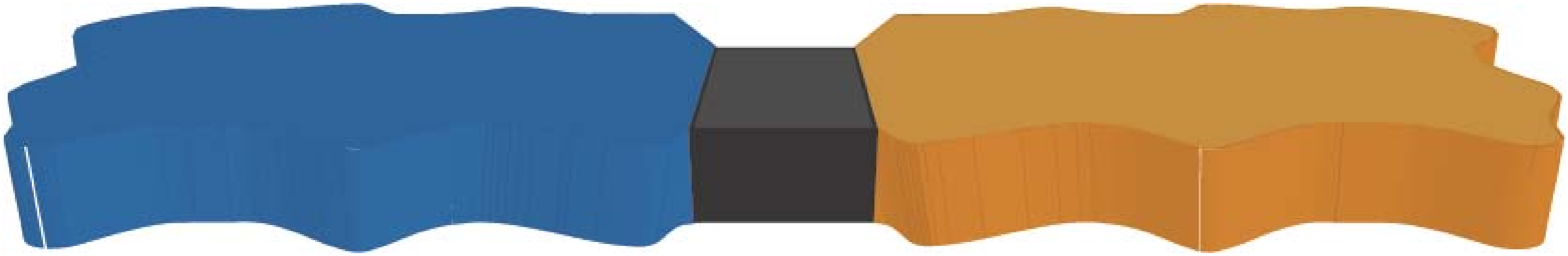
Quantum optics & RMT of $\mathcal{PT}$- symmetric resonators



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Dresden, 17 June 2011

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Motivation: Optical realisations of PT symmetry


LETTERS
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Observation of parity–time symmetry in optics

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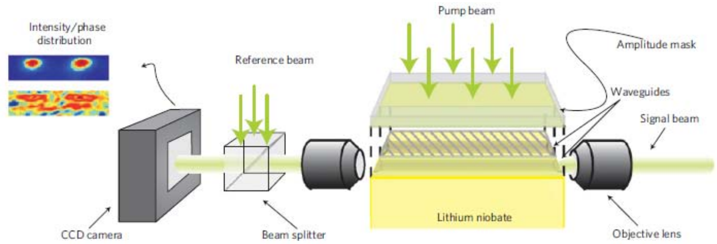


0. How do states form in complex (multimode) systems?
(Quantisation condition)

1. Can these systems be lasers?
(Quantum noise)

2. What would then be the laser threshold?
(How many eigenvalues are real or complex?
Count on average: Random matrix theory)

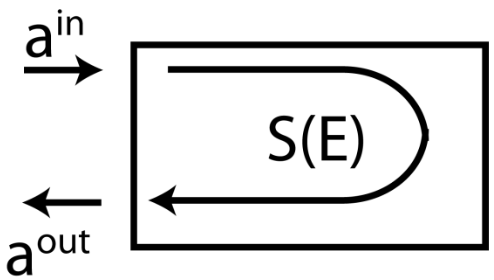
Tool: scattering theory

0. quantization in scattering theory

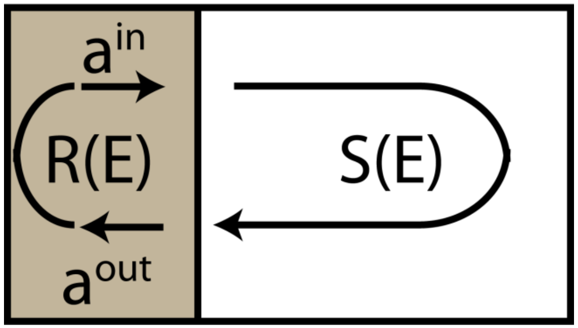
$$\det(H - E) = 0$$

Scattering matrix:

$$a^{\text{out}} = S(E)\,a^{\text{in}}$$

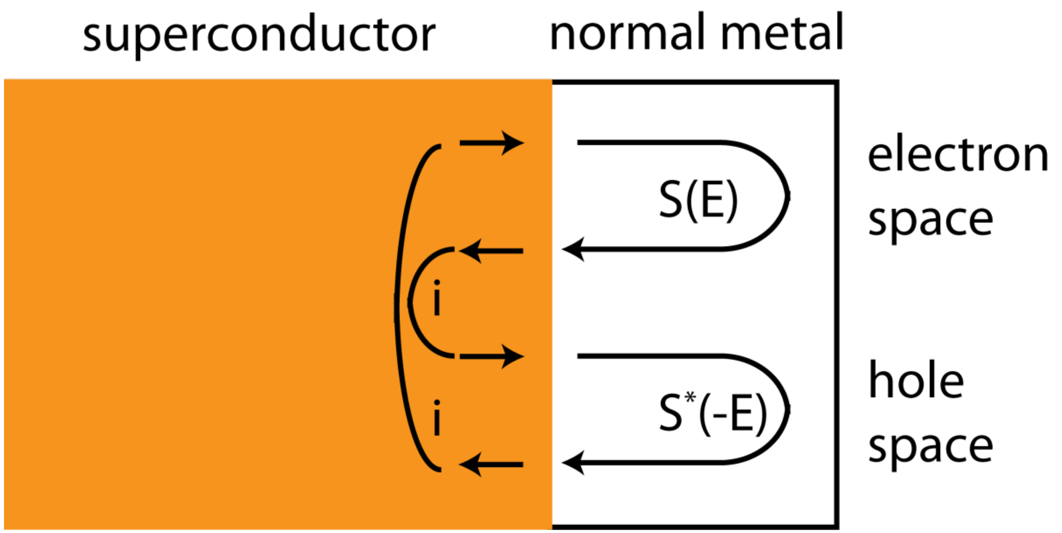


Internal scattering (transfer) *Smilansky/Bogomolny ~1990*



$$\det[S(E)R(E) - 1] = 0$$

Example: Andreev reflection



BdG Hamiltonian

$$\mathcal{H} = \begin{pmatrix} H & \Delta \\ \Delta & -H^* \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} H & \Delta \\ \Delta & -H^T \end{pmatrix}$$

$$\mathrm{Det}[S(E)S^*(-E) + 1] = 0$$

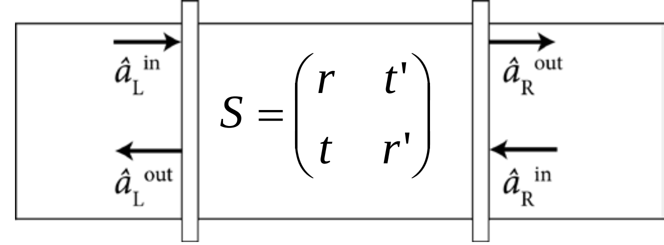
Beenakker 1993

Symmetries: TRS & particle-hole

$H^T = H^*$ (hermitian case…)

$H(-B) = H^T(B)$ (Onsager relation:) TRS for $B=0$ ($S=S^T$)

nonherm$\mathcal{PT}$-symmetry



$$a^{out} = S a^{in}$$

Identify symmetries: start from Hamiltonian

$$H = H_0 + iA(B) - i\Gamma(\mu) \quad \text{where } H_0 = H_0^* = H_0^T \text{ and}$$

$$A(B) = A^*(B) = -A^T(B) = -A(-B)$$

$$\Gamma(\mu) = \Gamma^*(\mu) = \Gamma^T(\mu) = -\Gamma(-\mu)$$

B (magn field): breaks $\mathcal{T}$

μ: loss/gain

Time reversal: $\mathcal{T}H = H^*, \quad \mathcal{T}\psi = \psi^* \quad \Rightarrow \quad \mathcal{T}S = [S^*]^{-1}$

Parity: $\mathcal{P}H(x) = H(-x), \mathcal{P}\psi(x) = \psi(-x) \Rightarrow \quad \mathcal{P}S = \sigma_x S \sigma_x$

$\mathcal{PT}$-symmetry $\mathcal{PT}S = \sigma_x [S^*]^{-1} \sigma_x$

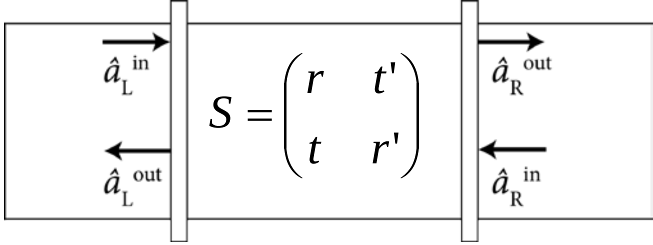
$$\mathcal{PT}S(E, B, \mu) = \sigma_x [S^*(E, B, \mu)]^{-1} \sigma_x \quad = \sigma_x S(E^*, -B, -\mu) \sigma_x$$

$S(E, -B, \mu) = S^T(E, B, \mu)$ *Onsager reciprocity*

$S(E, B, -\mu) = [S^+(E^*, B, \mu)]^{-1}$ *microreversibility*

HS, PRL 104, 233601 (2010)

nonherm$\mathcal{PT}$-symmetry



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Identify symmetries: start from Hamiltonian

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$\mathcal{PT}$-symmetry $\boxed{\mathcal{PT}S = \sigma_x [S^*]^{-1} \sigma_x}$

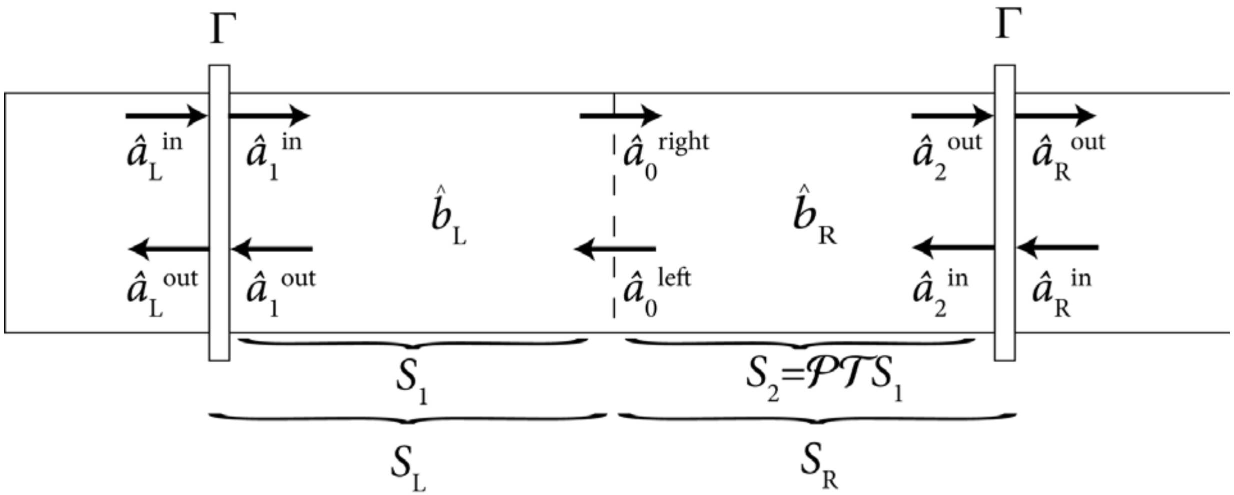
$$\mathcal{PT}S(E, B, \mu) = \sigma_x [S^*(E, B, \mu)]^{-1} \sigma_x \quad = \sigma_x S(E^*, -B, -\mu) \sigma_x$$

$S(E, -B, \mu) = S^T(E, B, \mu)$ *Onsager reciprocity*

$S(E, B, -\mu) = [S^+(E^*, B, \mu)]^{-1}$ *microreversibility*

HS, PRL 104, 233601 (2010)

build a $\mathcal{PT}$-symmetric system



$$S_\Gamma = -\begin{pmatrix} \sqrt{1-\Gamma} & i\sqrt{\Gamma} \\ i\sqrt{\Gamma} & \sqrt{1-\Gamma} \end{pmatrix}$$

Wave matching: $$\begin{pmatrix} a_L^{out} \\ a_0^{right} \end{pmatrix} = S_L \begin{pmatrix} a_L^{in} \\ a_0^{left} \end{pmatrix}, \qquad \begin{pmatrix} a_0^{left} \\ a_R^{out} \end{pmatrix} = S_R \begin{pmatrix} a_0^{right} \\ a_R^{in} \end{pmatrix}$$

Closed system (Γ→0):
codim-1 quantization condition

$$\det \mathrm{Im}(r'_L) = 0$$

HS, PRL 104, 233601 (2010)

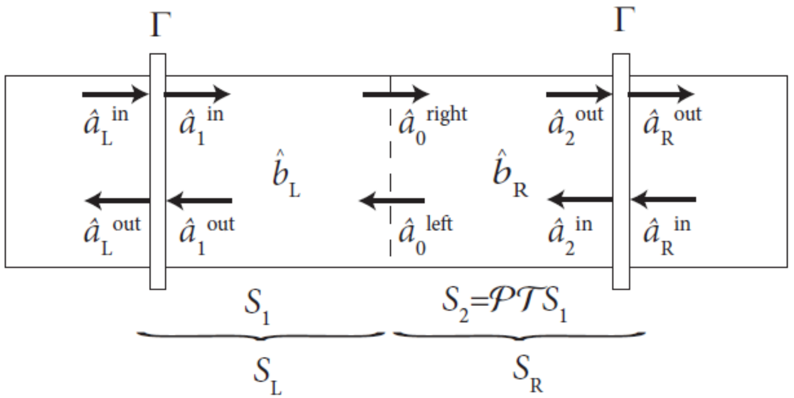
Example: 1d resonator

M Znojil 2001

Cannata et al 2007

$$S_L = -\begin{pmatrix} \frac{r_1+\sqrt{1-\Gamma}}{1+r_1\sqrt{1-\Gamma}} & \frac{it_1'\sqrt{\Gamma}}{1+r_1\sqrt{1-\Gamma}} \\ \frac{it_1\sqrt{\Gamma}}{1+r_1\sqrt{1-\Gamma}} & \frac{t_1t_1'\sqrt{1-\Gamma}}{1+r_1\sqrt{1-\Gamma}} - r_1' \end{pmatrix}$$

$$\det \mathrm{Im}(r_L') = 0$$



Ballistic, $\Gamma \to 0$ $\left[S_1 = \begin{pmatrix} 0 & t_1 \\ t_1 & 0 \end{pmatrix}\right]$: $\mathrm{Im}\, t_1^2 = 0$

= poles of full S matrix

$$S = \begin{pmatrix} \frac{\sqrt{1-\Gamma}(t_1^{*2}-t_1^2)}{t_1^2(1-\Gamma)-t_1^{*2}} & \frac{|t_1|^2\Gamma}{t_1^2(1-\Gamma)-t_1^{*2}} \\ \frac{|t_1|^2\Gamma}{t_1^2(1-\Gamma)-t_1^{*2}} & \frac{\sqrt{1-\Gamma}(t_1^{*2}-t_1^2)}{t_1^2(1-\Gamma)-t_1^{*2}} \end{pmatrix}$$

HS, PRL 104, 233601 (2010)

I. Quantum noise & lasing

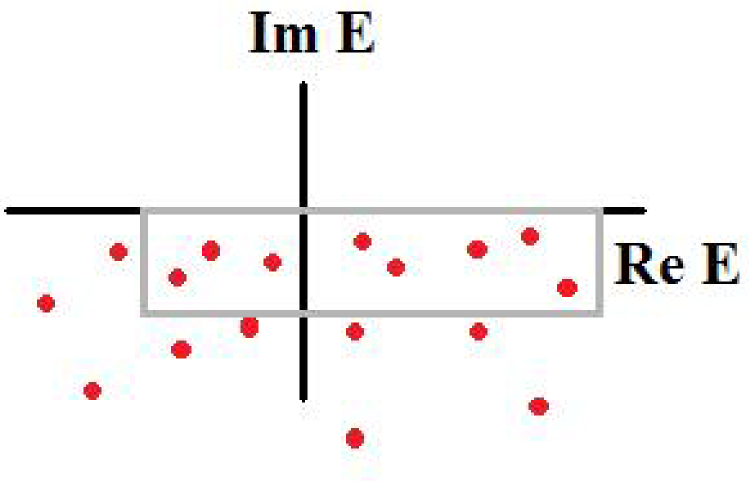
***Decay*: quasi-bound resonant states**

$$a^{\text{out}} = S \times 0$$

Poles at complex *E*:

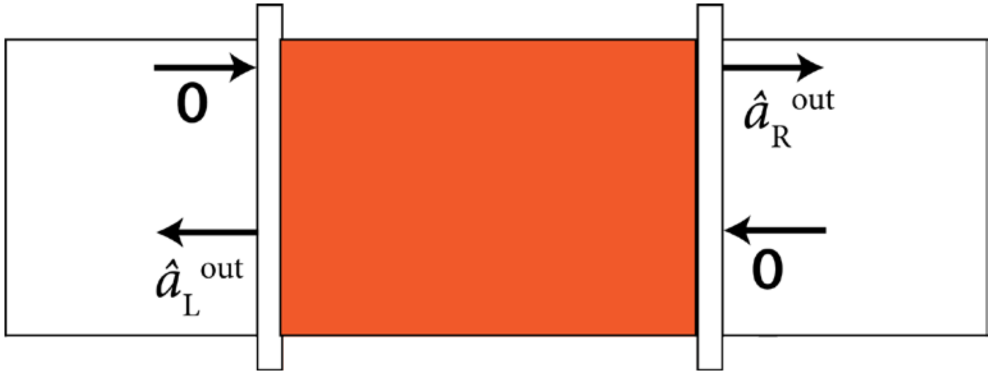
Im *E* = −Γ/2: decay rate

$$I(t) \propto \exp(-\Gamma t)$$



I. Quantum noise & lasing

***Laser*: counteracted by gain**

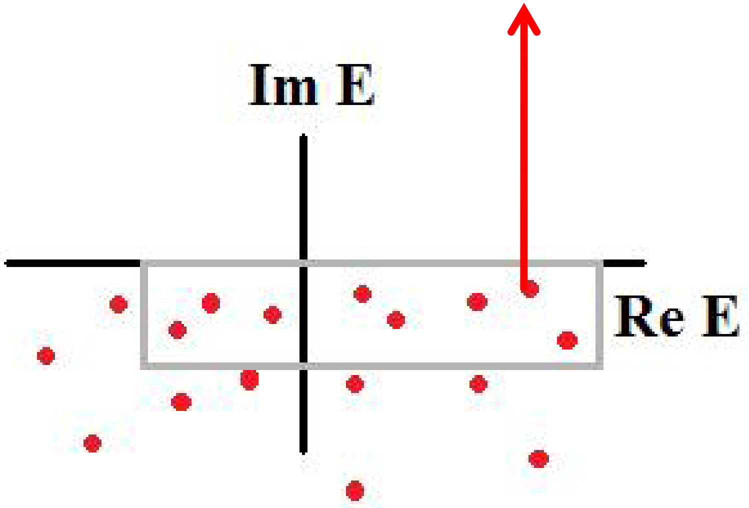


$$I(t) \propto \exp(-\Gamma t)$$

Γ<0: unstable,
saturation,
Laser

- ***k*= ω *n*/*c***
- **Im *n*>0: Im *k* >0: losses**
- **Im *n*<0: Im *k* <0: gain**

$$I(x) \propto \exp(-2\,\mathrm{Im}\,kx)$$



I. Quantum noise & lasing

***Laser*: counteracted by gain**

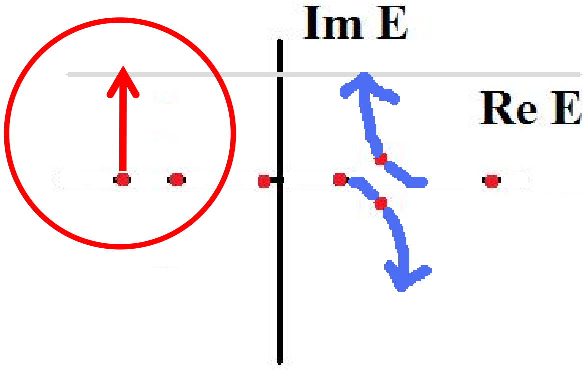
$$I(t) \propto \exp(-\Gamma t)$$

Γ<0: unstable, *saturation, Laser*

- $k = \omega\, n/c$
- Im n>0: Im k >0: losses
- Im n<0: Im k <0: gain

$$I(x) \propto \exp(-2\,\mathrm{Im}\,kx)$$

+ PT symmetry:

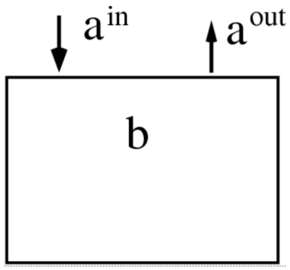


Laser threshold?

Quantum noise in scattering theory

Scattering matrix:

$$a^{\text{out}} = S a^{\text{in}}$$



Q-optics: 2nd quantization $a^{\text{out}} = S a^{\text{in}} + Q_1 b + Q_2 b^\dagger$

Commutation relations: $SS^\dagger + Q_1 Q_1^\dagger - Q_2 Q_2^\dagger = 1$

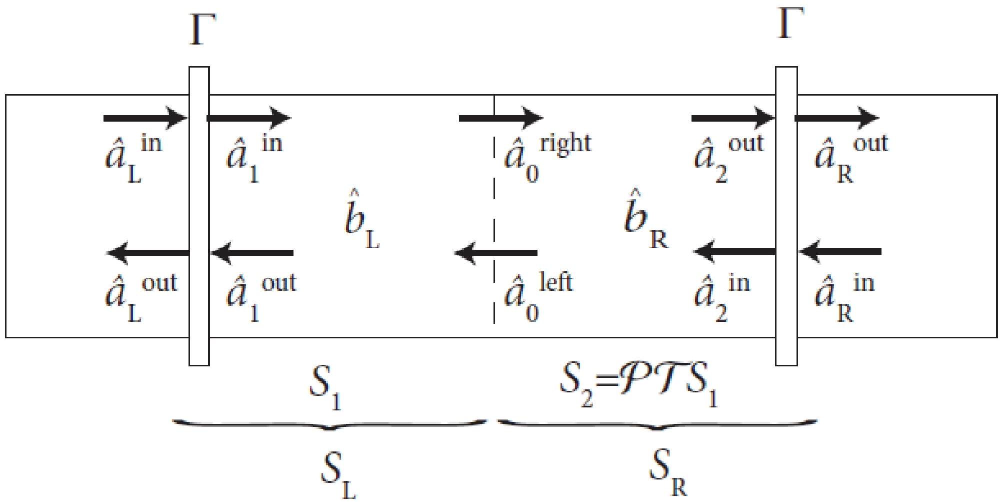
(Fluctuation-dissipation theorem)

Population inversion, no incoming radiation: $\langle b^\dagger b \rangle = 0$

$$I(\omega) = \frac{1}{2\pi} \sum_{m=1}^{N} \langle a_m^{\text{out}\dagger}(\omega) a_m^{\text{out}}(\omega) \rangle = (1/2\pi) \text{tr}(S^\dagger S - \mathbb{1})$$

HS et al, Physica A (2000)

Apply to PT: separate ampl. & abs. regions



Amplitudes □operators, incl. noise ops b, ***operator matching***

ballistic resonator, $\Gamma \to 0$ $I_L(E) = I_R(E) = \dfrac{\Gamma\left(|t_0|^{-2} - |t_0|^2\right)}{\pi|2i\tau(E-E_0)+\Gamma|^2}$

Line width $\Delta E = \Gamma/\tau$ with time of flight $\tau \approx 2L/c$

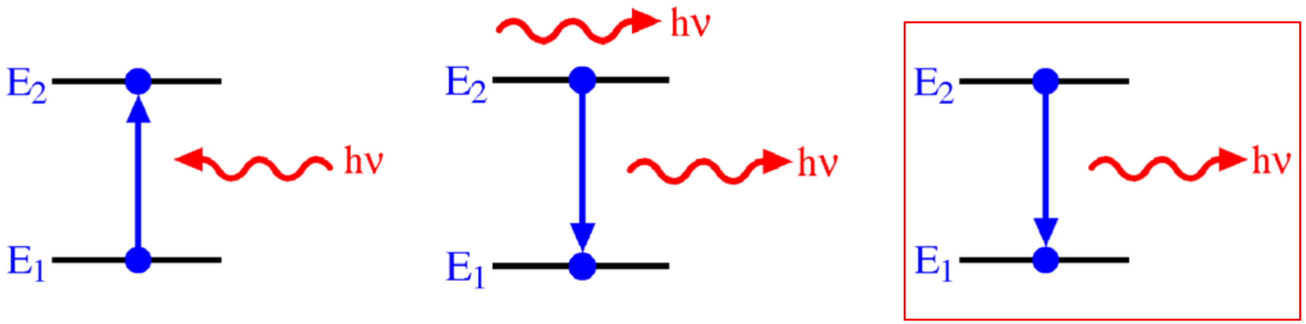
Total intensity finite! $I_{\text{tot}} = \int I(E)\,dE = \dfrac{|t_0|^{-2} - |t_0|^2}{\tau}$

□Internal photon density diverges, saturation □ (lasing)

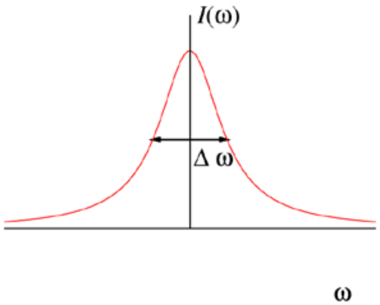
HS, PRL 104, 233601 (2010)

Physical origin: *quantum noise breaks microreversibility*

- Photons: discrete emission and absorption events



- – stimulated emission: coherent, noiseless
- – spontaneous emission: incoherent, noisy
- ➔ line broadening $\Delta\omega$, instability (lasing)



Formal origin: *Commutators of (external) bosonic (scattering field) operators are fixed*

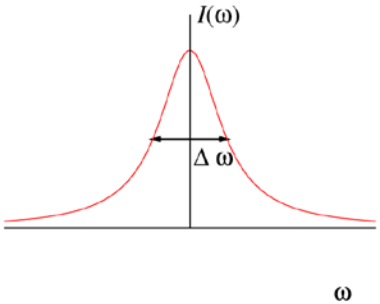
HS, PRL 104, 233601 (2010)

Relation to mode-orthogonality and complexity measures

Ordinary (homogeneous) lasers: Petermann factor
[HS PRA 2009]

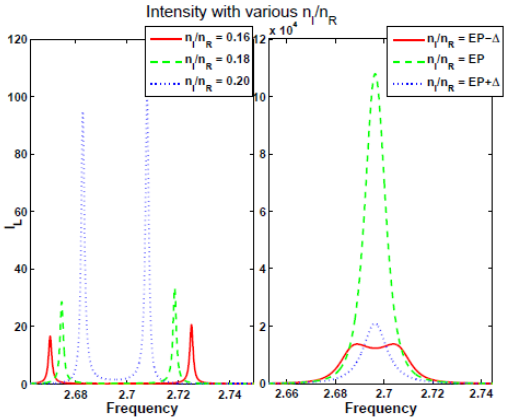
$$\Delta\omega = K\Gamma^2/(2I_{\text{total}})$$

$$K = \left| \frac{\int |\psi_0|^2 \mathrm{Im}\, n_0^2 d\mathbf{r}}{\int \psi_0^2 \, \mathrm{Im}\, n_0^2 d\mathbf{r}} \right|^2$$



PT @ isolated resonances:

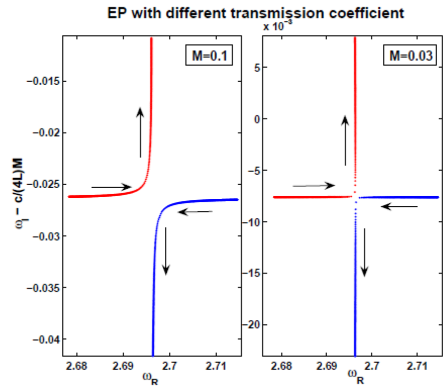
$$I_{\text{total}} \approx \frac{n_I}{n_R}\omega_R \frac{K_{\text{both}}}{K_{\text{gain}}^{1/2}}$$



PT @ exceptional points:

K diverges, (Lorentzian)^2

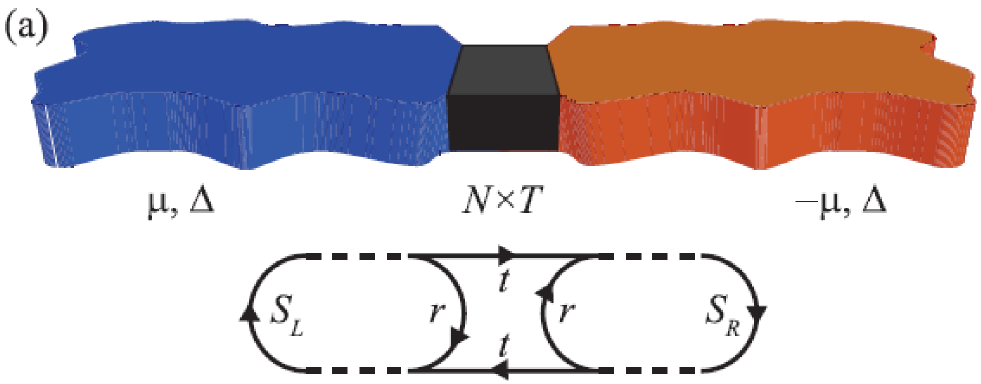
$$I_{\text{total}} \simeq \frac{2}{\Gamma^2}\frac{1}{n_R n_I}\omega_R \frac{K_{\text{gain}}^{3/2}}{K_{\text{both}}}$$



G Yoo, HS Sim and HS, in prep

II. Counting complex eigenvalues

Variant: closed system, with tuneable coupling T



Quantisation:

$$\det\left[\begin{pmatrix} r & t \\ t & r \end{pmatrix}\begin{pmatrix} S_L & 0 \\ 0 & S_R \end{pmatrix} - \mathbb{1}\right] = 0$$

Random-matrix theory:

$$S_L(E;\mu) = 1 - 2iV^\dagger(E - i\mu - H + iVV^\dagger)^{-1}V$$

$$S_R(E;-\mu) = [S_L^{-1}(E^*;\mu)]^* = 1 - 2iV^\dagger(E + i\mu - H^* + iVV^\dagger)^{-1}V$$

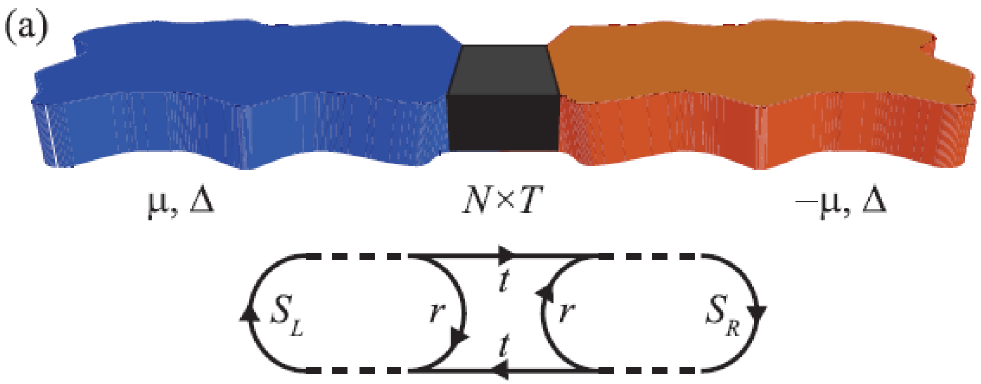
H **: *M*×*M*, 'random' (Gaussian ens),** V**: coupling (rank *N*)**

Need N<<M

HS, PRA 83, 030101(R) (2011)

II. Counting complex eigenvalues

Variant: closed system, with tuneable coupling T



Quantisation:

$$\det\left[\begin{pmatrix} r & t \\ t & r \end{pmatrix}\begin{pmatrix} S_L & 0 \\ 0 & S_R \end{pmatrix} - \mathbb{1}\right] = 0$$

Quantization condition: eigenvalues of eff Hamiltonian

$$\mathcal{H} = \begin{pmatrix} H - i\mu & \Gamma \\ \Gamma & H^* + i\mu \end{pmatrix}$$

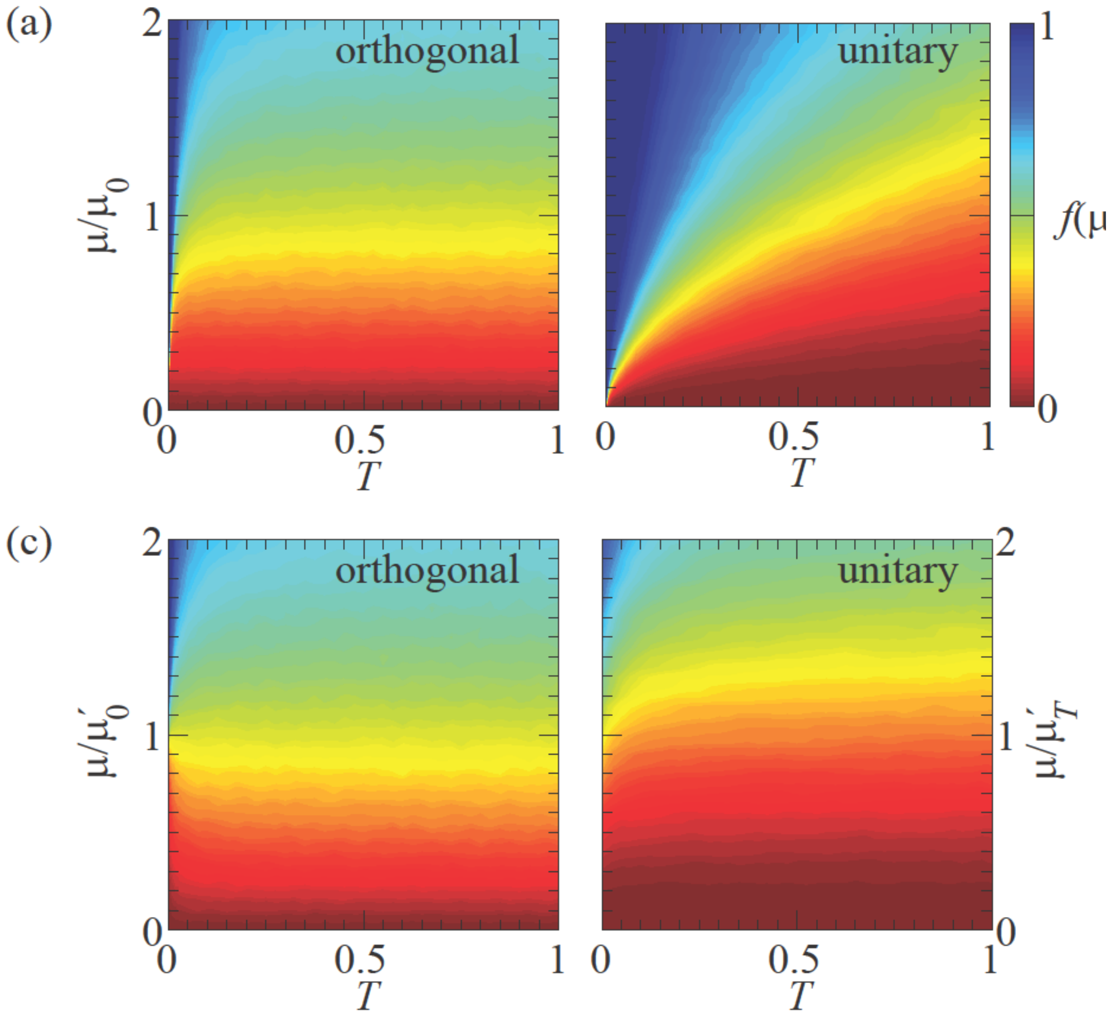
$$\Gamma = \mathrm{diag}(\gamma_m)$$

$$\gamma_m = [\sqrt{T}/(1+\sqrt{1-T})]\Delta M/\pi \equiv \gamma$$

BdG $\mathcal{H} = \begin{pmatrix} H & \Delta \\ \Delta & -H^T \end{pmatrix}$ **: Thouless energy *NT*δ**

HS, PRA 83, 030101(R) (2011)

B = 0 **B finite**



Crossover scales

$$\mu_{PT} \sim \sqrt{N}\Delta/2\pi \equiv \mu_0$$

B = 0: ~T-indep

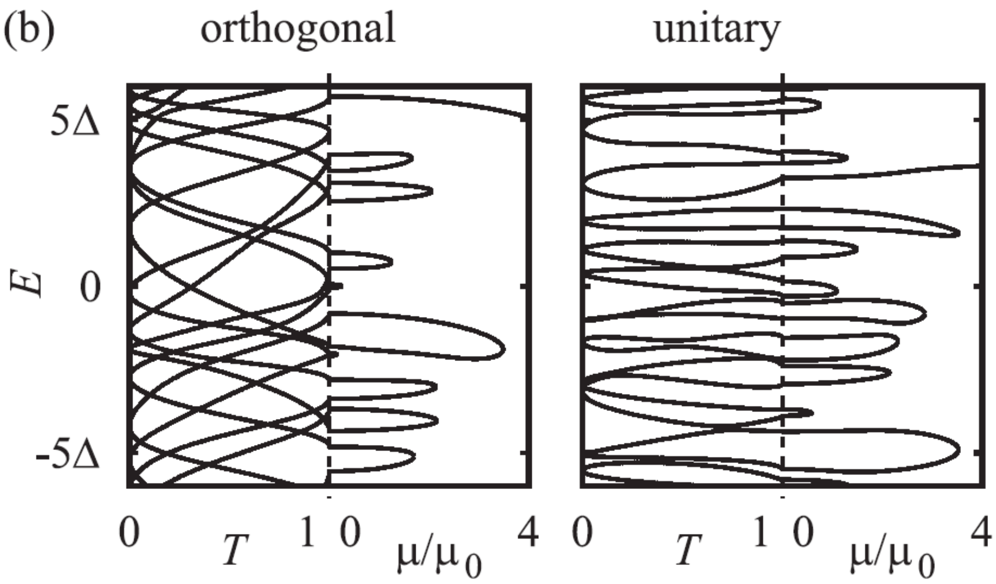
$$\mu_0' = \mu_0/\sqrt{1+1/NT}$$

B finite

$$\mu_{PT} \sim \sqrt{T}\mu_0 \equiv \mu_T'$$

HS, PRA 83, 030101(R) (2011)

$B = 0$ **B finite**



Origin: level repulsion

Parity basis

$$\mathcal{H}_\mathcal{P} = \begin{pmatrix} \mathrm{Re}H + \Gamma & i\,\mathrm{Im}H + i\mu \\ i\,\mathrm{Im}H + i\mu & \mathrm{Re}H - \Gamma \end{pmatrix}$$

$\mathcal{P}$&$\mathcal{T}$-symmetric in hermitian limit:

$$\mathcal{H}_\mathcal{P} = \begin{pmatrix} H + \Gamma & i\mu \\ i\mu & H - \Gamma \end{pmatrix}$$

□ two level sequences of $H \pm \Gamma$: $\varepsilon_k^\pm$ can cross or repel

(crossover scales then follow from almost-degenerate perturbation theory)

HS, PRA 83, 030101(R) (2011)

Summary

scattering theory, qm-optics & RMT of $\mathcal{PT}$-sym reson's

- **multidimensional quantisation condition (of codim 1)**
- **self-sustained sources of radiation;**
  closed limit: laser threshold (marginally instable)
- **direct measures of nonhermiticity/nonorthogonality**
- **spontaneous symmetry breaking:**
  enhances noise, reduces laser threshold,
  essentially coupling-independent

HS, PRL 104, 233601 (2010) [Scattering quantisation & Q noise]
HS, PRA 83, 030101(R) (2011) [RMT]
Chris Birchall and HS, in preparation (see poster) [RMT]
Gwangsu Yoo, Heung-Sun Sim and HS, in preparation [Q-noise, gen Petermann fact]