Plasma induced laser beam smoothing below the filamentation threshold

M. Grech
Centre Lasers Intenses et Applications, UMR 5107, Université Bordeaux 1, 351 cours de la Libération, 33405 Talence, France and Département de Physique Théorique et Appliquée, CEA/DAM-Ile-de-France, 91680 Bruyères le Chatel, France

V. T. Tikhonchuk
Centre Lasers Intenses et Applications, UMR 5107, Université Bordeaux 1, 351 cours de la Libération, 33405 Talence, France

G. Riazuelo
Département de Physique Théorique et Appliquée, CEA/DAM-Ile-de-France, 91680 Bruyères le Chatel, France

S. Weber
Centre Lasers Intenses et Applications, UMR 5107, Université Bordeaux 1, 351 cours de la Libération, 33405 Talence, France and Centre de Physique Théorique, UMR 7644, Ecole Polytechnique, 91128 Palaiseau, France

(Received 20 April 2006; accepted 25 July 2006; published online 21 September 2006)

This paper deals with a statistical approach for description of the laser field interaction with underdense plasmas and modification of the laser beam temporal coherence during its propagation through a plasma at power well below the filamentation threshold. The main properties of the plasma density perturbations driven by a randomized laser beam are derived from a stochastic wave equation. The laser spectral and angular broadening is shown to occur on a distance that depends essentially on the ratio of the average power in a speckle to the critical power for filamentation. The coherence time of the transmitted light is reduced to the plasma acoustic time of response to the laser. It is typically a few picoseconds. Dedicated diagnostics have been developed for the interaction code PARAX in order to analyze the laser and plasma statistical properties. The effect of the plasma length on the transmitted light coherence is found to be in good agreement with theoretical predictions. Forward stimulated Brillouin scattering is shown to play a key role in the laser coherence loss in this low-intensity regime. The limitations of the analytical model are discussed in terms of the deviation of the electric field distribution from the Gaussian statistics and creation of density-electric field correlations. This regime of laser induced incoherence is especially interesting in that the associated angular broadening is not as deleterious as observed for higher intensities. Moreover, beam smoothing can be achieved in low-density plasmas where energy losses due to absorption and backscattering are not too important. © 2006 American Institute of Physics.

I. INTRODUCTION

A detailed understanding of the characteristics of laser-plasma interaction is essential for inertial confinement fusion (ICF). The control of laser beam coherence is necessary to achieve a homogeneous ablation and a symmetric compression of the target. For this, optical smoothing techniques have been developed in the past decades. They consist in breaking the spatial and temporal coherence of the laser pulse. Spatial smoothing is achieved by using random phase plates (RPP) to redistribute the energy among a larger number of small hot spots, the so-called speckles. The obtained intensity distribution is highly inhomogeneous but has well-known, reproducible average properties. The temporal coherence also needs to be broken so that the time-integrated intensity distribution is smoothed. This is achieved by the phase modulation and dispersion of the laser beam by smoothing by spectral dispersion. The efficiency of smoothing techniques in reducing parametric instabilities, self-focusing, and Rayleigh-Taylor hydrodynamic instability has been proved. However, such techniques are expensive and face severe technological constraints.

In the last few years, it has been shown that the interaction of a spatially incoherent laser beam with an underdense plasma enhances spatio-temporal smoothing. This is an attractive possibility for an efficient control of the laser energy deposition. At sufficiently high intensities, the interplay between filament instability and forward stimulated Brillouin scattering (FSBS) has been identified as the dominant mechanism for plasma induced smoothing. This has been confirmed in theory, numerical calculations and experiments. However, this regime requires relatively high laser intensities and is accompanied by undesirable effects as enhanced backscattering and important angular spreading associated with the filamentation instability.

At lower intensities, below the critical power for filamentation, the propagation through a very low density
plasma, typically a few percent of the critical density, can also induce smoothing.\textsuperscript{19,20} This regime could be interesting for ICF applications since it is not associated with parasite effects of beam spreading and backscattering parametric instabilities. Moreover, the standard ICF laser pulse contains two parts, a low-intensity plateau followed by a higher-intensity peak. Parameters used in this paper correspond to the plateau regime. It has been suggested that the observed temporal coherence loss is due to multiple scattering of the transmitted light on self-induced density fluctuations. However, so far this effect has not well been understood. This work attempts a theoretical analysis of this phenomenon within a statistical approach and its numerical simulation using the three-dimensional interaction code PARAX.\textsuperscript{21}

Smoothing techniques introduce into the laser beam random spatial and/or temporal perturbations so that statistical methods are necessary to describe the electric field\textsuperscript{22,23} and the plasma perturbations.\textsuperscript{26} The laser electric field coherence properties and their modification during the propagation through a plasma are described by the correlation functions. The plasma density fluctuations are driven by a randomized laser beam. They consist of static and propagating large scale ion acoustic waves. Their interaction with a laser beam is responsible for the induced smoothing.

The paper is organized as follows. Section II introduces the statistical framework to describe the electric field correlation function. The main properties of the electric field correlation function in vacuum are addressed. The equation that governs the modification of this function through a random medium is derived. The effect of given density fluctuations is considered in terms of the modification of the laser coherence along the propagation direction. Section III considers the plasma density fluctuations driven by lasers. Using a simple wave model, the dynamics of plasma perturbations due to the ponderomotive force is studied analytically in two transverse dimensions for regular and randomized laser beams. Their spectra and level are derived. The effects on the density fluctuation level of electron-ion collisions and non-local transport are addressed within the statistical approach and compared to two-dimensional numerical simulations from PARAX. Section IV considers the system of coupled equations for the electric field and density correlation functions. The angular and spectral broadening of the transmitted light is derived within a perturbation approach. The general properties of the transmitted light are discussed. In Sec. V, three-dimensional simulations of the laser smoothing below the filamentation threshold are performed. Specific diagnostics have been used to characterize the modification of the light coherence along the propagation direction. Numerical results are compared to analytical results from the statistical model. The importance of FSBS from an ensemble of speckle in the induced coherence loss is shown and the main assumptions of the model are discussed. Finally, Sec. VI contains the conclusions of the present study.

II. STATISTICAL DESCRIPTION OF A RANDOM LASER BEAM

A. Field correlation function

This paper considers the laser electromagnetic field in the envelope approximation assuming that the complex amplitude $E(r,z,t)$ varies slowly over the laser wavelength and period. Moreover, it is assumed that $E$ is a stochastic quantity that can be characterized by its statistical average $\langle E \rangle$ and the correlation function\textsuperscript{22}

$$\Gamma_{EE}(R,2p,T,2\tau,z)$$

$$= \langle E(R+p,T+\tau,z)E^*(R-p,T-\tau,z) \rangle,$$  \hspace{1cm} (1)

where $R$, $z$, and $T$ are the macroscopic coordinates and time and $\rho$ and $\tau$ describe the correlations in the plane perpendicular to the beam propagation axis $z$ and in time.

The spatial correlations can be introduced in the laser beam before it enters the plasma with the RPP technique. Temporal and spatial behaviors are then decoupled and $\Gamma_{EE}(R,\rho,T,\tau,z)=\Gamma_{CE}(R,\rho)F(T,\tau)$, where $\Gamma_{CE}$ is the spatial correlation function and $F$ describes the temporal evolution. The purpose of this paper is to describe how temporal correlations are affected during the laser propagation through the plasma, assuming that spatial correlations are known at the plasma entrance, $z=0$. These initial correlations are assumed to follow Gaussian statistics with zero mean value.

B. Propagation equation for the electric field correlation function

The propagation of a laser beam through a plasma with electron density $n_0$ is described by the scalar paraxial wave equation:

$$\left(2ik_0\frac{\partial}{\partial z} + \nabla^2 \right)E(r,t) = \frac{\omega_p^2}{c^2} \delta n(r,t) E(r,t),$$  \hspace{1cm} (2)

where $\delta n$ is the density perturbation, $\omega_p = (e^2 n_0/m_e e_0)^{1/2}$ is the plasma frequency, $k_0 = (\omega_0/c)(1-n_0/n_e)^{1/2}$ is the laser wave number, $n_c = e_0 \omega_0^2 m_e/e^2$ is the critical density, and $c$ the light velocity in vacuum. This paraxial description accounts for small-angle refraction and laser beam scattering in the near-forward direction.

It is widely used to describe the propagation of regular laser beams.\textsuperscript{7,21} It also applies to smoothed beams but its usage becomes more complicated. Indeed, solving this equation for a randomized beam involves disparate scales in time and space, and therefore requires powerful computational tools. We will present the results of numerical simulations using the code PARAX later in this article. However, from a theoretical point of view, it is more efficient to deal with average quantities.

A propagation equation for the electric field correlation function (1) is derived from the paraxial equation in Appendix A.

For this, a diffusion approximation is introduced that assumes the electric field distribution taken in plane $z_1$ to be independent on density fluctuations taken in plane $z_2 > z_1$. It is justified if the longitudinal coherence length of density fluctuations, $L_C$, is much shorter than the distance $\Lambda_C$ [see
Eq. (9) below] over which the nonlinearity is significant. It should be noted that this diffusion approximation, discussed in Ref. 27, is different from the one used in transport theory. It considers the multiple scattering of the laser light on density fluctuations as a random process, leading to an equation for the field correlation function:

$$
\left( \partial_z - \frac{i}{k_0} \nabla_R \cdot \nabla_p \right) \Gamma_{EE} = - [\partial_z \ln Y(z)] \Gamma_{EE},
$$

(3)

where

$$
Y(z) = \left\langle \exp \left\{ - \frac{k_0 n_0}{2 n_c} \int_0^{z} \left[ \delta n \left( \mathbf{R} + \frac{\mathbf{p}}{2}, T + \frac{\tau - \tau'}{2} \right) - \delta n \left( \mathbf{R} - \frac{\mathbf{p}}{2}, T - \frac{\tau - \tau'}{2} \right) \right] d\tau' \right\} \right\rangle.
$$

(4)

The diffusion approximation assumes that the laser beam crosses many density fluctuations along its propagation, i.e., \( z \gg L_c \), so that the integral \( \int_0^{z} \delta n(z) d\tau \) should follow a Gaussian distribution, according to the central limit theorem. Equation (3) then takes the following form:

$$
\left( \frac{\partial}{\partial z} - \frac{i}{k_0} \nabla_R \cdot \nabla_p \right) \Gamma_{EE} = \frac{\sqrt{2\pi}}{4} L_c k_0^2 \left( \frac{n_0}{n_c} \right)^2 [D^N(\mathbf{p}, T, \tau) - D^N(0, T, 0)] \Gamma_{EE},
$$

(5)

where the plasma density correlation function \( D_N \) is given as

$$
D_N(2\mathbf{p}, T, 2\tau) = \langle \delta n(\mathbf{R} + \mathbf{p}, T + \tau, z) \delta n(\mathbf{R} - \mathbf{p}, T - \tau, z) \rangle.
$$

(6)

If the beam is wide enough so that the density fluctuations propagating with the acoustic velocity \( c_a \) have no time to traverse it (\( T < 2c_a L_0 \)) and assuming that they are homogeneously distributed, one can neglect macroscopic effects. Then, \( D_N \) depends only on the relative position \( \mathbf{p} \). The situation is different for time. Since nonstationary processes are studied, it is important to account for the variable \( T \).

The same considerations can be applied to the equation for the field-field correlation function \( \Gamma_{EE} \):

$$
\left( \frac{\partial}{\partial z} - \frac{i}{2k_0} (\nabla_R^2 + \nabla_p^2) \right) \Gamma_{EE} = \frac{\sqrt{2\pi}}{4} L_c k_0^2 \left( \frac{n_0}{n_c} \right)^2 [D^N(\mathbf{p}, T, \tau) + D^N(0, T, 0)] \Gamma_{EE}.
$$

(7)

For the RPP-created laser beams, \( \Gamma_{EE} \) is negligible in comparison with \( \Gamma_{EE}^* \). Equations (5) and (7) show that this property is conserved during the beam propagation.

Let us assume that the density correlation follows the Gaussian distribution:

$$
D_N(\mathbf{p}, T, \tau, z) = \delta n_0^2(z) \exp(- \frac{\mathbf{p}^2}{2\rho_0^2} - \tau^2/2\tau_0^2).
$$

(8)

A part of the laser light is scattered on these perturbations. Its coherence time is strongly reduced to the order of \( \tau_M \), while the coherence radius varies much less provided that \( \rho_M \approx \rho_0 \). The scattered light intensity increases a distance estimated from Eq. (5):

$$
\Lambda_e^{-1} = k_T^2 L_c (n_0/n_c)^2 \delta n_0^2.
$$

(9)

This corresponds to the medium length needed for the laser beam smoothing. For larger distances, the laser beam characteristic spatial correlation length \( \rho_e(z) \) and the coherence time \( \tau_e(z) \) vary with \( z \) as follows:

$$
\rho_e(z) = \rho_0 \left( 1 + \frac{z}{\Lambda_e \rho_0^2} \right)^{-1/2}, \quad \tau_e(z) = \tau_M \sqrt{\frac{\Lambda_e}{z}}.
$$

(10)

The coherence radius of the light is slightly reduced along the propagation because of the multiple scattering, which reduces the coherence time. Figure 1 presents the solution of Eq. (5) for the electric field spectrum as a function of the propagation distance. The spectral broadening of the laser beam is symmetric, which is a characteristic feature of the multiple scattering process on independently created density.
fluctuations. The effect of the induced smoothing can be described in terms of the increase of the fraction of scattered light with frequency outside of the monochromatic, incident peak; i.e., with $|\omega| > 1/T$:

$$F_s = \frac{\int_{|\omega'| > 1/T} d\omega' |E(\omega')|^2}{\int d\omega' |E(\omega')|^2}.$$ 

(11)

The evolution of $F_s$ along the $z$ axis can be approximated by the curve presented in the inset of Fig. 1:

$$F_s(z) \approx 0.82[1 - \exp(-z/\Lambda_c)].$$

(12)

It is suitable for $z/\Lambda_c < 2$, and it will be compared with the results of numerical simulations in Sec. V.

III. PLASMA DENSITY FLUCTUATIONS Driven By A LASER

A. Enhanced fluctuations in laser driven plasmas

Plasma fluctuations responsible for laser multiple scattering could be created by the laser beam itself due to thermal effects and/or the ponderomotive force. Small-amplitude density perturbations can be described within a linear acoustic wave model. Assuming plasma quasineutrality and neglecting plasma heating, the linear response in a transverse plane to a laser hot spot with the intensity $I(r, t)$ is described by the wave equation

$$\left(\partial_t^2 + 2 \gamma_{ei} \partial_t - u_r^2 \nabla^2 \right) \delta n(r, t) = \frac{c_s^2}{2n_i T_e c} \nabla^2 \delta I(r, t),$$

(13)

where $u_r^2 = c_s^2 + 3 v_T^2$, $c_s^2 = Z T_e / m_i$ is the ion acoustic wave velocity, and $v_T^2 = T_i / m_i$ is the ion thermal velocity, which depends only on the ion temperature $T_i$ and mass $m_i$. $\gamma_{ei}$ is the ion acoustic wave damping rate and $\delta I$ is an operator that accounts for the thermal and collisional effects on the excitation of density fluctuations. It is important if the speckle width is larger or comparable to the electron-ion mean free path $\lambda_{ei}$. Its Fourier spectrum is detailed in Appendix B.

In most situations of practical interest, $T_i \ll T_e$, so that $v_T \approx c_s$. The characteristic time for ion acoustic wave damping is much larger than the ion acoustic time, and often, than the duration of the simulation. Moreover, the spatio-temporal laser beam smoothing appears as the consequence of the interaction of speckles with density fluctuations driven by their neighbors. Therefore, it would be efficient provided the ion acoustic wave is not damped before it interacts with many speckles. For these reasons, the ion acoustic damping is neglected in what follows.

In general, the density perturbation created by the laser contains several components. Quasistationary density depletions are embedded in the location of stationary intensity maxima. They are created during the characteristic acoustic time $t_s \approx \rho_0 / c_s$. Density bumps move away from the intensity peaks with the velocity $c_s$. Finally, in the case of a nonstationary illumination, density depressions are released when the laser pressure is not strong enough to maintain the depression. They also propagate with the velocity $c_s$. The propagating perturbations are the ion acoustic waves transferring the perturbations across the laser beam and leading to its eventual temporal smoothing.

In the case of a spatially smoothed beam, the intensity distribution consists of many randomly distributed spikes. It can be characterized by its correlation function $C_I(\rho) = \langle |R + 1/2 \rho|^2 |R - 1/2 \rho|^2 \rangle$. Two methods can be used to describe it: either the density correlation function is deduced directly by averaging the solutions of Eq. (13), or a stochastic equation for the correlation function $\rho_n$ is derived from Eq. (13).

Let us consider the solution in two transverse dimensions (2TD) of Eq. (13) in the case of a ponderomotive coupling and assuming that the laser source is instantaneously turned on at $T=0$. By taking the average of the products of the solutions of Eq. (13), one then finds the following expression for the density correlation function

$$\bar{D}_G(k, T, \omega) = \frac{\bar{C}_I(k) T}{2(n_i T_e c)^3} \left\{ G_1 \text{sinc}(2\omega T) + \sum_{\pm} \{ G_2 \text{sinc}[2(\omega \pm c, k) T] + G_3 \text{sinc}[2(\omega \pm c, k) T] \} \right\},$$

(14)

where $\bar{C}_I(k)$ is the Fourier transform of the intensity correlation function and $\text{sinc}(x) = \sin(x) / x$. The first term in the curly brackets reads

$$G_1(T, k) = 1 + \frac{1}{2} \cos(2c_s kT).$$

(15)

It results from two contributions: on one hand, the correlations between stationary depressions, and on the other hand, the correlations between bumps propagating in the same direction. The second term,

$$G_2(T, k) = \frac{1}{4},$$

(16)

denotes the correlations between bumps and corresponds to the acoustic free mode of the plasma ($\omega = \pm c_s k$). Its amplitude is a constant. The third term reads

$$G_3(T, k) = -2 \cos(c_s kT).$$

(17)

It describes the correlations between propagating bumps and stationary depressions. The particularity of this component is that its frequency $\omega = \pm c_s k / 2$ is half the ion-acoustic frequency.

This correlation function describes the density perturbations driven by a step-like laser. It can be generalized to a pulse with a characteristic rise-time $t_m$. A similar treatment shows that the bump amplitude decreases as $1/t_m$, whereas its width increases linearly with $t_m$.

One can also obtain an evolution equation for the density correlation function by taking the average of the product of two Eqs. (13). In the case in which the damping is small, one finds
where $\tilde{\Gamma}_i$ is the spatio-temporal intensity correlation function in the Fourier space. This approach allows one to consider an arbitrary spatially and/or temporally smoothed beam provided its intensity correlation function is known. For a spatially smoothed beam, Eq. (14) is the solution to Eq. (18). For a beam with a coherence time $\tau_c \ll \rho_0 / c_s < T$, the induced density correlation function reads

$$
\tilde{D}_N(T, k, \omega) = -\frac{\sqrt{\pi} \tau_c c_s^3 k^3 \tilde{A}_k^2 C_1(k)}{8(n_c T, c)^2 \omega} \times \exp \left( -\frac{\tau_c^2 \omega^2}{4} \right) \sum_{\pm} (-1)^{\pm} \sin^2 \left[ \left( \omega \pm c_s k \right) T \right] \left( \omega \pm c_s k \right)^2.
$$

For such a short correlation time, stationary depressions do not exist and one can see that the excited resonance $\omega = \pm c_s k / 2$ disappears.

Finally, as the macroscopic time $T$ is much longer than the characteristic correlation time, Eq. (18) can be reduced to a second order in time equation by assuming $|\omega^2 - c_s^2 k^2| \ll \omega^2$:

$$
\left[ \tilde{\sigma}_I^2 + \frac{(\omega^2 - c_s^2 k^2)^2}{\omega^2} \right] \tilde{D}_N = \frac{c_s^2 k^3}{(n_c T, c)^2 A_k^2 \tilde{\Gamma}_i}.
$$

This equation can also be derived directly from Eq. (13) in the envelope approximation.

### B. Properties of density fluctuations

The density correlation function defined by Eq. (18) contains all the general properties of the density fluctuations driven by a randomized laser beam. It depends on three variables: $T$, $\omega$, and $k$, and it is difficult to calculate it numerically and to visualize it. The reduced correlation functions are easier to manipulate and also contain important information. Their properties are discussed below.

#### 1. Time-averaged k-spectra

By averaging Eq. (14) over time $T$ and integrating over $\omega$, one finds the $k$-spectrum of RPP-driven fluctuations:

$$
\langle \delta \tilde{n}^2(k) \rangle = \frac{\tilde{C}_1(k)}{8(n_c T, c)^2} \left[ 3 + \text{sinc}(2c_s k T) - 4 \text{sinc}(c_s k T) \right],
$$

while, from Eq. (19), one finds the $k$-spectrum of fluctuations driven by a spatially and temporally smoothed laser beam:

$$
\langle \delta \tilde{n}^2(k) \rangle = \frac{\sqrt{\pi} c_s^3 k^3 \tau_c T \tilde{C}_1(k) \exp \left( -\frac{c_s^2 k^2}{4 \tau_c^2 k^2} \right)}{16(n_c T, c)^2 \tilde{C}_1(k)}.
$$

Figure 2(a) shows normalized $k$-spectra of density fluctuations driven by a RPP beam for time moments $T=10 \rho_0 / c_s$ and $50 \rho_0 / c_s$. Figure 2(b) corresponds to the case of a temporally incoherent beam with the coherence time $\tau_c = 0.1 \rho_0 / c_s$ and $\tau_c = 0.5 \rho_0 / c_s$. In both panels, the normalized Gaussian intensity $k$-spectrum is shown. In the RPP case, the maximum wavelength is given by the laser spatial properties and the minimum wavelength depends on time. The spectrum then spreads from $k = (c_s T)^{-1}$ to $k \approx \rho_0^{-3}$. For density perturbations driven by a spatially and temporally incoherent beam, the density fluctuations have a narrow $k$-spectrum with width of the order of $\rho_0^{-1}$. The spectrum has a maximum for $k_M=2(\rho_0^2 + c_s^2 k_s^2)^{-1/2}$ and no small-wavelength perturbations are excited. A similar behavior has been observed by Brantov et al.\textsuperscript{26}

#### 2. Time-averaged frequency spectra

In the same way, integration over wave numbers $k$ and averaging over $T$ allows one to access the $\omega$-spectra. For RPP-driven fluctuations in 2TD, one finds
3. Level of density fluctuations

The level of density fluctuations can be derived by integrating the density correlation function over \( k \) and \( \omega \). For a RPP laser beam, in the 2TD ponderomotive case, the square of the density perturbation level reads

\[
\langle \delta n^2(T) \rangle = \frac{(f)^2 c_s T}{4 \rho_0 (n_c T_c)^2} \left[ h(2 c_s T / \rho_0) - 2 h(c_s T / \rho_0) \right],
\]

where

\[
h(x) = \sqrt{\pi} e^{x^2} \exp(-x^2) = 2 \int_0^\infty e^{t^2-x^2} dt.
\]

For a temporally incoherent laser beam with the coherence time shorter than the acoustic time, the square of the fluctuation level can be estimated from Eq. (19):

\[
\langle \delta n^2(T) \rangle = \frac{(f)^2 c_s T}{2 \rho_0 (n_c T_c)^2} \frac{\tau_c c_s \rho_0}{\rho_0 + \tau_c c_s^2}.
\]

In this case, the fluctuation level does not saturate, and for a time longer than \( \rho_0 / c_s \), it increases as the square root of the time. The non-stationary speckles continuously excite density fluctuations, therefore their level gradually increases.

These analytical expressions have been compared to the 2TD PARAX simulations of the plasma response to a random laser beam. The basic characteristics of the code are presented in Appendix B. The results for the RPP case are shown in Fig. 4. A beam with the average intensity \( 2 \times 10^{13} \text{ W/cm}^2 \) was focused in a helium plasma with the density \( n_0 = 0.05 n_c \) and \( T_e = 500 \text{ eV} \). The calculated k- and \( \omega \)-spectra are in a good agreement with theory, as well as the average level of fluctuations.

The temporal growth of the density fluctuations driven by spatially and temporally incoherent laser beam is presented in Fig. 5. The beam coherence time \( \tau_c = 4.1 \text{ ps} \) is shorter than the plasma acoustic time \( \tau_a = 8 \text{ ps} \). Its average intensity is \( 3.7 \times 10^{13} \text{ W/cm}^2 \). The characteristic increase of the fluctuation level as \( \sqrt{T} \) is observed provided that the characteristic correlation time is shorter than the acoustic time.

The thermal effects related to the inverse bremsstrahlung absorption and plasma heating have also been accounted for in some simulations. According to Eq. (13), the density perturbations were enhanced by a factor

\[
\gamma_r = 1 + 32 \mid^{\rho_0} 5/4 \exp(-\rho_0^2 / 4 c_s^2 T_c).
\]

This estimate is in agreement with the simulation results shown in Fig. 6.

IV. LASER BEAM PROPAGATION IN SELF-INDUCED DENSITY FLUCTUATIONS

A. Hypothesis of the Gaussian statistics

This section presents the analysis of the set of coupled Eqs. (5) and (18), which provide a self-consistent description for the modification of the laser coherence due to self-excited density perturbations. The coupling of these two equations requires a relation between the intensity and the electric field correlation functions. This is possible provided the electric field follows the Gaussian statistics. The intensity correlation function, which is fourth order in the field, can then be writ-
ten as a bilinear combination of the electric field correlation functions: \( \langle \rho, \tau \rangle = \langle I \rangle^2 + |\Gamma_{EE}|^2 + |\Gamma_{EE}^*|^2 \). The contribution of \( \Gamma_{EE} \) is negligible, as discussed in Sec. II, and the mean intensity \( \langle I \rangle \) does not participate in the excitation of density perturbations. Thus, only the term with \( \Gamma_{EE}^* \) contributes to the interaction.

The Gaussian hypothesis is justified for a random laser beam propagating in vacuum because of the large number of speckles and due to the central limit theorem. The statistics could be affected by nonlinear effects during the propagation through a plasma. However, for powers below the filamentation threshold, one can assume that there is no physical mechanism that can modify the statistics on distances shorter than \( \Lambda_c \). This assumption is compared to numerical simulations and it is discussed in the following section.

The system of Eqs. (5) and (20) defines the characteristic level of density fluctuations \( \delta n \approx \gamma_2 \langle I \rangle / n_c T_e \) discussed in Sec. III B 3. Correspondingly, the characteristic length \( \Lambda_c \) (9) becomes inversely proportional to the square of the laser intensity. It is convenient to express it in terms of the ratio of the average power in a speckle: \( P_{sp} = \pi \rho_0^2 \langle I \rangle \) to the critical power for the speckle self-focusing:

\[
P_c = \left( 8 \pi c^2 / \omega_0^2 \right) n_c T_e (1 - n_d / n_c)^{1/2} n_e / n_0.
\]

In the case of laser driven density perturbations, the expression of \( \Lambda_c \) becomes

\[
\Lambda_c = \frac{0.066 L_R}{\gamma_2^2 (1 - n_d / n_c) \bar{P}^2}.
\]

The ratio \( \bar{P} = P_{sp} / P_c \) is the key scaling parameter: for a given ratio \( P_{sp} / P_c \), the laser-plasma interaction has similar behavior. This will be demonstrated in the following section.
neous/H20849 first order, the system for the perturbed components reads

\[ \Gamma^{-1}_{EE}(T,k,\omega) = 2\pi p_0^2 z \frac{\pi}{\Lambda_c} \delta(\omega) \exp\left(-\frac{p_0^2 k^2}{6}\right) \]

\[ -\frac{3p_0}{c_s} \cos(2\omega T) \xi_{\rho k}(\omega) + \frac{\rho_0}{4c_s} \xi_{\rho k}(\omega) \xi_{\rho k}(\omega) \exp(-\kappa^2/2) \]

where for the 2TD case, \( \xi_{\rho k}(\nu) = \pi|\nu|I_0(\nu)\exp(-\nu^2/4) \) and \( I_0 \) is the modified Bessel function of the first kind.

Each component of the density correlation function gives rise to a branch of scattered light. The static density depressions induce a reduction of the transverse coherence of the transmitted light without frequency broadening. The last two terms describe the scattering from moving density perturbations. They are responsible for the reduction of spatial coherence and frequency broadening. The induced component in the transmitted spectrum (second term) would be difficult to observe as it requires a resolution time shorter than the acoustic time \( \rho_0/c_s \). The last term in Eq. (32) accounts for the scattering on the density perturbations that propagate transversally to the beam with \( c_s \). It dominates the frequency broadening effect.

This solution is real, which corresponds to a symmetric broadening of the transmitted light in \( k \) and \( \omega \). Furthermore, the function \( \xi_{\rho k}(\rho_0\omega/c_s) \) has a maximum for \( k = \omega/c_s \). That means that the spatial and temporal dependencies are mixed, which is the requirement for any smoothing technique.

The time-averaged \( k \)-spectrum is obtained by integrating the field correlation function over \( \omega \). As \((2\pi)^{-1}\int d\nu\xi_{\nu}(\nu) = 1/2|\nu|\exp(-\nu^2/4)\), the transmitted light coherence width is reduced from \( \rho_0 \) to \( \rho_0/\sqrt{3} \). This result follows also from Eq. (10) by taking for the density transverse correlation length \( \rho_M = \rho_0/\sqrt{2} \), which corresponds to the speckle characteristic width. Therefore, the angular broadening of the transmitted light is enhanced by a factor of \( \sqrt{3} \).

The spectral properties of the transmitted light follow from Eq. (32) by integration over \( k \). As \((2\pi)^{-3}\int d\kappa^2\xi_{\kappa}(\kappa) = 1/2|\nu|\exp(-\nu^2/4)\), the spectral broadening of the light is symmetric. The coherence time \( \rho_0/c(\gamma/2) \) agrees with the estimate of Eq. (10) by taking for the density correlation time \( \tau_M = \rho_M/c_s \). That is, the characteristic coherence time of the transmitted light is given by the density response time.

V. NUMERICAL SIMULATIONS OF LASER BEAM SMOOTHING

A. Incident light and plasma properties

Three-dimensional simulations of the interaction were performed to verify the theoretical model and to study the effect of plasma induced smoothing at low laser intensities for the experimentally relevant conditions. In addition to standard PARAX diagnostics, several new ones have been developed, inspired by Schmitt and Afeyan’s work. The laser and plasma parameters are characteristic for present day experiments. The Gaussian laser beam with the wavelength \( \lambda_0 = 1.053 \mu m \) was focused in the center of a 2 mm long helium plasma through a RPP with square elements, which provides a focal spot containing half the energy in a 230 \( \mu m \) large square. The average intensity was \( I_0 = 3 \times 10^{13} \) W/cm\(^2\). The speckles have a Gaussian shape with width \( \rho_0 = 4.3 \mu m \). The laser power brought by an average speckle was therefore \( P_{sp} = \pi\rho_0^2 I_0 = 17 \) MW. The plasma density was varied from 1% to 5% of critical density, the electron temperature was \( T_e = 500 \) eV, and the ion temperature 50 eV. The ion acoustic velocity was \( c_s = 0.17 \mu m/ps \), which leads to the plasma response time.
$\rho_0/c_i \approx 25$ ps. The critical power for self-focusing $P_c$ was decreased from 1600 to 330 MW as the density was increased. Therefore, the ratio $\tilde{P} = P_{sp}/P_c$ was increased from 1% to 5%. The characteristic length for plasma induced smoothing $\Lambda_c$ Eq. (29) was decreased from 78 to about 1 mm.

The electric field correlation function of the incident field is shown in Fig. 7(a). Its real part is in agreement with the analytical predictions while the imaginary part accounts for a small asymmetry of the speckle pattern and is negligible. It has also been verified that the function $\Gamma_E(\rho)$ is zero at the plasma entrance. The shape of the time correlation function is characteristic for a temporally coherent laser beam: it is real and depends on the macroscopic time $T$, as shown in Fig. 7(b). The time-averaged spatio-temporal spectrum $|E(k,\omega)|^2$ is presented in Fig. 7(c). The spatial and temporal components are decoupled: the spatial component is large with the width $\approx \rho_0^{-1}$, whereas the temporal component is very narrow with the width given by $1/T$.

### B. Numerical simulations of the beam temporal smoothing

#### 1. Properties of the transmitted light

A visual evidence of the temporal beam smoothing in a plasma is presented in Fig. 8. It shows the temporal evolution of the intensity of the transmitted light $I(x, t)$ at the central section $y=0$. In panel a ($n_0/n_c=1\%$), the speckle pattern has no temporal modifications. The distance $\Lambda_c = 78$ mm in this case is too large and the plasma does not affect the laser beam. In panel b ($n_0/n_c=3\%$), the intensity distribution presents some temporal fluctuations on time scale of about 50–100 ps. Finally, in panel c, the intensity distribution is strongly modified with time. Here, the characteristic distance $\Lambda_c \approx 2.5$ mm is comparable to the plasma length and the smoothing effect is rather efficient. The contrast of the time-integrated intensity distribution

$$C_{int} = \langle I_{\perp}^{-1} \rangle \sqrt{\langle I_{\parallel}^{-1} \rangle - \langle I_{\perp}^{-1} \rangle}$$

presents a quantitative characteristic of the smoothing effect. Here $\bar{I}$ is the time-averaged intensity and the spatial average in Eq. (33) is taken over the beam cross section at a given distance $z$. It is known that, for a temporally coherent beam with a Gaussian statistics, this contrast is equal to 1 and therefore its reduction characterizes the smoothing effect.

Figure 9 shows the contrast evolution along the propagation direction for different conditions of interaction. In order to reduce the data volume, the average was calculated in one transverse direction ($x, y=0$) instead of in the whole transverse plane. The gray curve corresponds to the low-density case in which the contrast stays almost constant. The dashed line corresponds a 3% of $n_c$ plasma density, where the contrast is reduced to about 70%. For the case of the plasma density of $n_0=0.05n_c$, the contrast is strongly reduced to less than 50% after the distance of 2 mm. Thermal effects enhance even more the smoothing efficiency. The characteristic length is therefore reduced by a factor $\gamma_T^2 \approx 2.2$ and the contrast is reduced to about 38%. At the end of the simu-
tion box the contrast reduction is saturated. This is due to the diffraction of the beam, as discussed in the following paragraph.

2. Modification of the temporal correlation function

Figure 10 presents the real and imaginary parts of the time correlation function $C_{EE}(T, \tau)$ for the case in which $\Lambda_c = 2.5$ mm. The real part is strongly modified during the propagation. Whereas the function contains only the macroscopic scale $T$ at $z=0$, a smaller scale appears for larger values of $z$. It characterizes the coherence time, which is shown in Fig. 11. It is reduced from 250 ps at the beginning of the interaction to about 20 ps after a few hundred $\mu$m of interaction, which is in good agreement with the analytical prediction $\rho_0/(c_s^2) \approx 18$ ps. The coherence time does not evolve so much in time, but it is reduced as the interaction length increases. This reduction is explained by the reduction of the speckle radius with $z$ due to the multiple scattering. To underline this effect, the time integrated speckle radius is calculated as function of $z$, and the corresponding time $\rho_s(z)/(c_s^2)$ is shown in Fig. 11.

The calculated real part of the temporal correlation behaves according to theoretical predictions shown in Fig. 1(b). The only difference is due to the fact that the average intensity varies with $z$ because of the diffraction effect that modifies the value for $\tau=0$. Otherwise, in both figures the correlation function consists of two parts: the homogeneous part decreases with $z$, while the perturbation with the coherence time about $\rho_0/(c_s^2)$ increases on the characteristic length $\Lambda_c$.

Moreover, the temporal correlation function contains a growing imaginary part that becomes comparable to the real part at the distance of 800 $\mu$m. This effect is not described by the current model of multiple scattering presented in Sec. IV, where the correlation function remains real along the propagation axis. Although the imaginary part of the correlation function has no effect on the coherence time, it implies that the real and imaginary parts of the electric field, $E_R$ and $E_I$ are correlated. Indeed, the imaginary part of the temporal correlation function reads

$$\text{Im} \Gamma_{EE}(T, 2\tau) = \langle E_R(T+\tau)E_I(T-\tau) \rangle - \langle E_R(T-\tau)E_I(T+\tau) \rangle.$$  

(34)

It is not zero if the real and imaginary parts of the electric field have a different behavior on the time scale $\approx \rho_0/(c_s^2)$, where the imaginary part of $\Gamma_{EE}$ has an extremum. Further-
more, as the temporal electric field correlation function contains an imaginary part, the spectral density of energy,

\[ \langle |E(\omega)|^2 \rangle = T^{-1} \int d\tau T_{EE^*}(T, \omega), \]

is no longer symmetric. Whereas the model of multiple scattering leads to a symmetric spreading of the transmitted light spectrum, numerical simulations suggest an asymmetric spreading of the frequency spectrum as \( z \) increases.

3. Spectral properties of the transmitted light

The evolution of the laser spectral energy density along the propagation direction is shown in Fig. 12 in the case in which \( \Lambda_c \approx 2.5 \) mm [panel (a), ponderomotive case] and \( \Lambda_c \approx 1.1 \) mm [panel (b), accounting for thermal effects]. It is calculated from the temporal evolution of the electric field in a transverse direction. One observes that, as \( z \) increases, the intensity of stray light is reduced. The scattered component grows with \( z \) and a spectral broadening of the order of \( 0.25 \) ps\(^{-1} \approx 2 \pi c_s / p_0 \) as well as a redshift \( \delta \omega \approx 0.1 \) ps\(^{-1} \) are present. This redshift is characteristic of FSBS and has already been observed in numerical simulations at higher powers of the order of \( P_c \).\(^{13,14} \) It has also been observed in the numerical simulations at powers below the filamentation threshold by Michel.\(^{29} \) Moreover, the amplitude of the straight light (\( \omega=0 \)) decreases more quickly if thermal effects are accounted for. This shows the importance of such effects on the density fluctuation level and their effects on the modification of the laser coherence properties.

In the case of ponderomotive coupling, the transversally resolved frequency spectrum of the light is shown in Fig. 13. The incident light [panel (a)] contains in the \( x \) direction two very different scales. The macroscopic one is the beam envelope \( \approx 400 \) \( \mu \)m, the microscopic one is the speckle width, which is a few micrometers. The spectrum has a narrow distribution in frequencies with the width \( \approx 1/T \). After \( 800 \) \( \mu \)m propagation, a spreading and redshift appear at the center of the beam. It means that the light at the center of the focal spot is already partially smoothed, whereas, for larger values of \( x \), the laser light coherence properties are not yet modified. As \( z \) increases one observes an increase of the intensity of the scattered light and the spectral broadening extends to the beam edges. This is due to the macroscopic properties of the intensity distribution provided by the RPP.

FIG. 11. Evolution along the propagation direction of the coherence time measured from the numerical 2TD electric field correlation function for \( T=30 \) ps (circles), \( T=130 \) ps (squares), and \( T=230 \) ps (diamonds). The gray line shows the theoretical coherence time calculated from the measured coherence width of the electric field \( (p_0/c_s \sqrt{2}) \). This corresponds to the case in which \( \Lambda_c \approx 2.5 \) mm.

FIG. 10. Real (top) and imaginary (bottom) parts of the temporal electric field correlation function for \( T=33 \) ps (a),(b); and \( T=130 \) ps (c),(d). For \( z=0 \) \( \mu \)m (gray curve), \( z=800 \) \( \mu \)m (dashed curve), \( z=1200 \) \( \mu \)m (dot-dashed curve), and \( z=2000 \) \( \mu \)m (solid curve).
The intensity at the center of the beam is higher, thus, the light at the center of the focal spot needs a shorter length than at edges to be smoothed.

The light that is scattered in the middle of the beam has both, a reduced temporal coherence and a reduced coherence width. It diverges faster than the straight light, and, after a certain distance, enhances the density fluctuations at the beam edges and thus increases the smoothing efficiency.

The $\omega$-$k$ spectrum of the light is presented in Fig. 14 for the same conditions of interaction. It shows how spatio-temporal correlations are created as the beam propagates deeper into the plasma. After 400 $\mu$m [panel (a)], the laser light contains a quasisymmetric frequency spread component. However, the part of the energy outside of the central peak, i.e., $|\omega| \leq T^{-1}$, is relatively small. It increases for larger values of $z$ where the laser light undergoes both a stronger frequency broadening and a redshift. The scattered waves are

![FIG. 12. Frequency spectrum of the laser electric field in the ponderomotive case (a) and accounting for thermal effects (b), for $z=0$ (gray curve), 800 $\mu$m (dashed curve), 1600 $\mu$m (dot-dashed curve), and 2000 $\mu$m (solid curve). The corresponding smoothing length is about 2.5 mm in the ponderomotive case (a) and 1.1 mm with thermal effects (b).](image1)

![FIG. 13. Light spectrum resolved in the transverse direction for $z=0$ (a), 800 $\mu$m (b), and 2000 $\mu$m (c). This corresponds to the case in which $\Lambda_s \approx 1.1$ mm.](image2)
filling the ion acoustic cone as it is suggested by the statistical model presented in Sec. IV. A similar behavior has already been reported by Schmitt and Afeyan\textsuperscript{13} but for much higher laser intensities where the average power in a speckle was of the order of the self-focusing threshold. It was explained as a combined effect of FSBS and filamentation. In present conditions, the average power in a speckle is well below the self-focusing threshold. Consequently, the angular spread does not increase \cite{footnote1}, while the $\omega$-spectrum is significantly broadened. The speckle width calculated from the spatial electric field correlation function decreases from 4.3 $\mu$m at the entrance to 3 $\mu$m at the exit. That corresponds to increase of the beam aperture angle from 1.2° to 2.5° at distance of 2 mm. This is ten times smaller than the aperture angle reported in Ref. 13.

C. Efficiency of the plasma induced smoothing and the role of FSBS

The theory of multiple scattering predicts that the smoothing length \cite{footnote29} depends essentially on the ratio $\hat{P} = P_{sp}/P_c$. It has been verified in PARAX simulations that for the same ratio $\hat{P} = 0.05$ but for different $n_0$ and $\langle I \rangle$ the smoothing length and the fraction of scattered light $F_S$ is almost the same if the thermal effects are switched off. This is illustrated in Fig. 15 for the case in which $\Lambda_c = 2.5$ mm with $n_0/n_c = 0.05$ and $\langle I \rangle = 3 \times 10^{13}$ W/cm$^2$ (circles), $n_0/n_c = 0.025$ and $\langle I \rangle = 6 \times 10^{13}$ W/cm$^2$ (diamonds) and $n_0/n_c = 0.01$ and $\langle I \rangle = 1.5 \times 10^{14}$ W/cm$^2$ (squares). It has been verified that the thermal effects on the plasma induced smoothing can be described by the factor $\gamma_T$ \cite{footnote27}. Indeed, one can see in Fig. 15 that the evolution of $F_S$ is almost the same by accounting for the thermal effects (up triangles) than by multiplying the intensity by a factor $\gamma_T$ in the ponderomotive case (down triangles). The corresponding smoothing length is $\Lambda_c = 1.1$ mm.

FIG. 15. Evolution along the propagation axis of the part of the energy outside of the monochromatic peak for different smoothing lengths: $\Lambda_c = 2.5$ mm (circles, diamonds, squares) and $\Lambda_c = 1.1$ mm (up and down triangles).
Considering in more detail the case in which \( n_0 = 0.05 n_e \), and accounting for thermal effects, one can define three characteristic regions of interaction. In the first, i.e., 500 \( \mu \text{m} \), the first speckles induce some density fluctuations and the light is scattered on them. The temporal coherence is reduced but the fraction of scattered light is small. The density fluctuation level is constant in time as it can be seen in Fig. 16(b) and in Fig. 17. In the second region, typically from 500 to 1500 \( \mu \text{m} \), the scattered intensity increases with \( z \) and attains the saturation level of 50\%–60\%. The perturbation approach that has been developed in Sec. V does not hold here. Indeed the scattered light with a reduced coherence plays an important role in the excitation of density perturbations. They are continuously excited by the fluctuating ponderomotive force and increase in time from \( \approx 0.8\% \) at \( t = 30 \text{ ps} \) to 1.5\% at \( t = 260 \text{ ps} \) as \( \sqrt{t} \) (cf. Fig. 17). This induces a further reduction of the characteristic length \( \Lambda_c \) and therefore increases the induced smoothing efficiency. In the third region the diffraction of the beam takes over and it reduces the average laser intensity [panel (a)]. Density fluctuations are no longer excited [panel (b)], and the smoothing efficiency saturates [panel (c)].

The enhanced scattering in the second region is associated with spectral broadening and redshift of the laser light. These features are characteristic for FSBS.\(^ {13,14} \) In contrast to the case considered by Schmitt and Afeyan\(^ {13} \) and Maximov \textit{et al.}\(^ {14} \) in which it was seeded by the filament instability, the present study shows that initial RPP-driven fluctuations and associated frequency broadening of the laser light in the first region serve as a strong seed for the FSBS. As a consequence, the laser beam does not suffer filamentation and such undesirable effects as enhanced backscattering and angular spreading.

Analytical predictions for FSBS were proposed in Ref. 14 using the random phase approximation (RPA) technique. The convective amplification was derived for scattered light with large wave numbers, for the light scattered outside of the incident cone. This approach is not appropriate to describe the current situation in which the \( k \)-spectrum is not so much broadened. Indeed, it was underlined in Ref. 14 that the RPA does not hold in the incident cone where incident and scattered light could be partially correlated.

Nevertheless, the characteristic gain length for FSBS, \( L_{\text{FSBS}} \), can be estimated from Ref. 14 by setting the correlation time to \( \rho_0/c_s \). One then obtains

\[
L_{\text{FSBS}}^{-1} = \pi n_0 \frac{\rho_0}{n_e c_s n_p T_e},
\]

which, for our simulation parameters, is of the order of 1 mm. In a more general way, one shows from Eqs. (29) and (36) that \( \Lambda_c/L_{\text{FSBS}} \approx \tilde{P}^{-1} \), so that the multiple scattering on self-induced density fluctuations can not be observed without FSBS in the low-intensity regime. It just serves as a seed for it. As it is amplified over the whole interaction length, it quickly enters the nonlinear regime of saturation. The temporal evolution of the instability is mainly governed by the temporal increase of the density perturbations driven by a temporally incoherent beam, as described in Sec. II, which leads to the reduction of the effective length for smoothing.

The multiple scattering as well as FSBS activities strongly depend on the ion acoustic wave damping.\(^ {30} \) In plasmas with \( ZT_e/T_i \leq 3 \), where \( \gamma_i \approx 0.3k_e c_s \), the density perturbations are damped before they interact with neighboring speckles. As a result, numerical simulations show that multiple scattering does not occur, FSBS has not been excited and no induced incoherence is observed.

D. Limitations of the model of multiple scattering

The key role of FSBS observed in numerical simulations prompts us to revisit the main assumptions that have been used in the model presented in Sec. IV. In what follows, the differences between the interaction of laser beams with self-induced and already existing density perturbations are discussed. A RPP laser beam was creating the density fluctuations during about 30 ps and then the coupling term in the acoustic wave equation was instantaneously turned off at \( t = 0 \). Thus, for \( t > 0 \), the laser beam propagates through already existing fluctuations without exciting new ones. Provided that the acoustic damping is negligible, the density fluctuation level stays constant for time shorter than \( L_0/c_s \) and the laser beam undergoes the multiple scattering on density fluctuations with the level of about 0.6\%.

The simulation results are presented in Fig. 18. One observes that the redshift in the frequency spectrum has disappeared (panel a), the imaginary part of the electric field correlation function is negligible. The spectral broadening is symmetric and the efficiency of the smoothing has been reduced by a factor of about 2 (panel b). The resulting evolution of \( F_\xi \) is in good agreement with theoretical prediction of Eq. (12). These results demonstrate that a continuous excitation of density fluctuations by FSBS enhances the smoothing efficiency. This effect is not captured by the perturbation approach of Eqs. (5) and (20).

An important assumption of the model presented in Sec. IV is that the electric field distribution in the focal spot follows the Gaussian statistics. This holds true if one considers the propagation either in a vacuum or through medium with a slow nonlinearity, \( \rho_0/c_s \gg \tau_e \). Under such conditions, Garnier \textit{et al.}\(^ {24} \) proved that the Gaussian statistics is preserved. In the contrary, in Ref. 25, the authors show that the main effect of the propagation through an instantaneously responding medium, \( \rho_0/c_s \gg \tau_e \), is to destroy the Gaussian statistics, leading to a contrast enhancement.

The departure from the Gaussian statistics was observed in numerical simulations from the beam instantaneous contrast: \( C = (\langle I \rangle^2 - \langle I \rangle^2) / \langle I \rangle^2 \). In the case in which the real and imaginary parts of the electric field are statistically independent, a Gaussian field distribution is characterized by the exponential intensity distribution: \( p(I) = \langle I \rangle^{-1} \exp(-I/\langle I \rangle) \), and the corresponding instantaneous contrast is equal to 1. In the region of the first 500 \( \mu \text{m} \), in full PARAX simulations, the contrast increases from 1 to about 1.5 (cf. Fig. 19); that is, the Gaussian statistics is lost. Indeed, in the case of RPP laser beams, the coherence time is given by the pulse duration, typically a few hundred picoseconds. It is therefore much
longer than the response time of the medium, typically a few picoseconds, and the nonlinearity destroys the Gaussian statistics. The Gaussian statistics of a transverse distribution of hot spots holds if there is only one microscopic transverse scale. However, even below the filamentation threshold, the stationary speckles undergo a focusing in the density depression they dig. This modifies the field distribution and two different scales appear: the reduced size of the speckles and the average distance between them. This physical mechanism is responsible for enhancement of the instantaneous contrast and thus, for deviation from the Gaussian statistics. It is not accounted for in the current model for multiple scattering.

On the contrary, in the case in which the source term is switched off in the acoustic equation, the contrast stays close to 1 within 5%. The Gaussian properties are conserved. This is a clear demonstration of the effect of the instantaneous nonlinearity on breaking the Gaussian statistics.

After propagating on a distance of the order of one-tenth of the temporal coherence of the beam is reduced to the time of nonlinear response. Numerical simulations show that the contrast returns back to 1 at distances larger than 1000 μm; that is, the temporal smoothing restores the Gaussian statistics. This case, in which \( t_c \) is reduced to almost \( \tau_c \), is intermediary between the fast and slow nonlinearity case discussed above. It is therefore difficult to make any theoretical predictions; however, numerical simulations suggest that the Gaussian properties is restored.

As it has been discussed, the reason for the increase of instantaneous contrast is the speckle focusing in density depressions. This occurs provided that density depressions and intensity local maxima are correlated. These correlations are neglected in the construction of Eq. (5) that describes the electric field correlation function. This is the major difference between the propagation through self-induced and al-

FIG. 16. Evolution along the propagation axis of the laser average intensity in a circular zone containing 2/3 and 1/2 of the total energy (a), of the density fluctuations level (b), and of the part of the laser energy outside of the monochromatic peak (c). Dashed curves: \( t = 30 \) ps, dot-dashed curves: end of the simulation \( (t = 260 \) ps), solid curves: time averaged over the whole simulation duration. This case corresponds to \( \Lambda_c \approx 1.1 \) mm in which thermal effects are accounted for.

FIG. 17. Temporal evolution of the density fluctuation level for \( z = 0 \) (gray curve), 400 μm (dashed curve), 1200 μm (dot-dashed curve), and 2000 μm (solid curve). The characteristic evolution as \( \sqrt{t} \) of fluctuations driven by a temporally incoherent beam is shown by the dotted line. The parameters are the same as in Fig. 16.
already existing density fluctuations. A diagnostic for these correlations has been proposed by Schmitt and Afeyan. Figure 20 illustrates the evolution of these correlations during the propagation through the plasma. In the first 500 μm, the correlation between density depressions and intensity maxima increases. It is of the form: $\delta n \propto I$. It corresponds to the region where the instantaneous contrast is increasing. As soon as the induced smoothing becomes efficient, for $z > 1000$ μm, the correlation between intensity and density perturbations is then reduced and the instantaneous contrast decreases to 1. Similar behavior has been observed at powers above the self-focusing threshold in Ref. 13. This again confirms that the effect of multiple scattering is rather universal as soon as the density perturbations attain a sufficiently high level. In the case of decoupled density fluctuations, there is no correlation between intensity and perturbations and the Gaussian statistics is conserved along the whole interaction length (cf. Fig. 19).

VI. CONCLUSIONS

The problem of modification of coherence properties of a partially incoherent beam during its propagation through a randomly fluctuating medium is considered in this paper. Using a stochastic field equation, the effect of such density fluctuations on the spectral (temporal) and angular (spatial) properties of a laser beam was addressed. It is found that, after having propagated through a certain distance $\Lambda_r$, the temporal and spatial coherences of the transmitted light are reduced. The length $\Lambda_r$ depends essentially on the amplitude of the fluctuations and on their characteristic scales. These density perturbations can be excited by an external source or by the laser beam itself. In the latter case, they are driven by the ponderomotive force and the inhomogeneous heating associated with the random distribution of hot spots. Their average properties are derived from the analysis of the correlation function. The thermal effects are shown to play a non-negligible role in the excitation of density perturbations for conditions of recent experiments.

The set of coupled equations for the electric field and plasma density correlation functions shows, within a perturbation approach, the characteristic features of the transmitted light spectral and angular broadening. The multiple scattering requires a certain distance to affect the laser coherence, which depends essentially on the speckle Rayleigh length and the average power in a speckle normalized to the critical power for filamentation. The analytical model shows a symmetric frequency broadening of the transmitted light without strong angular spreading.

Three-dimensional numerical simulations using the interaction code PARAX have been performed and compared to
predictions of the statistical model. A spatially incoherent (RPP) beam with an average intensity well below the filamentation threshold was propagated through a low-density millimetric size plasma. Appropriate diagnostics have been used to analyze the smoothing effect. The spectral broadening distance is in good agreement with the model predictions. The coherence time of the transmitted light is reduced to the ion acoustic time \( \rho_{01}/c_s \) and angular beam divergence is enhanced less than twice.

Numerical simulations show that the spectral broadening of the scattered light is associated with a redshift, which was not predicted by the model, and which is characteristic for FSBS. Multiple scattering on RPP-driven density fluctuations occurs in the first speckle length and serves as a strong seed for FSBS that grows along the whole interaction length. The reduced coherence time of the scattered light is responsible for a continuous excitation of the density perturbations that increases as the square root of time. As a consequence, the smoothing effect is enhanced and its efficiency increases.

The redshift in the frequency spectrum of the transmitted light is associated to deviation from the Gaussian statistics for the electric field distribution. This deviation is due to the fast nonlinearity of the plasma. The intensity-density correlations that occur when the RPP laser beam is not already smoothed are not accounted for in the model and are responsible for the departure from the Gaussian statistics. When the temporal coherence of the beam is broken, these correlations are reduced and the Gaussian statistics is restored. This corresponds to the case in which the coherence time of the laser light is of the order of the nonlinearity time of response.

The use of low-intensity lasers smoothed by only the RPP enables to avoid deleterious effects of filamentation instability and to obtain an efficient smoothing in the same time. The temporal coherence properties are modified while the angular beam spreading stays almost the same. Moreover, RPP-driven density fluctuations serve as a much stronger seed for FSBS than thermal noise for backward scattering instabilities that grow in the more intense hot-spots. That is, plasma induced laser beam smoothing can operate under conditions where backscattering instabilities are not yet excited. This low-intensity regime of interaction appears as an interesting possibility to control the laser coherence properties in the ICF context. Parameters chosen in this article, \( \Lambda_{\rho} \approx 0.3 \text{ MW} \), are characteristic for current laser facilities and for the plateau stage of standard ICF pulses.

ACKNOWLEDGMENTS

One of the authors (M.G.) would like to thank Claude Gouédard for fruitful discussions on the statistical properties of partially incoherent laser beams. Usage of the computing center Centre de Calcul Recherche et Technologie (CCRT) of the CEA is acknowledged.

APPENDIX A: DERIVATION OF THE EQUATION FOR THE ELECTRIC FIELD CORRELATION FUNCTION

Let us consider a correlation of electric fields at two positions and at two instants \((r_1,t_1)\) and \((r_2,t_2)\) denoted as + and −, respectively. The equation for the product of two fields \(E_+(z)E_-^*(z)\) follows from Eq. (2):

\[
2ik_0\beta_z - \frac{\omega_{p0}^2}{c^2}[\delta n^+(z) - \delta n^-(z)]\frac{E_+(z)E_-^*(z)}{} = (\Delta^+_{\rho} - \Delta^-_{\rho})E_+(z)E_-^*(z).
\]  

(A1)

By considering the right-hand side as a known function, the formal solution to Eq. (A1) reads...
It is now assumed that the longitudinal coherence length of density fluctuations $L_C$ is much shorter than the propagation length, $z\gg L_C$, and that the electric field in the $z_1$ plane is independent of density fluctuations in the plane $z_2>z_1$. Thus, according to the integration limits in Eq. (A2), one can decouple the electric field and density components when taking the average:

$$\langle E_+(z)E^*_+(z)\rangle = \langle E_+(0)E^*_+(0)\rangle \langle \phi(z) \rangle + \frac{i}{2k_0} \int_0^z dz' \left\{ \exp\left[-i\frac{\omega_0^2}{2k_0c^2}z' \right] \langle \delta n' \rangle \right\} (\Delta^+ - \Delta^-) \langle E_+(z')E^*_+(z') \rangle,$$

leading to

$$\langle E_+(z)E^*_+(z)\rangle = \langle E_+(0)E^*_+(0)\rangle \langle \phi(z) \rangle + \frac{i}{2k_0} \int_0^z dz' \left\{ \frac{\langle \phi(z') \rangle}{\langle \phi(z) \rangle} (\Delta^+ - \Delta^-) \right\} \langle E_+(z')E^*_+(z') \rangle,$$

The differential form of the previous equation reads

$$\left( \frac{\partial}{\partial z} - i \frac{\omega_0^2}{2k_0c^2} \right) \Gamma_{EE'} = \partial_z \ln Y(z) \Gamma_{EE'},$$

where

$$Y(z) = \exp\left[-i\frac{\omega_0^2}{2k_0c^2}z \right] \langle \delta n \rangle + \frac{R + \frac{\rho}{2}T + \frac{\tau}{2}z'}{2} \langle \delta n' \rangle - \frac{R - \frac{\rho}{2}T - \frac{\tau}{2}z'}{2} \langle \delta n' \rangle \right\}.$$

The propagation through density fluctuations is considered as a diffusion process over many randomly distributed density fluctuations, the diffusion approximation ensures, according to the central limit theorem, the integral $\int_0^z \delta n(z)dz$ should follow Gaussian statistics. As a consequence, the expression of $\ln Y(z)$ can be simplified to:

$$Y(z) = \exp\left[-i\frac{\omega_0^2}{8k_0^2c^2} \right] \int_0^z \left[ D_N(0,T = t_1,0) + D_N(0,T = t_2,0) \right] \langle E(k,z,t) \rangle,$$

Finally, using more relevant quantities $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and $\mathbf{p} = \mathbf{r}_1 - \mathbf{r}_2$, $T = (t_1 + t_2)/2$ and $\tau = t_1 - t_2$ and writing $\Delta^+ - \Delta^-$ as $2\nabla_{\mathbf{r}} \nabla_{\mathbf{p}}$, one obtains Eq. (5).

**APPENDIX B: THE INTERACTION CODE PARAX**

The code PARAX simulates the propagation of an electromagnetic wave through an underdense plasma within the paraxial approximation. The equation for the slowly varying envelope of the laser electric field is, in Fourier space for the transverse coordinate:

$$\left[ \partial_t + \frac{\omega}{c} \kappa_{IB} + \frac{i}{2k_0} \mathbf{c}^2 - i\frac{\omega_0^2}{2k_0c^2} \langle \delta n(k,z,t) \rangle - 1 \right] \langle E(k,z,t) \rangle = 0,$$

where $\kappa_{IB} = \nu_{ei} n_i n_e$ is the electromagnetic wave damping due to electron-ion collisions. In simulations presented here, the plasma density and charge state are low so that the absorption by inverse bremsstrahlung is negligible. Equation (B1) generalizes Eq. (2) and is valid for $L_R \ll c/\tau$.

The plasma density perturbations induced by the laser ponderomotive force and thermal effects are treated within an ion-acoustic wave model. In the absence of a transverse plasma flow, it reads

$$\left( \sigma_d^2 + 2\gamma_d \sigma_d + \frac{\zeta^2}{2} \right) \ln\langle \delta n(k,z,t) \rangle = \frac{\epsilon_0}{2n_e T_e} A_k \int d\mathbf{k}' E(k',z,t)E^*(k'-\mathbf{k},z,t).$$

$\gamma_d$ is the ion acoustic wave damping and accounts for the Landau damping due to both electrons and ions and for the ion-ion collision effects. The $A_k$ operator is derived from Brantov et al. and reads

$$A_k = \frac{1}{2} + \frac{0.88Z^{5/7}}{(k|\lambda_{ei}|)^{4/7}} + \frac{2.54Z}{1 + 5.5(k|\lambda_{ei}|)^2}.$$

This equation is similar to Eq. (13) except for the logarithm on the left-hand side, which allows to saturate the density depressions and to ensure the positivity of the density in transient regimes. As this paper is interested in the low-intensity regime, this precaution is not necessary.

---


