Microwave fidelity studies by varying antenna coupling

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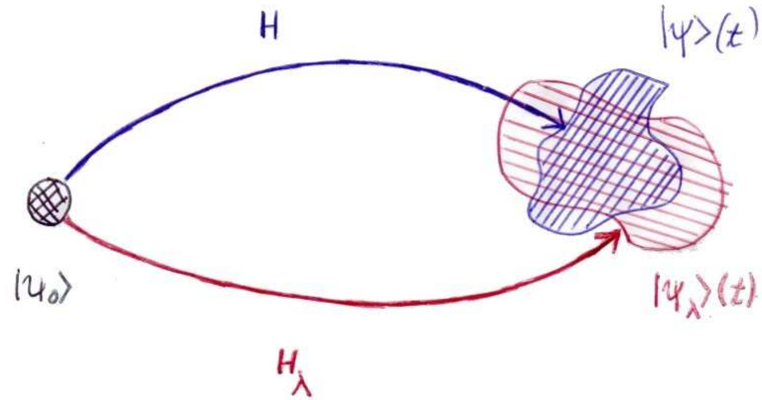
[B. Köber, U. Kuhl, H.-J. St., T. Gorin, T. Seligman, D. Savin, PRE 82, 036207 (2010)]

FOR760

Fidelity

Introduced by Peres 1985 as a measure of the stability of quantum motion:

- Calculate the propagation of an initial pulse $|\psi_0\rangle$ under the influence of two slightly different Hamiltonians $H = H_0$ and $H_\lambda = H_0 + \lambda V$



- Fidelity amplitude defined as overlap integral

$$f_\lambda(t) = \langle \psi_\lambda(t) \,|\, \psi(t) \rangle = \langle \psi_0 | e^{-\imath H_\lambda t} e^{\imath H_0 t} | \psi_0 \rangle$$

- Fidelity, as originally introduced by Peres:

$$F_\lambda(t) = |f_\lambda(t)|^2$$

Experimental realisations

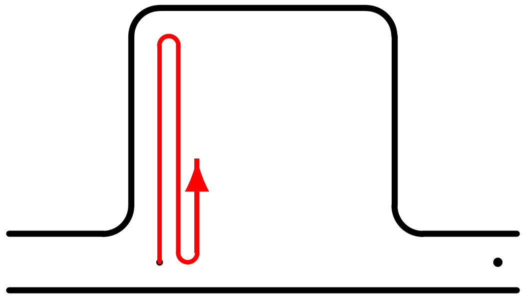
- Spin-Echo experiments in NMR (Levstein, Usaj, Pastawski 1998)

  Measures the nuclear magnetisation averaged over the whole sample forward and backward in time is measured!

  But wave functions are not accessible!

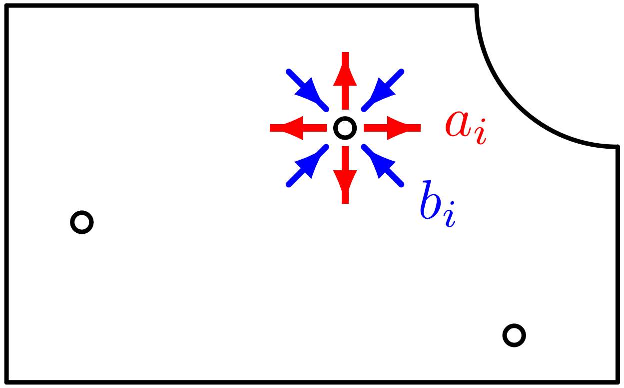
- Microwave billiards (Marburg group 2005 - 2011)

  Allows in principle to measure wave functions by scanning with a probe antenna through the system

  Drawback: the probe antenna introduces a perturbation comparable in size with the effect to be studied!

Scattering matrix



$$b_n = \sum_m S_{nm} a_m$$

Scattering matrix $S = (S_{nm})$:

- S_{nn}: reflection amplitude at antenna n
- $S_{nm}, n \neq m$: transmission amplitude between antennas n and m

Unique property of microwave experiments:

All components of S accessible!

Standard scattering experiments usually yield cross-sections only!

The scattering fidelity

Introduced by us as a substitute for the ordinary fidelity:

$S_{ab}(\omega), S_{ab}^{(\lambda)}(\omega)$: scattering matrix elements for the unperturbed and the perturbed system, resp.

$$\hat{S}_{ab}(t) = \int S_{ab}(\omega) e^{\imath\omega t} d\omega$$

Scattering fidelity defined as

$$f_{ab}^{(\lambda)}(t) = \langle \hat{S}_{ab}^{(\lambda)*}(t) \hat{S}_{ab}(t) \rangle \Big/ \sqrt{\langle \hat{S}_{ab}^{(\lambda)*}(t) \hat{S}_{ab}^{(\lambda)}(t) \rangle \langle \hat{S}_{ab}^{*}(t) \hat{S}_{ab}(t) \rangle}$$

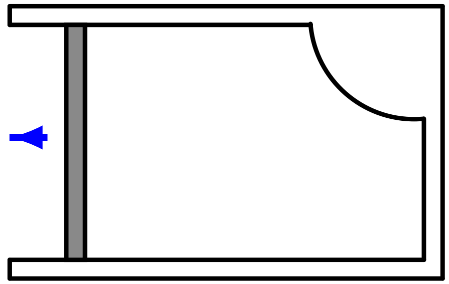
$\langle \cdots \rangle$: ensemble average

For weak antenna coupling and chaotic systems the scattering fidelity reduces to the ordinary fidelity:

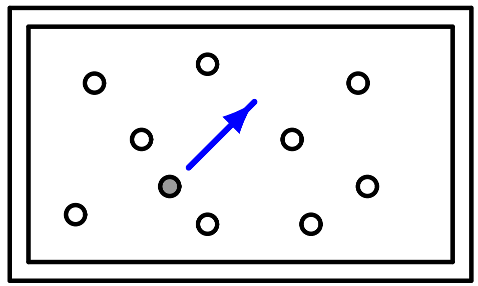
$$f_{ab}^{(\lambda)}(t) \to f_\lambda(t)$$

Possible parameter variations

- Shift of a wall (with T. Seligman, T. Gorin)
  Global variation, modeled by
  $H = H_0 + \lambda V$, with H_0, V from GOE
  Gaussian or exponential decay

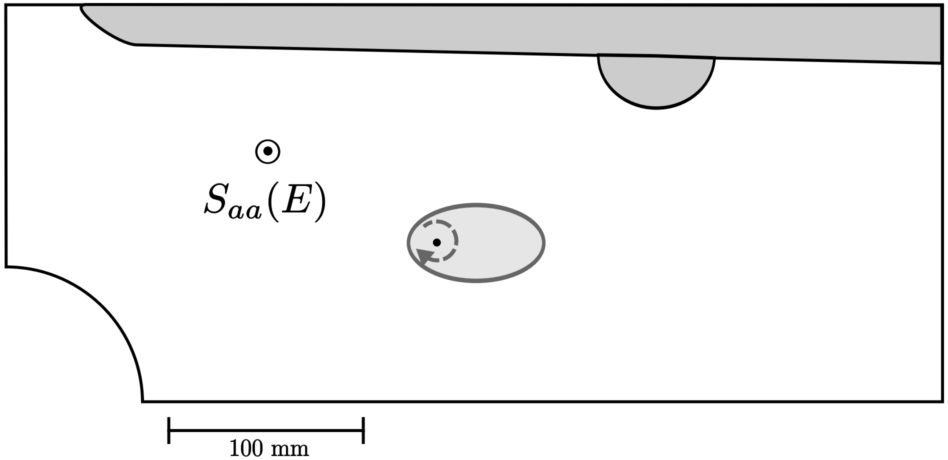


- Shift of an impurity
  Local variation, modeled by
  $H = H_0 + \lambda V V^\dagger$, with H_0, V from GOE
  Algebraic decay



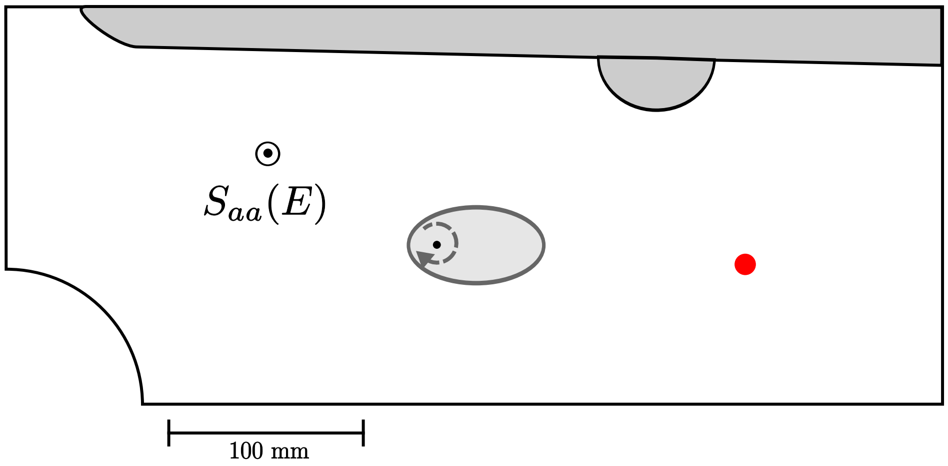
- Wall deformation (with K. Richter, A. Goussev)
  Local variation, semi-classical description
  Algebraic decay

Variation of a channel coupling



- bouncing balls suppressed by insets
- ensemble average by rotatable ellipse

Variation of a channel coupling

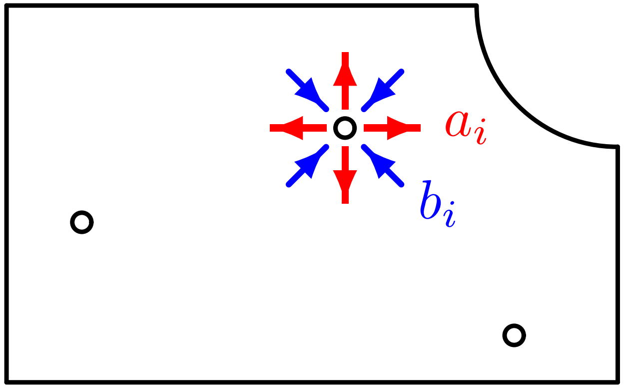


Antenna with different terminators:

- hard wall reflection
- open end reflection
- 50 Ω load

- bouncing balls suppressed by insets
- ensemble average by rotatable ellipse
- terminators from calibration kit

The effective Hamiltonian



$$b_n = \sum_m S_{nm} a_m$$

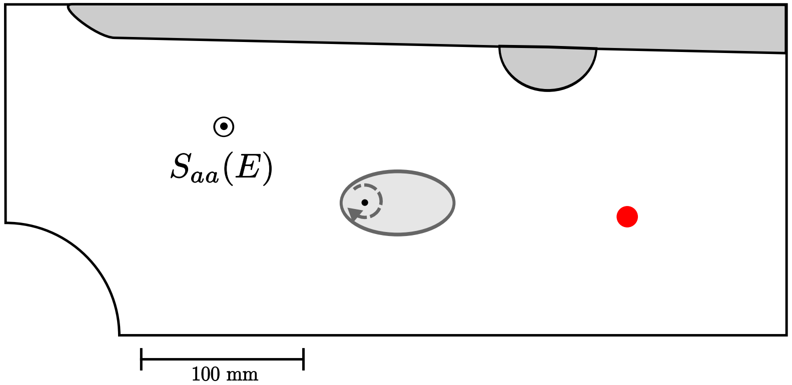
Scattering theory yields

$$S = \frac{1 - \imath W^\dagger G W}{1 + \imath W^\dagger G W}\,, \quad G = \frac{1}{E - H}$$

$W = (W_{nk})$: Matrix carrying the information on the coupling of the kth antenna to the nth eigenfunction

Point-like coupling: $W_{nk} \sim \psi_n(r_k)$

The effective Hamiltonian (*cont.*)



$$\begin{pmatrix} b_A \\ b_C \end{pmatrix} = \begin{pmatrix} S_{AA} & S_{AC} \\ S_{CA} & S_{CC} \end{pmatrix} \begin{pmatrix} a_A \\ a_C \end{pmatrix}$$

A: measuring antenna
C: variable antenna

Terminator relates a_C to b_C: $$a_C = e^{-(\alpha+\imath\varphi)} b_C$$

$\Longrightarrow S$ matrix reduces to a 1D matrix for the measuring antenna:

$$S_{AA} = \frac{1 - \imath W_A^\dagger G_{\text{eff}} W_A}{1 + \imath W_A^\dagger G_{\text{eff}} W_A}, \quad G_{\text{eff}} = \frac{1}{E - H_{\text{eff}}}$$

$$H_{\text{eff}} = H - \imath \lambda_T W_C W_C^\dagger, \quad \lambda_T = \tanh \frac{\alpha + \imath\varphi}{2}$$

The effective Hamiltonian (*cont.*)

$$H_{\text{eff}} = H - \imath\lambda_T W_C W_C^\dagger\,, \quad \lambda_T = \tanh\frac{\alpha+\imath\varphi}{2}$$

Normalized coupling matrix: $V = W_C/\sqrt{W_C^\dagger W_C}$

With $\lambda_C = W_C^\dagger W_C$ it follows

$$H_{\text{eff}} = H - \imath\lambda V V^\dagger\,, \quad \lambda = \lambda_T\lambda_C$$

λ_C can be determined experimentally via transmission coefficient

$$T_C = 1 - |\,\langle S_{CC}\rangle\,|^2 = \frac{4\lambda_C}{(1+\lambda_C)^2}$$

Special cases of H_{eff}

$$H_{\text{eff}} = H - \imath\lambda VV^\dagger\,, \quad \lambda = \lambda_T\lambda_C$$

$$\lambda_T = \tanh\frac{\alpha + \imath\varphi}{2}\,, \quad \lambda_C = W_C^\dagger W_C$$

- 50Ω load
  nothing comes back, $\alpha \to \infty$: $\lambda_T = \tanh\frac{\alpha + \imath\varphi}{2} \to 1$

$$H_{\text{eff}} = H - \imath\lambda_C VV^\dagger$$

- open end or hard wall reflection
  everything comes back, $\alpha = 0$: $\lambda_T = \tanh\frac{\imath\varphi}{2} = \imath\tan\frac{\varphi}{2}$

$$H_{\text{eff}} = H + \tan\tfrac{\varphi}{2}\lambda_C VV^\dagger$$

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  $$H_{\text{eff}} = H + \tan\frac{\varphi}{2}\lambda_C VV^{\dagger}$$

φ unknown, but $\varphi_{\text{oe}} = \varphi_{\text{hw}} + \pi$! Hence $(\lambda_T)_{\text{hw}} = 1/(\lambda_T)_{\text{oe}}$

Yields relation between the three coupling constants:

$$\boxed{\lambda_{\text{hw}}\lambda_{\text{oe}} = \lambda_{50\Omega}^2 = \lambda_C^2}$$

Theoretical description

Fidelity $\qquad f_{ab}^{(\lambda}(t) \sim \langle \hat{S}_{ab}^{(\lambda)*}(t) \hat{S}_{ab}(t) \rangle$

Parametric cross-correlation function!

Main result (D. Savin):

Parametric cross-correlation function can be expressed in terms of an autocorrelation function with an effective parameter:

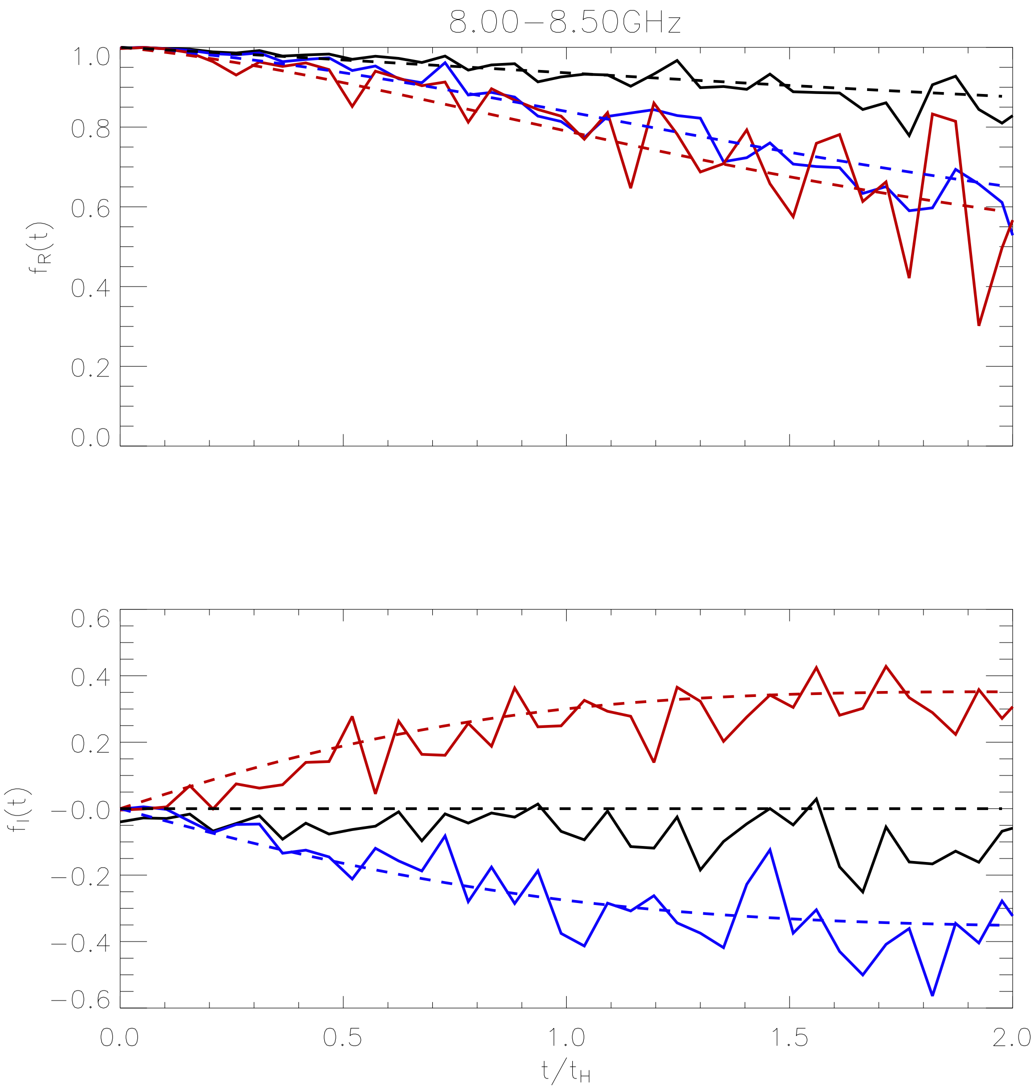
$$\langle \hat{S}_{ab}^{(\lambda_1)*}(t) \hat{S}_{ab}^{(\lambda_2)}(t) \rangle = \langle \hat{S}_{ab}^{(\lambda_{\text{eff}})*}(t) \hat{S}_{ab}^{(\lambda_{\text{eff}})*}(t) \rangle$$

λ_{eff} related to λ_1, λ_2 via

$$\frac{4\lambda_{\text{eff}}}{(1+\lambda_{\text{eff}})^2} = \frac{2(\lambda_1^* + \lambda_2)}{(1+\lambda_1^*)(1+\lambda_2)}$$

Results from VWZ paper (Verbarschoot *et al.* 1985) applicable!

Results for fidelity amplitude



unperturbed system:
no antenna

perturbed system:
antenna with terminator

From fidelity decay:

- $\lambda_{\mathrm{oe}} = 0.19\imath$
- $\lambda_{\mathrm{hw}} = -0.23\imath$
- $\lambda_{50\Omega} = 0.20$
- $\sqrt{\lambda_{\mathrm{oe}}\lambda_{\mathrm{hw}}} = 0.21$

From reflection:

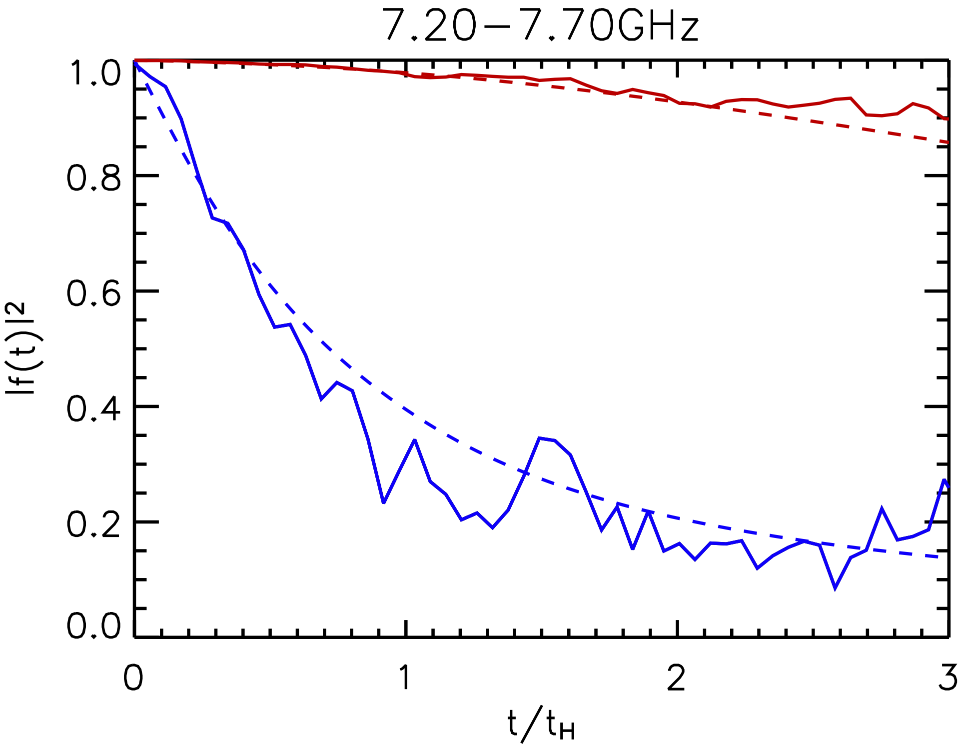
- $\lambda_C = 0.20$

Fidelity for reflecting terminator

hard wall reflection: $\lambda_T = \imath \tan \frac{\varphi}{2}$

open end reflection: $\lambda_T = -\imath \cot \frac{\varphi}{2}$

$\varphi = 2\pi l/\lambda = 2\pi \nu l/c$, l: effective length



—: hard wall

—: open end

solid: experiment

dashed: VWZ model

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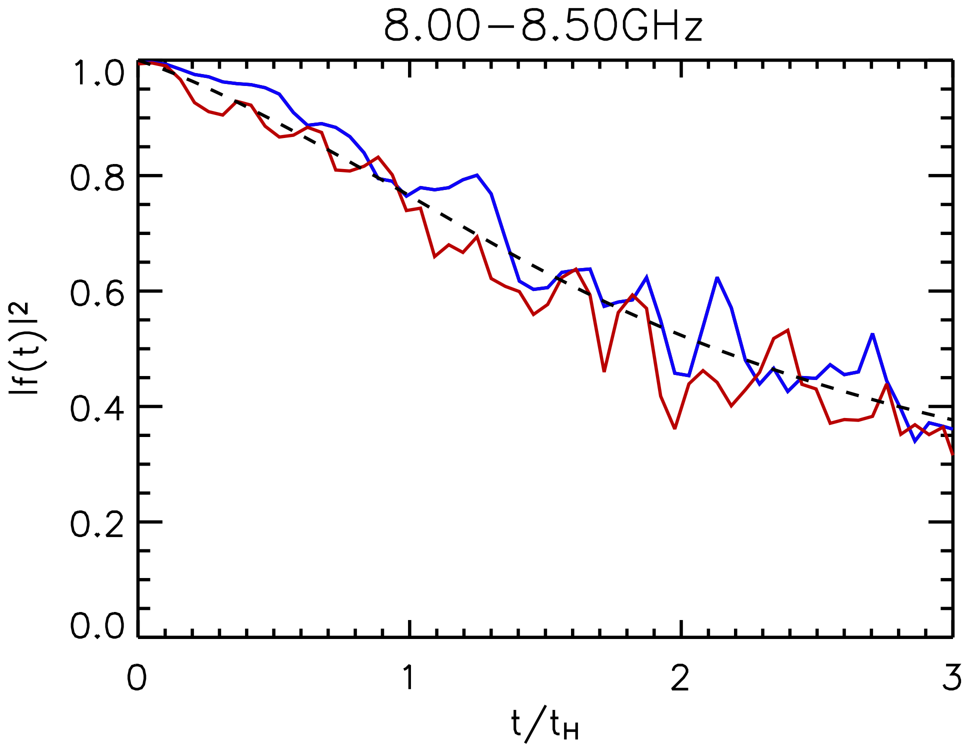

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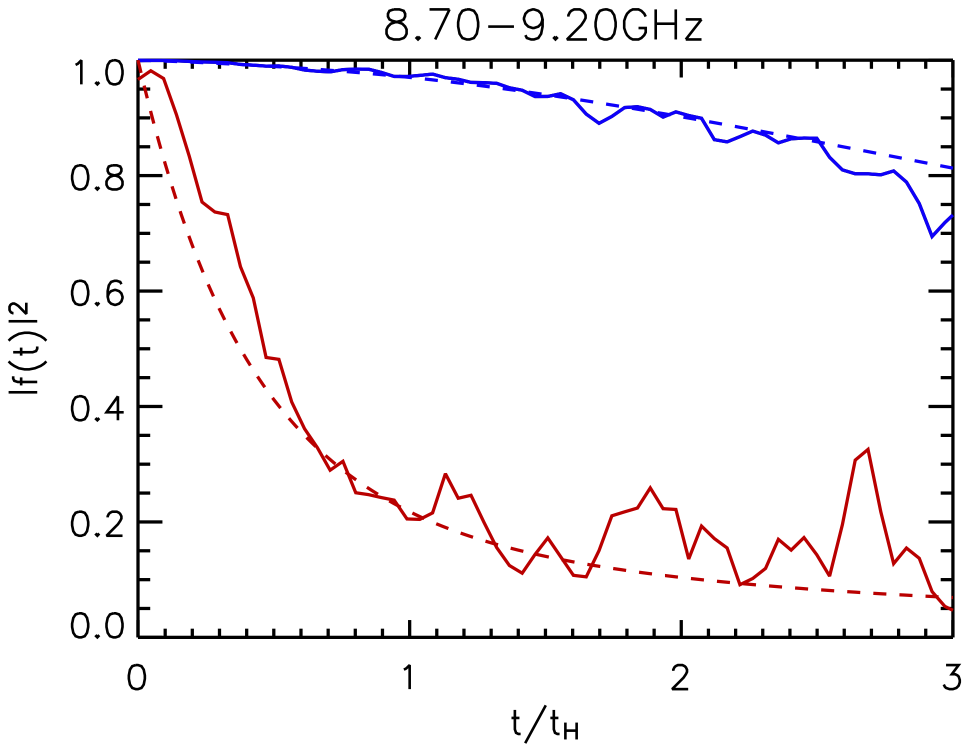

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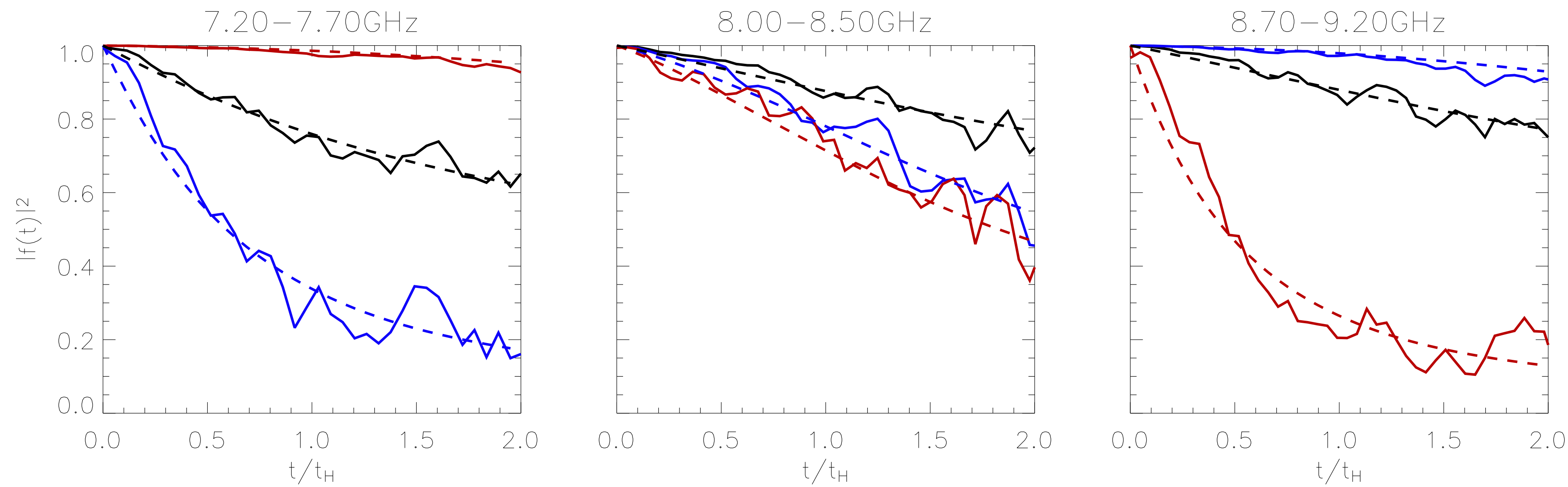
Collected results



$\lambda_{\rm oe} = 0.65\imath$ | $\lambda_{\rm oe} = 0.19\imath$ | $\lambda_{\rm oe} = 0.05\imath$

$\lambda_{\rm hw} = -0.04\imath$ | $\lambda_{\rm hw} = -0.23\imath$ | $\lambda_{\rm hw} = -0.83\imath$

$\lambda_{50\Omega} = 0.37$ | $\lambda_{50\Omega} = 0.20$ | $\lambda_{50\Omega} = 0.21$

$\sqrt{\lambda_{\rm oe}\lambda_{\rm hw}} = 0.16$ | $\sqrt{\lambda_{\rm oe}\lambda_{\rm hw}} = 0.21$ | $\sqrt{\lambda_{\rm oe}\lambda_{\rm hw}} = 0.20$

$\lambda_C = 0.19$ | $\lambda_C = 0.21$ | $\lambda_C = 0.24$

Conclusions

- Description of the billiard with variable antenna in terms of an effective Hamiltonian
- Explicit expressions of the coupling parameters in terms of terminator properties
- Description of the scattering fidelity, a parametric cross-correlation function, in terms of an autocorrelation function with an effective parameter, thus reduction to the VWZ problem
- Quantitative agreement between experiment and theory

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General problem:

- Fidelity decays for closed and open systems hardly discernible
- Reliable results only for a perfectly controllable situation
- Usually an open channel simultaneously acts as a scatterer

Thanks!

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U. Kuhl

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T. Seligman, Cuernavaca, Mexico
D. Savin, Brunel, UK

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