Entanglement, quantum measurement and quantum information with Rydberg atoms and cavities

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Quantum and classical interacting systems

"Rest of the world"

isolated system

$|\psi_{A,B}\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$

State preparation

Coupling $\rightarrow$ entanglement

Measurement:
- randomness
- EPR correlation
Quantum entanglement and Cavity QED

- Principle of cavity QED experiments:
  - Two level atoms interacting with a single mode of a high Q cavity
  - The "strong coupling" regime: coupling >> dissipation
Probing EPR entanglement and complementarity

Illustrating Bohr-Einstein dialog
Central concept: complementarity

**Massive slits**: insensitive to collisions with single particles
**Interferences**: mater behave as waves
Experiment performed with photons, electrons, atoms, molecules.

**Light slits**: recoil of the slit monitors which path information
No interferences: mater behave as particles
The "Schrödinger cat"

• Elementary formulation of the problem:
  
  Superposition principle in quantum mechanics:
  → Any superposition state is a possible state
  → Schödinger: this is obviously absurd when applied to macroscopic objects such as a cat!!

Up to which scale does the superposition principle applies?
Schrödinger cat
and quantum theory of measurement

• Hamiltonian evolution of a microscopic system coupled to a measurement apparatus:

\[ |\Psi_{at+...+chat}\rangle = \frac{1}{\sqrt{2}}\left( |\text{Entangled atom-meter state}\rangle + |\text{Entangled atom-meter state, measurement}\rangle \right) \]

→ Entangled atom-meter state
Problem: real meters provide one of the possible results not a superposition of the two
→ too much entanglement in QM?
Need to add something?
NO! "decoherence" does the work
Entanglement and quantum information

Can one do something "useful" from the strangeness of quantum logic?

- **Yes:**
  - Quantum cryptography: deos work
  - Quantum communication: teleportation
  - Quantum algorithms:
    - Search problems (Grover)
    - Factorization (Shor)

- **How?**
  by manipulating qubits with a quantum computer

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Classical bit: 0 or 1

Qubit: two level system 0 and 1

\[ \frac{1}{\sqrt{2}}[a|0\rangle + b|1\rangle] \]
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- **Practically: extreeeeemely difficult to realize: a quantum computer manipulates huge Schrödinger cat states.**
  - Anyway interesting for understanding
    the essence and the limits of quantum logic
1. Rydberg atoms in a cavity: achieving the strong coupling regime

2. Rabi oscillation in vacuum: entanglement and complementarity at work

4. Rabi oscillation a mesoscopic field: Schrödinger cat state and decoherence
1. Rydberg atoms in a cavity: achieving the strong coupling regime

One photon and one atom in a box:

- Photon box: superconducting microwave cavity
- “circular” Rydberg atoms
The cavity

a "photon box":
- superconducting Niobium mirrors
- microwave photons:
  $\lambda=6\ mm$, $\nu_{\text{cav}}=51\ GHz$
- photon lifetime:
  $T_{\text{cav}}=1\ ms$ ($Q = 3.10^8$)

- one mode
- a few trapped photons
"Circular Rydberg atoms":

\[ l = |m| = n - 1 \]

- radiative lifetime: 30 ms
- dipôle: \( d = 1500 \) u.a.
- ideal closed two level system
Laser velocity selection

Circular atom preparation:
- 53 photons process
- pulsed preparation
0.1 to 10 atoms/pulse

Cryogenic environment
T=0.6 to 1.3 K
\(\text{weak blackbody radiation}\)

\(^{85}\text{Rb}\)

State selective detector
One atom = one click
Experimental setup

Atom preparation

Detection

Lasers

Atomic beam
Resonant atom-field coupling

\[ \Omega_0/2\pi = 50 \text{ kHz} \]

\[ T_{\text{rabi}} = 20 \text{ \mu s} \]

\[ |e,0\rangle \rightarrow \cos \left( \frac{\Omega_0 t}{2} \right) |e,0\rangle - i \sin \left( \frac{\Omega_0 t}{2} \right) |g,1\rangle \]

→ Coherent Rabi oscillation
Single photon induced Rabi oscillation

\[ \omega_{ge} = \omega_{cav} \]

Coherent Rabi oscillation replaces irreversible damping by spontaneous emission

\[ \Omega_0 = 47 \, \text{kHz} \]
\[ T_{\text{Rabi}} = 20 \, \mu\text{s} \]
Vacuum Rabi oscillation and quantum gates

\[ g,1 \]  
\[ e,0 \]  
\[ \Omega_0 = 47 \text{ kHz} \]
\[ T_{Rabi} = 20 \mu s \]

- Phase gate, QND detection of a single photon
- Atom-field state exchange
- EPR pair preparation

\[ \omega_{ge} = \omega_{cav} \]
Tailored preparation of tree entangled qubits

\[ \frac{1}{\sqrt{2}} \left( |0,0,0\rangle - |1,1,1\rangle \right) \]

- Rabi oscillation in C
- Classical \( \pi/2 \) pulse
- Detection
2. Rabi oscillation in vacuum: entanglement and complementarity at work
Complementarity in the Ramsey interferometer

Classical microwave fields acts as "beam-splitters" for the internal atomic state.

Classical $\pi/2$ pulses

Ramsey fringes signal with two classical pulses: results from a two path interference
From classical to quantum beam splitters:

- \( \pi/2 \) pulse in a large coherent state: \( \bar{n} \gg \sqrt{n} \)

\[
|\alpha_e\rangle \approx |\alpha_g\rangle \approx |\alpha\rangle
\]

\[
|e\rangle \otimes |\alpha\rangle \to \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle) \otimes |\alpha\rangle
\]

When \( \alpha \) is large enough,
one more photon in the field does not
make any difference on the field state
\( \Rightarrow \) NO which path information stored in the field: "classical beam splitter"

- \( \pi/2 \) pulse in vacuum: \( \alpha=0 \)

\[
|e\rangle \otimes |0\rangle \to \frac{1}{\sqrt{2}}(|e,0\rangle + |g,1\rangle)
\]

Atom-field EPR pair:
Hagley et al. PRL 79,1 (1997)

The photon number is a perfect label of the atomic state
\( \Rightarrow \) FULL which path information stored in the field: "quantum beam splitter"
Resonant interaction with a coherent field as "beam-splitter" in a Ramsey interferometer

Classical $\pi/2$ pulses

$\pi/2$ pulse in C

$\frac{1}{\sqrt{2}}(|e\rangle\otimes|\alpha_e\rangle + |g\rangle\otimes|\alpha_g\rangle)$

Entangled EPR pair

Graphs showing $P_g$ for different values of $n$ and $\alpha$: $n=0$, $\alpha=0$, $n=0.31$, $\alpha=0.56$, $n=2$, $\alpha=1.41$, $n=12.8$, $\alpha=3.6$
Quantitative interpretation in term of atom-cavity entanglement

- Variation of fringe Visibility $V$:

\[
R_1 \quad |e\rangle \otimes |\alpha\rangle \quad R_2
\]

\[
\frac{1}{\sqrt{2}} (|e\rangle \otimes |\alpha_e\rangle + |g\rangle \otimes |\alpha_g\rangle)
\]

Reduced atom density matrix:

\[
\rho_{at} = \frac{1}{2} \begin{pmatrix}
1 & \langle \alpha_e | \alpha_g \rangle^* \\
\langle \alpha_e | \alpha_g \rangle & 1
\end{pmatrix}
\]

\[
V = |\langle \alpha_e | \alpha_g \rangle| \cdot \eta
\]

$\eta$: saturated contrast at large $n$
4. Rabi oscillation in a mesoscopic field: Schrödinger cat state and decoherence
Coherent field states

- **Number state:** $|N\rangle$
- **Quasi-classical state:**
  $$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_N \frac{\alpha^N}{\sqrt{N!}} |N\rangle \quad ; \quad \alpha = |\alpha| e^{i\Phi}$$

- **Photon number distribution**
  $$P(N) = e^{-\bar{N}} \frac{\bar{N}^N}{N!} \quad ; \quad \bar{N} = |\alpha|^2$$

- **Phase space representation**
  $$\Delta N \cdot \Delta \Phi > 1$$

- **Poisson distribution**
  ![Poisson distribution graph]

- **Phase space diagram**
  ![Phase space diagram]
Rabi oscillation in a classical field

\[ |\psi_{at}(0)\rangle = |e\rangle \]

\[ \rightarrow |\psi_{at}(t)\rangle = \cos(\Omega_R t/2) |e\rangle - i \sin(\Omega_R t/2) |e\rangle \]

\[ = \frac{1}{\sqrt{2}} \left( e^{-i\Omega_R t/2} |\psi_{at,+}\rangle + e^{+i\Omega_R t/2} |\psi_{at,-}\rangle \right) \]

\[ |\psi_{at,+}\rangle = 1/\sqrt{2} (|e\rangle + |g\rangle) \]

\[ |\psi_{at,-}\rangle = 1/\sqrt{2} (|e\rangle - |g\rangle) \]

Rabi oscillation results from a quantum interference between probability amplitudes of the two atomic "eigenstates"
Rabi oscillation in a mesoscopic field

\[ |\psi(0)\rangle = |\alpha\rangle \otimes |e\rangle \]

\[ \rightarrow |\psi(t)\rangle \approx \frac{1}{\sqrt{2}} \left( e^{-i\Omega_R t/2} |\alpha_+ (t)\rangle \otimes |\psi_{at,+} (t)\rangle + e^{+i\Omega_R t/2} |\alpha_- (t)\rangle \otimes |\psi_{at,-} (t)\rangle \right) \]

\[ = \frac{1}{\sqrt{2}} \left( e^{-i\Omega_R t/2} |\psi_+ (t)\rangle + e^{+i\Omega_R t/2} |\psi_- (t)\rangle \right) \]

Rabi oscillation frequency: \[ \Omega_R = \Omega_0 \sqrt{\bar{N}} + 1 \quad \text{where} \quad \bar{N} = |\alpha|^2 \]
Graphical representation of the atom-field state in the complex plane:

\[ |\psi_+(t)\rangle \approx |\alpha_+(t)\rangle \otimes |\psi_{at,+}(t)\rangle \]

\[ |\alpha_+(t)\rangle = |\alpha e^{-i\Phi(t)}\rangle \]

\[ |\psi_{at,+}(t)\rangle = \frac{1}{\sqrt{2}} \left( e^{-i\Phi(t)} |e\rangle + |g\rangle \right) \]

\[ \Phi(t) = \frac{\Omega_0 t}{4\sqrt{N}} \]
Graphical representation of the atom-field state in the complex plane:

\[ |\psi_-(t)\rangle \approx |\alpha_-(t)\rangle \otimes |\psi_{at,-}(t)\rangle \]

\[ |\alpha_-(t)\rangle = |\alpha e^{i\Phi(t)}\rangle \]

\[ |\psi_{at,-}(t)\rangle = \frac{1}{\sqrt{2}} \left( e^{i\Phi(t)} |e\rangle - |g\rangle \right) \]

\[ \Phi(t) = \Omega_0.t/4\sqrt{N} \]
Graphical representation of the atom-field state in the complex plane:

- **Entangled atom-field state:**

\[ |\psi(t)\rangle \approx \frac{1}{\sqrt{2}} (e^{-i\Omega R \cdot t/2} |\psi_+(t)\rangle + e^{i\Omega R \cdot t/2} |\psi_-(t)\rangle) \]

A Schrödinger cat state: Field phase "measures" the atomic state

\[ \Phi(t) = \Omega_0 t / 4\sqrt{N} \]

[Diagram showing the complex plane with arrows indicating states and a Schrödinger cat state.]
Measured phase distribution of the "Schrödinger cat" field state

- Coherent field containing 10 to 40 photons
- atom 1: prepares the "cat state"
- atom 2: measures the phase distribution of the field

|g>  Atom 1  P_g

33 injected photons
- no atom 1
- atom 1: 335m/s
- atom 1: 200m/s
Wigner of the field state

- 15 injected photons
- \( V = 335 \) m/s
- \( T_{\text{cav}} = 800 \) \( \mu \)s

Wigner function can be measured:

- Lutterbach and Davidovich, PRL 78 2547 (1997)
- P. Bertet et al., PRL 89, 200402 (2002)
Rabi oscillation entanglement and complementarity

\[ |\psi(t)\rangle \approx \frac{1}{\sqrt{2}} \left( e^{-i\Omega t/2} |\alpha_+ (t)\rangle \otimes |\psi_{at,+}(t)\rangle + e^{+i\Omega t/2} |\alpha_- (t)\rangle \otimes |\psi_{at,-}(t)\rangle \right) \]

Rabi oscillation results from a quantum interference between \( |\psi_{at,+}\rangle \) and \( |\psi_{at,-}\rangle \)

No oscillations as soon as: \( \langle \alpha_+ (t)|\alpha_- (t)\rangle \approx 0 \)  \( \) complementarity again!

Collapse of Rabi oscillation
Preparation of a phase cat
Field states coincide again: revival of Rabi oscillation
signature of the coherence of the "cat" state
Demonstration of coherence by induced revival

Atom-field entanglement

$\pi$ rotation of the atomic state

Field phase evolution is reversed
Recombinaison of the two field components:
Revival of Rabi oscillation

Morigi et al PRA 65, 040102
Induced revival signal

-20 -10 0 10 20 30 40 50 60
Interaction time

Transfer

Π Pulse

22 µs

18.5 µs

23.5 µs
Perspectives

• Rydberg atoms and superconducting cavities
  A two cavity experiment

  ▲ EPR pair of Schrödinger cat states (non-locality at the mesoscopic scale)
    - Non-locality and decoherence

  ▲ Complex manipulation of quantum information
    - Quantum feedback
    - Elementary algorithm
    - Error correction code
The team

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Les bizarrerie quantiques: intrication, mesure et décohérence

- **Limite classique-quantique**
  - Pas de superpositions à notre échelle
  - Le "chat de Schrödinger"

- **Décohérence**
  - Couplage inévitable d’un système macroscopique à un environnement

\[
\frac{1}{\sqrt{2}} (|\text{chat}\rangle + |\text{mort}\rangle) \Leftrightarrow
\]

- Nous n’observons qu’une toute petite fraction des états possibles

- Quelques états stables (base préférée)
- Les superpositions quantiques de ces états sont très rapidement détruites
Rabi oscillation in a small coherent field

\[ |\alpha|^2 = 0.85 \text{ photon} \]
Rabi oscillation in a small coherent field: observing discrete Rabi frequencies

Fourier transform of the Rabi oscillation signal

Discrete peaks corresponding to discrete photon numbers

Direct observation of field quantization in a "box"
Rabi oscillation in a small coherent field: Measuring the photon number distribution

\[ P_g(t) = \sum_N P(N) \frac{1}{2} \left( 1 - \cos(\Omega_0 t \sqrt{N + 1}) e^{-t/\tau} \right) \]

▲ Fit of \( P(n) \) on the Rabi oscillation signal:

▲ accurate field statistics measurement
Rabi oscillation in small coherent fields

Observation de la distribution de phase du champ: "detection homodyne"

1. Préparation du champ à mesurer:
   Un état cohérent $|\alpha\rangle$ par exemple

2. Addition d'un champ de même amplitude et de phase variable $\theta$

3. Un atome absorbe le champ résiduel: mesure $P(0)$

Interférence destructive pour $\theta=0$.

![Diagram showing the process of detecting phase distribution in a coherent state and absorption by an atom.](image-url)

signal d'absorption par un atome

$|g\rangle$  

33 photons

![Graph showing the absorption signal $P_g$ as a function of phase $\theta$.](image-url)
Détection des deux composantes de phase

\[ \Phi = \frac{\Omega_0 \cdot t_{\text{int}}}{4 \sqrt{N}} \]

Atomes rapides
N grand

Atomes "lents"
N petit