Potential shapes, non-hermiticity and the spontaneous breakdown of $\mathcal{PT}$ symmetry

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The basic properties of quantum mechanical potentials are determined by their shape. In this respect complex $\mathcal{PT}$-symmetric potentials allow for a wider variation than real potentials. Recently we analysed the asymptotic properties of several exactly solvable $\mathcal{PT}$-symmetric potentials [1]. It turned out that potentials possessing the same real component may have completely different characteristics if their imaginary potential component is different. This was the case with the $\mathcal{PT}$-symmetric Scarf II and Rosen–Morse II potentials, which possess asymptotically vanishing and non-vanishing imaginary component, respectivaly. The Scarf II potential exhibits a number of features characteristic of other $\mathcal{PT}$-symmetric potentials including the spontaneous breakdown of $\mathcal{PT}$ symmetry, quasi-parity and two series of bound states, etc. These features are shared by a number of $\mathcal{PT}$-symmetric potentials possessing rather different potential shapes. On the contrary, the Rosen–Morse II potential showed neither of the above features. In particular, the spontaneous breakdown of $\mathcal{PT}$ symmetry did not occur in this case, rather with increasing non-hermiticity the energy eigenvalues were gradually shifted to the positive domain. These properties of the Rosen–Morse II potential are reminescent of the purely imaginary $\mathrm{i}x^3$ potential, the classic example of $\mathcal{PT}$-symmetric quantum mechanics.

Motivated by these results, we consider further $\mathcal{PT}$-symmetric potentials mainly form the exactly solvable class to investigate how the structure of the imaginary (and real) potential component influences the basic properties of these systems. We pay special attention to the question whether the increasing non-hermiticity results in the spontaneous breakdown of $\mathcal{PT}$ symmetry or not.

[1] G. Lévai: Int. J. Theor. Phys. **50** (2011) 997.