REALITY OF THE QUANTUM NORMAL FORM GENERATED BY PT-SYMMETRIC PERTURBATIONS

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Abstract. Consider the classical Hamiltonian $\mathcal{H}_\varepsilon(\xi, x) = \omega_1\xi_1 + \ldots + \omega_l\xi_l + \varepsilon\mathcal{V}(\xi, x), \varepsilon \in \mathbb{R}$, defined on the phase space $\mathbb{R}^l \times \mathbb{T}^l$, where $\mathcal{V}$ is a periodic potential with real Fourier coefficients $\mathcal{V}_k$ satisfying the condition: $\mathcal{V}_{2k} = \mathcal{V}_{-2k}$ and $\mathcal{V}_{2k+1} = -\mathcal{V}_{-2k-1}$, for all $k \in \mathbb{Z}$. Its (Weyl) quantization yields the non self-adjoint PT-symmetric operator $H_\varepsilon = -i\hbar[\omega_1\frac{\partial}{\partial x_1} + \ldots + \omega_l\frac{\partial}{\partial x_l}] + \varepsilon V$ acting on $L^2(\mathbb{T}^l)$. We consider a class of perturbations $\mathcal{V}$ for which the quantum normal form is uniformly convergent with respect to $\hbar$, and prove that the PT-symmetry implies the reality of its symbol. This entails the explicit contruction of a similarity transformation mapping H_ε into a self-adjoint operator, as well as of an exact quantization formula for its real spectrum. For $\hbar = 0$ the construction reduces to a classical complex canonical transformation mapping the complex Hamiltonian $\mathcal{H}_\varepsilon(\xi, x)$ into a real one.