Semiclassical Wigner dynamics

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Abstract

Phase-space representations allow to discuss quantum dynamics in a conceptual framework as close as possible to classical (symplectic) mechanics, supporting the development and interpretation of semiclassical approximations. I present a synopsis of representations of quantum states and their time evolution in phase space, focussing on the Wigner function and its propagator. The Wigner function stands out as a one-to-one representation of the projective Hilbert space (the Hilbert space of the density operator). In particular, it preserves the full information on the quantum state including coherences. Special cases are discussed, such as non-standard topologies (cylindrical, toroidal) of phase space.

The unitary time evolution of quantum states in terms of the Wigner function is generated by an evolution equation equivalent to the von-Neumann equation for the density operator. As to finite time, a dynamical group, largely analogous to the classical group generated by the Liouville operator, is based on the propagator of the Wigner function. Its close-to-classical structure greatly facilitates the construction of semiclassical approximations. We consider various levels, from the full classical case, the Liouville operator, through a semiclassical approximation based on the van-Vleck propagator, to an improved uniform approximation in terms of Airy functions derived in a phase-space path-integral formalism. Applications to spectral statistics, propagation of entanglement, and quantum dissipation are indicated.

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