {\rm Re} \left(\hat{W}_{ij} \right) \\ &= \langle \Phi_i^{\rm B} | H_{\rm B} | \Phi_j^{\rm B} \rangle \\ &+ \frac{1}{2\pi} \sum_{\rm C} \mathcal{P} \int_{\epsilon_{\rm C}}^{\infty} {\rm dE}' \; \frac{\hat{\gamma}_i^{\rm C} \hat{\gamma}_j^{\rm C}}{{\rm E} - {\rm E}'} \end{split}$$

principal value integral

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$$\mathrm{Im} \left\{ \langle \Phi_{i}^{\mathsf{B}} | \mathsf{H}_{\mathrm{eff}} | \Phi_{j}^{\mathsf{B}} \rangle \right\} = \mathrm{Im} \left(\hat{\mathsf{W}}_{ij} \right) = -\frac{1}{2} \sum_{\mathsf{C}} \hat{\gamma}_{i}^{\mathsf{C}} \hat{\gamma}_{j}^{\mathsf{C}}$$

$$\frac{1}{\mathsf{residuum}}$$

The role of boundary conditions

Schrödinger equation

$$(\mathsf{H}_{\mathrm{eff}} - \mathsf{z}_{\lambda}) \, \Phi_{\lambda} = \mathbf{0}$$

 $H_{\rm eff}$ energy dependent

► resonance states (inside the energy window): H_{eff} non-Hermitian $z_{\lambda} = E_{\lambda} - \frac{i}{2}\Gamma_{\lambda}$ complex, in general Φ_{λ} complex and biorthogonal, in general $\longrightarrow \langle \Phi_{\lambda}^* | \Phi_{\lambda'} \rangle = \delta_{\lambda\lambda'}$

► discrete states (beyond the energy window): H_{eff} Hermitian $z_{\lambda} = E_{\lambda}$ real, i.e. $\Gamma_{\lambda} = 0$ but $E_{\lambda} \neq E_{\lambda}^{B}$ Φ_{λ} orthogonal $\longrightarrow \langle \Phi_{\lambda} | \Phi_{\lambda'} \rangle = \delta_{\lambda\lambda'}$

Scattering wave function INSIDE the system

$$\begin{split} \Psi_{c}^{E} &= \xi_{c}^{E} + \sum_{\lambda=1}^{N} (1 + G_{P}^{(+)} H_{PQ}) \Phi_{\lambda} \cdot \frac{\langle \Phi_{\lambda}^{*} | H_{QP} | \xi_{c}^{E} \rangle}{E - z_{\lambda}} \\ &\rightarrow \Psi_{c}^{E} _{int} = \sum_{\lambda=1}^{N} c_{\lambda E}^{c} \Phi_{\lambda} \quad ; \\ &c_{\lambda E}^{c} &= \frac{\langle \Phi_{\lambda}^{*} | H_{QP} | \xi_{c}^{E} \rangle}{E - z_{\lambda}} = \frac{\langle \xi_{c}^{E} | H_{PQ} | \Phi_{\lambda} \rangle}{E - z_{\lambda}} \end{split}$$

The $\Psi_{c \text{ int}}^{E}$ can be represented in the set $\{\Phi_{\lambda}\}$ of biorthogonal wavefunctions (eigenfunctions of H_{eff}):

$$\begin{split} |\Psi_{c \text{ int}}^{\mathsf{E},\mathsf{R}}\rangle &=& \sum_{\lambda}c_{\lambda\mathsf{E}}^{\mathsf{c}}|\Phi_{\lambda}^{\mathsf{R}}\rangle = \sum_{\lambda}c_{\lambda\mathsf{E}}^{\mathsf{c}}|\Phi_{\lambda}\rangle \\ \langle\Psi_{c \text{ int}}^{\mathsf{E},\mathsf{L}}| &=& \sum_{\lambda}c_{\lambda\mathsf{E}}^{\mathsf{c}}\langle\Phi_{\lambda}^{\mathsf{L}}| = \sum_{\lambda}c_{\lambda\mathsf{E}}^{\mathsf{c}}\langle\Phi_{\lambda}^{*}| \end{split}$$

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Biorthogonal wavefunctions and

exceptional points

in the case of a two-level system

Mathematics: Exceptional points

Consider a family of operators of the form

$$\mathsf{T}(\varkappa) = \mathsf{T}(0) + \varkappa \mathsf{T}'$$

 \varkappa – scalar parameter T(0) – unperturbed operator \varkappa T' – perturbation

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Number of eigenvalues of $T(\varkappa)$ is independent of \varkappa with the exception of some special values of \varkappa (exceptional points) where (at least) two eigenvalues coalesce

Example:

$$\mathsf{T}(\varkappa) = \begin{pmatrix} 1 & \varkappa \\ \varkappa & -1 \end{pmatrix} \qquad \qquad \mathsf{T}(\varkappa = \pm \mathsf{i}) \to \text{eigenvalue } \mathsf{0}$$

T. Kato, Perturbation theory for linear operators

2×2 Hamiltonian of a physical system

► For example: 2 × 2 Hamiltonian

$$\mathsf{H}(\omega) = \left(\begin{array}{cc} \epsilon_1 & \omega \\ \omega & \epsilon_2 \end{array}\right)$$

 ϵ_i – eigenenergies of individual states ω – interaction via the environment

Eigenvalues

$$\mathsf{E}_{\pm} = rac{\epsilon_1 + \epsilon_2}{2} \pm \mathsf{Z} \; ; \quad \mathsf{Z} \equiv rac{1}{2} \sqrt{(\epsilon_1 - \epsilon_2)^2 + 4\omega^2}$$

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Physical meaning of Z

Re(Z) ↔ level repulsion in energy shift of the states in energy

 $Im(Z) \leftrightarrow width bifurcation$

change of the time scale being characteristic of each state

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The two eigenvalue trajectories cross when

$$\frac{\epsilon_1 - \epsilon_2}{2\omega} = \pm i$$

These crossing points may be called exceptional points:

$\mathsf{E}_+ = \mathsf{E}_- \equiv \mathsf{E}_0$

Here, Re(Z)
ightarrow 0~ AND ~Im(Z) ightarrow 0~

- The crossing points can be found by varying two parameters
- As a function of only one parameter, the two states (usually) avoid crossing at the critical value of the parameter

Open quantum system (with decaying states)

 $\epsilon_1 \neq \epsilon_2$ and ω complex; $Im(\epsilon_i) < 0$

$$E_{1,2} = e_{1,2} - \frac{i}{2} \gamma_{1,2}$$

 e_i – position in energy of the state γ_i – width (inverse lifetime) of the state

 $\operatorname{Re}(\mathsf{Z})\gg\operatorname{Im}(\mathsf{Z})$: level repulsion $\gamma_{1,2}
eq 0$ usually

Re(Z) \ll Im(Z): width bifurcation $\gamma_{1,2} = 0$ is possible (P-symmetry) (bound state in the continuum) *PT* symmetric models (with stable states)
 (using the formal equivalence between the Schrödinger equation and the optical wave equation)

experimental results: Guo et al PRL 103, 093902 (2009) Rüter et al, Nature Physics 6, 192 (2010)

$$\begin{aligned} &\operatorname{Re}(\epsilon_{1}) = \operatorname{Re}(\epsilon_{2}) = e \\ &\operatorname{Im}(\epsilon_{1}) = -\operatorname{Im}(\epsilon_{2}) \text{ ('gain' and 'loss')} \\ &\omega_{12} = \omega_{21}^{*} \text{ ; } \mathsf{Z} = \sqrt{|\omega|^{2} - \operatorname{Im}(\epsilon)^{2}} \\ &\operatorname{E}_{1,2} = e \pm \frac{i}{2} \gamma_{1,2} \\ &\operatorname{Re}(\mathsf{Z}) \gg \operatorname{Im}(\mathsf{Z}): \ \mathcal{PT} \text{ symmetry } (\gamma_{1,2} = 0) \\ &\operatorname{Re}(\mathsf{Z}) \ll \operatorname{Im}(\mathsf{Z}): \ \mathcal{PT} \text{ symmetry breaking } (\gamma_{1,2} \neq 0) \end{aligned}$$

In models for open quantum systems and in \mathcal{PT} symmetric models exceptional points cause, in their neighborhood, non-trivial changes in the system

An exceptional point defines a threshold of spectroscopic redistribution and of \mathcal{PT} symmetry breaking, respectively

Eigenfunctions at the exceptional point

Eigenfunctions are linearly dependent

 $\phi^{\mathrm{cr}}_+ \rightarrow \pm \mathrm{i} \phi^{\mathrm{cr}}_- \qquad \phi^{\mathrm{cr}}_- \rightarrow \mp \mathrm{i} \phi^{\mathrm{cr}}_+$

analytical studies: Rotter, PRE 64, 036213 (2001) Günther et al, JPA 40, 8815 (2007) numerical studies: Magunov et al, JPB 34, 29 (2001)

 Agreement with experimental results: encircling exceptional points or critical points of avoided level crossings

$$\phi^{\mathrm{cr}}_+ \mid = \mid \phi^{\mathrm{cr}}_- \mid$$

Dembowski et al., PRL 86, 787 (2001) Experimentally: wavefunctions (including phases) are restored after four surroundings

Phases of the eigenfunctions

Eigenfunctions are biorthogonal

$$\langle \phi_{\mathbf{k}}^{*} | \phi_{\mathbf{l}}
angle \equiv \delta_{\mathbf{k},\mathbf{l}} \qquad \langle \phi_{\mathbf{k}} | \phi_{\mathbf{k}}
angle \equiv \mathbf{A}_{\mathbf{k}} \geq 1$$

Borderline cases

- two distant levels: wavefunctions are orthogonal

$$\langle \phi_{\mathsf{k}}^{*} | \phi_{\mathsf{k}}
angle pprox \langle \phi_{\mathsf{k}} | \phi_{\mathsf{k}}
angle = \mathsf{A}_{\mathsf{k}} pprox 1$$

 two crossing levels: wavefunctions are linearly dependent

$$\phi_{\mathbf{k}}
ightarrow \pm \mathbf{i} \ \phi_{\mathbf{l}} \qquad \langle \phi_{\mathbf{k}} | \phi_{\mathbf{k}}
angle = \mathbf{A}_{\mathbf{k}}
ightarrow \infty$$

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Mathematically,

$$\langle \phi_{\mathbf{k}}^* | \phi_{\mathbf{l}} \rangle = \mathbb{C}$$

Physically: for distant levels,

$$\langle \phi_{\mathsf{k}}^{*} | \phi_{\mathsf{k}}
angle o \langle \phi_{\mathsf{k}} | \phi_{\mathsf{k}}
angle = 1$$

Therefore,

$$\mathbb{C} = 1 \rightarrow \operatorname{Im}(\mathbb{C}) = 0$$

 $\rightarrow~$ the phases of the two eigenfunctions $\phi_{\bf k}$ relative to one another are

not rigid

in approaching an exceptional point

Phase rigidity

$$\mathbf{r}_{\mathbf{k}} \equiv \frac{\langle \phi_{\mathbf{k}}^* | \phi_{\mathbf{k}} \rangle}{\langle \phi_{\mathbf{k}} | \phi_{\mathbf{k}} \rangle} = \mathbf{A}_{\mathbf{k}}^{-1} \implies \mathbf{1} \geq \mathbf{r}_{\mathbf{k}} \geq \mathbf{0}$$

 Experimental proof: in the regime of overlapping resonances with avoided level crossings

 $1 > \mathsf{r}_{\mathsf{k}} > 0$

Dembowski et al., PRL 90, 034101 (2003) Experimentally: the phase difference between two modes is π at large distance and $\pi/2$ at the singularity

\mathcal{PT} symmetry breaking as well as spectroscopic redistribution

are accompanied by non-rigid phases of the eigenfunctions of the Hamiltonian in the vicinity of an exceptional point

Biorthogonal wavefunctions and exceptional points in the many-level system

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The wavefunction $\Psi^{\text{E}}_{c~\mathrm{int}}$ inside the system

The Ψ^E_{c int} can be represented in the set of the biorthogonal wavefunctions {Φ_λ} (eigenfunctions of the non-Hermitian Hamiltonian H_{eff}):

$$\begin{split} |\Psi^{\mathsf{E},\mathsf{R}}_{\mathsf{c} \ \mathrm{int}}\rangle &=& \sum_{\lambda} \mathsf{c}^{\mathsf{c}}_{\lambda\mathsf{E}} |\Phi^{\mathsf{R}}_{\lambda}\rangle = \sum_{\lambda} \mathsf{c}^{\mathsf{c}}_{\lambda\mathsf{E}} |\Phi_{\lambda}\rangle \\ \langle\Psi^{\mathsf{E},\mathsf{L}}_{\mathsf{c} \ \mathrm{int}}| &=& \sum_{\lambda} \mathsf{c}^{\mathsf{c}}_{\lambda\mathsf{E}} \langle\Phi^{\mathsf{L}}_{\lambda}| = \sum_{\lambda} \mathsf{c}^{\mathsf{c}}_{\lambda\mathsf{E}} \langle\Phi^{*}_{\lambda}| \end{split}$$

Normalization

$$\langle \Psi^{\text{E},\text{L}}_{\text{c int}} | \Psi^{\text{E},\text{R}}_{\text{c int}} \rangle = \sum_{\lambda} \ (c^{\text{c}}_{\lambda\text{E}})^2 = \mathbb{C} \ \rightarrow \ 1$$

corresponds to a rotation with

$$\operatorname{Re}(c_{\lambda E}^{c}) \operatorname{Im}(c_{\lambda E}^{c}) = 0$$

Biorthogonality

$$\langle \Psi^{\mathsf{E},\mathsf{R}}_{\mathsf{c}~\mathrm{int}} | \Psi^{\mathsf{E},\mathsf{R}}_{\mathsf{c}~\mathrm{int}} \rangle = \sum_{\lambda} \ (\mathsf{c}^{\mathsf{c}}_{\lambda\mathsf{E}})^2 \langle \Phi_{\lambda} | \Phi_{\lambda} \rangle \equiv \sum_{\lambda} \ (\mathsf{c}^{\mathsf{c}}_{\lambda\mathsf{E}})^2 \ \tilde{\mathsf{A}}_{\lambda}$$

 \blacktriangleright Phases of the $\Psi^{\text{E}}_{c \ int}$ are not rigid

$$\tilde{\rho} = \left| \frac{\langle \Psi_{c \text{ int}}^{\mathsf{E},\mathsf{L}} | \Psi_{c \text{ int}}^{\mathsf{E},\mathsf{R}} \rangle}{\langle \Psi_{c \text{ int}}^{\mathsf{E},\mathsf{R}} | \Psi_{c \text{ int}}^{\mathsf{E},\mathsf{R}} \rangle} \right|; \quad 1 \geq \tilde{\rho} \geq 0$$

(summation over all overlapping resonance states)

Approaching a region with several exceptional points

 $ilde{
ho} < 1$

Definition of the phase rigidity of the $\Psi_{c \text{ int}}^{E}$

• in analogy to the definition of the r_{λ} for two levels

$$\rho = \left| \frac{\int d\mathbf{r} \ \Psi_{c \ int}^{E}(\mathbf{r})^{2}}{\int d\mathbf{r} \ |\Psi_{c \ int}^{E}(\mathbf{r})|^{2}} \right|$$
$$= \left| \frac{\int d\mathbf{r} ([\operatorname{Re}\Psi_{c \ int}^{E}]^{2} - [\operatorname{Im}\Psi_{c \ int}^{E}]^{2})}{\int d\mathbf{r} ([\operatorname{Re}\Psi_{c \ int}^{E}]^{2} + [\operatorname{Im}\Psi_{c \ int}^{E}]^{2})} \right|$$

due to the existence of exceptional points

 $1 \ge
ho \ge 0$

 the phase rigidity ρ describes the internal impurity of the open quantum system which is caused by exceptional points

Phase rigidity ρ and alignment

• due to $1 \ge \rho \ge 0$ a few states of the system can align, step by step, with the scattering states of the environment

full alignment is reached for $\rho = 0$

i.e when the spectroscopic overlap between $\Psi^{\text{E}}_{c \ int}$ and ξ^{E}_{c} is maximal

- alignment of a few states Ψ^E_{c int} with the channel wavefunctions ξ^E_c (c = 1, ..., C) occurs by trapping other resonance states
 i.e. by (partial or complete) decoupling them from the environment
- formation of short-lived aligned states is possible due to width bifurcation, i.e. ultimately due to the existence of neighboring exceptional points

System and environment

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Alignment of wavefunctions

 The wavefunctions Ψ^E_{c int} are the solutions of the scattering problem inside the system (localized)

The scattering wavefunctions ξ_c^E (c = 1, ..., C) are the (extended) wavefunctions of the environment

- Alignment of some wavefunctions Ψ^E_{c int}
 with the channel wavefunctions ξ^E_c (c = 1, ..., C)
 - \longrightarrow formation of short-lived states AND
 - \rightarrow resonance trapping:

all but the aligned resonance states are

hierarchically decoupled (trapped)

from the continuum (environment)

The resonance trapping phenomenon is proven experimentally: Persson et al., PRL 85, 2478 (2000)

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Alignment of the wavefunctions

of the system to the scattering wavefunctions of the environment (by trapping of other resonance states)

is equivalent to a dynamical phase transition

in a realistic many-level quantum system under the influence of the environment

Phase transitions under the influence of the environment

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Phase transition

• Toy model $H_{eff} = H_0 + i\alpha VV^+$:

one-channel case

all exceptional points accumulate in one point Heiss, Müller, Rotter, PRE 58, 2894 (1998)

→ a second-order phase transition may occur order parameter: Γ_0/M (first derivative jumps at α_{cr}) fluctuations at α_{cr} : $\Gamma_0 = |\gamma_{01}|^2/A_0$ $(|\gamma_{01}|^2 \rightarrow \infty \text{ in approaching } \alpha_{cr})$ Jung, Müller, Rotter, PRE 60, 114 (1999)

• Realistic system with $H_{eff} = H_B + V_{BC}G_C^{(+)}V_{CB}$:

hierarchical trapping of resonance states in the regime of overlapping resonances

many exceptional points, however no accumulation point

see e.g. Iskra, Rotter, Dittes, PRC 47, 1086 (1993)

 \rightarrow dynamical phase transition

regime at low level density (small coupling via the continuum) differs from regime at high level density (narrow resonances superimposed on broad resonances)

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Can the transition regime between the two phases be observed experimentally?

phase rigidity is given by

$$0 \leq \rho \equiv \left| \frac{\int d\mathbf{r} \, \Psi_{c \text{ int}}^{\mathsf{E}}(\mathbf{r})^{2}}{\int d\mathbf{r} \, |\Psi_{c \text{ int}}^{\mathsf{E}}(\mathbf{r})|^{2}} \right| \leq 1$$

- \blacktriangleright in the transition regime, many exceptional points $\rightarrow~\rho<1\rightarrow$ partial alignment of some states
- when all exceptional points accumulate in one point, the transition regime shrinks to one point in the parameter space
- when the exceptional points are distributed over a finite parameter range, the transition region is observable: e.g. whispering gallery modes cause enhanced transmission through the system see e.g. Bulgakov, Rotter, Sadreev, PRE 74, 056204 (2006) and PRB 76, 214302 (2007)

- Alignment of resonance states (and resonance trapping)
 - is a collective phenomenon to which all resonance states in a large energy region contribute (i.e. it is a global phenomenon, not a local one)
 - is environmentally induced
 - causes an enhancement of observable values in the parameter range where $\rho < 1$
 - causes a reduction of the number of long-lived resonance states (characteristic of exceptional points)

Summary

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Dynamical phase transitions

- are caused by width bifurcation under the influence of the environment
- are possible due to non-rigid phases of the functions $\Psi^{\text{E}}_{c~\mathrm{int}}$

$\mathbf{1} > \rho > \mathbf{0}$

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in the regime of many overlapping resonances

- are (mathematically) directly related to the existence of exceptional points in a finite range of the control parameter
- consist in spectroscopic redistribution and \mathcal{PT} symmetry breaking, respectively, starting at a few neighbored exceptional points

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 are observed experimentally in different quantum systems and in systems equivalent to them Appendix related to the poster

Dynamical phase transitions in quantum mechanics

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Niels Bohr 1936

In the atom and in the nucleus we have indeed to do with two extreme cases of mechanical many-body problems for which a procedure of approximation resting on a combination of one-body problems, so effective in the former case. loses any validity in the latter where we, from the very beginning, have to do with essential collective aspects of the interplay between the constituent particles

At that time were known:

narrow compound nucleus states at high excitation energy and shell model states in atoms at low excitation energy

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The Nobel Prize in Physics 1963

Maria Goeppert-Mayer and J. Hans D. Jensen for their discoveries concerning nuclear shell structure

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The nuclear shell model

- shell model describes individual nuclear states at low level density
- shell closures in nuclei differ from those in atoms since the forces differ from one another
- the residual forces of the shell model are of two-body type
- finite lifetimes of nuclear states are calculated perturbatively, i.e.
 - the shell model Hamiltonian is assumed to be Hermitian
 - particle decay is described by tunneling of the particle through a barrier
 - feedback from the continuum onto the shell model states is not taken into account

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Description of the compound nucleus states

- the question of the Hermiticity of the Hamiltonian is avoided by considering a certain distribution of the states
- the narrow compound nucleus states at high level density are described by means of random ensembles, mostly by the GOE
- the semicircle of the GOE describes the states in the middle of the spectrum (far from thresholds)
- GOE is related to quantum chaos, Bohigas et al, PRL 48, 1086 (1982)
- ▶ the time scales of compound nuclear resonances and direct reaction part are well separated
 → Feshbach unified theory of nuclear reactions

Can the compound nucleus be described by the shell model?

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Hamilton operator of the system

assumed to be hermitian in the shell model

finite lifetimes of the states (decay widths) calculated perturbatively

no feedback from the continuum onto the shell model states

hermiticity (or non-hermiticity) of the Hamiltonian not considered in the random ensemble

consideration of average lifetimes instead of lifetimes of individual states

no consideration of threshold effects and no feedback from the continuum

Compound nucleus states and shell model

- ► GOE is NOT related to a two-body random ensemble: it contains many-body forces → collective features
 - \rightarrow states of the GOE differ from shell model states (with two-body forces)
- GOE does not contain any threshold effects
- GOE contains level repulsion, however single resonances do not decay according to an exponential law

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Harney et al, Ann.Phys.(NY) 220, 159 (1992)

Question

Is the statement by Niels Bohr really wrong?

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Main problems of nuclear physics studies

- no direct control parameter due to the strong nuclear forces control of average values as function of energy is possible
- different interpretations of experimental data are possible by means of different sets of fitting parameters

e.g. problems in the identification of doorway states, Jeukenne and Mahaux, Nucl.Phys.A 136, 49 (1969)

nuclear forces are residual forces that are not derived from first principles but are described by means of parameters fitted to many nuclei

Problems of the random ensembles with experimental results

 mean compound nucleus lifetime does, experimentally, not increase with increasing excitation energy (regime of overlapping resonances)

Experiment: Kanter et al, Nucl.Phys.A 299, 230 (1978), Theoretical explanation by means of a dynamical phase transition: Rotter, Rep.Prog.Phys. 54, 635 (1991); J.Phys.A 42, 153001 (2009)

reduced neutron widths do, experimentally, not follow the Porter-Thomas distribution Koehler et al, Phys.Rev.Lett. 105, 072502 (2010)

Nuclear physics: State of the art 2011

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Dynamical phase transitions prove Bohr's statement

a dynamical phase transition takes place in the regime of overlapping resonances

The properties of low-lying nuclear states (described by the shell model) and those of highly excited nuclear states (described by random ensembles) differ fundamentally from one another

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- Uwe Günther, Dresden Boris Samsonov, Tomsk

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J. Phys. A 42 (2009) 153001 (Topical Review)

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