Interplay between system and environment in quantum mechanics

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Abstract

The phases of the eigenfunctions ϕ_k of a non-Hermitian Hamilton operator are, as a function of a certain parameter, rigid only far from exceptional points. In the case of a two-level system, the phase rigidity $r_k \equiv \frac{\langle \phi_k^* | \phi_k \rangle}{\langle \phi_k | \phi_k \rangle} = \frac{[\mathrm{Re}\ \phi_k]^2 - [\mathrm{Im}\ \phi_k]^2}{[\mathrm{Re}\ \phi_k]^2 + [\mathrm{Im}\ \phi_k]^2}$ of the eigenfunctions approaches zero in the very neighborhood of an exceptional point. This causes a nonlinear behavior of the eigenvalue trajectories: an avoided crossing of, respectively, discrete and narrow resonance states appears.

The role of exceptional points in many-level systems can be studied by using the Feshbach projection operator technique. In this case, the phase rigidity $\rho \equiv \left| \frac{\int dr([\mathrm{Re}\Psi^E_{c\ \mathrm{int}}]^2 - [\mathrm{Im}\Psi^E_{c\ \mathrm{int}}]^2)}{\int dr([\mathrm{Re}\Psi^E_{c\ \mathrm{int}}]^2 + [\mathrm{Im}\Psi^E_{c\ \mathrm{int}}]^2)} \right|$ of the scattering wavefunction $\Psi^E_{c\ \mathrm{int}}$ inside the system (being the part of the scattering wavefunction localized inside the system) approaches zero in a region with many exceptional points. Here, width bifurcation occurs and, finally, an environmentally induced dynamical phase transition appears: a few states of the system align to the scattering states of the environment while the remaining ones decouple from the environment. The dynamical phase transition is a collective (global) phenomenon to which all states of the system in a large energy region contribute.

When all exceptional points accumulate in one point, the (dynamical) transition region between the two different phases shrinks to one point in the parameter space. When however the exceptional points are distributed over a finite parameter range (what is mostly the case in realistic systems), the outstanding properties of the transition region are observable. Most remarkable feature is the enhanced transmission through the system in the corresponding parameter range with $\rho < 1$. Dynamical phase transitions are observed experimentally in different systems (including $\mathcal{PT}$ symmetric ones) by varying one or more parameters.

References

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