Self-recompression of laser filaments exiting a gas cell

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A complete experiment of pulse compression mediated by femtosecond filaments is numerically simulated. Spatiotemporal distortions experienced by a filament formed in an argon cell with subsequent propagation inside the exit glass window and an air path are examined. Millimeter propagation ranges in silica preserve the spatial shape of the filament. However, the pulse duration strongly broadens due to the combination of high dispersion in glass and Kerr nonlinearity. We show that, despite this harmful broadening, on-axis pulse compression can be restored in the final stage of atmospheric propagation and can even be reestablished in vacuum. The key process for refocusing the beam is the introduction of an intensity-dependent spatial curvature in the pulse phase accumulated inside the silica window.

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I. INTRODUCTION

In the mid-90s, first experiments on the long-range propagation of femtosecond (fs) laser pulses were performed [1]. Intense infrared pulses with a duration of about 100 fs were launched in the atmosphere, where they produced narrow filaments, extending several meters along the optical path and localizing more than 10% of the energy in the near-axis area. Similar results were later reported upon smaller length scales for dense media such as silica [2,3] or water [4,5]. Filamentation results from the initial self-focusing of light, which originates from the nonlinear Kerr response of the medium and leads to an increase in the laser intensity. This intensity growth is then saturated by the defocusing action of the electron plasma created by photoionization of the ambient atoms. Alternatively, it can be halted by chromatic dispersion, e.g., in water, depending on the pulse peak power. Understanding the complex dynamics of ultrashort filaments is crucial for potential applications such as supercontinuum generation, pulse compression, generation of high-order harmonics, remote sensing or even material processing [6–9].

Among the previous applications, pulse compression is one of the most challenging topics in nonlinear optics. Considerable progress has been reported by many authors in this field [10–16]. It is nowadays possible to use filaments to compress pulses down to the single cycle limit, potentially without any postcompression techniques. For evidencing this “self-compression,” experiments currently employ cells filled with noble gas at pressures controlling the ratio of input power over the critical power for self-focusing. However, such cells are terminated by silica windows oriented at Brewster’s angle, and furthermore diagnostics are typically placed several tens of centimeters, even meters, away from the cell. Therefore, besides propagation in its original medium, a filament undergoes two additional propagation stages that have been neglected in almost all numerical simulations so far. Until recently [17], neither crossing the silica window nor further propagation in air was previously devoted much attention. In the following, we will show that these seemingly unimportant propagation steps display rather rich dynamics, which can actually explain the remarkable robustness of the self-compression mechanism observed in experiments.

Several issues concerning the filament dynamics through the gas-window interface remain unsolved. First, the pulse entering the glass sample is subject to a nonlinear Kerr index being 1000 times higher than in the gas. Displaying several thousands of critical powers, the beam crossing the exit window should thus immediately break up into smaller spots by modulational instability [18–21]. Second, the much stronger dispersion of glass is highly detrimental to temporal compression, as group-velocity dispersion on the order of several hundreds of fs2/cm can easily double the duration of sub-10 fs pulses over very short (mm) distances in glass [22].

For a clarification of these issues, we perform numerical simulations through all nonlinear stages undergone by the pulse in a real experiment. These simulations successively describe compression in a gas cell, propagation through the cell exit window, and final propagation in air before the pulse reaches the diagnostics. By doing so, we show that the temporal pulse profiles are severely broadened by a factor much larger than 2 while they propagate in the glass window over distances as short as 0.5 mm. In the dielectric medium, dispersion acts jointly with the Kerr response to strongly stretch the pulse in time. Surprisingly, output filaments recompress in air and restore temporal profiles with few-cycle durations [17]. This amazing phenomenon also takes place when the pulse exiting the window propagates in vacuum. The purpose of this work is to investigate this “self-restoration” mechanism in detail. We will show that self-restoration follows from the nonlinear spatial phase accumulated inside the glass window, which then refocuses the beam into the last propagation medium. Behind the focus, pulse time slices with large waist diffract more slowly than those with smaller waist, which leads to on-axis recompression.

The paper is organized as follows. In Sec. II we discuss the model equations. Section III is devoted to the filament evolution along the three characteristic stages of the gas-cell experiment, with particular emphasis on the gas-glass-gas interfaces. The intrapulse dynamics is discussed together with spectra and related phases. By means of a two-scale
variational approach, in Sec. IV, we clarify the mechanism by which the pulse accumulates a nonlinear spatially dependent phase shift in the window. This phase buildup is source of further refocusing and recompression in the second gas. Section V concludes this work.

II. PROPAGATION MODEL

We assume a linearly polarized electric field $\mathcal{E} e^{i k_0 z - i \omega_0 t} + c.c.$, where $\mathcal{E}(x,y,z,t)$ denotes its envelope, normalized such that $I = |\mathcal{E}|^2$ is the pulse intensity. The wave number in the medium $k_0 = n_0 k_0$ involves the linear refractive index $n_0 = n(o_0)$ at central frequency $o_0$. Following the standard model for pulse propagation, this envelope is coupled to the free electron density $\rho(x,y,z,t)$ created by photoionization, through an extended nonlinear Schrödinger equation [7].

$$\frac{\partial \mathcal{E}}{\partial t} = \frac{i}{2 k_0} \nabla^2 \mathcal{E} + i D \mathcal{E} - \frac{k_0}{2 n_0^2 \rho c} \mathcal{E} - \frac{\sigma}{2} \mathcal{E} + \frac{U_j W(I) (\rho_{in} - \rho)}{2 l} \mathcal{E} + i \frac{\delta o_0}{c} n_2 T \int \mathcal{R}(t - t') |\mathcal{E}(t')|^2 dt' \mathcal{E},$$

$$\mathcal{R}(t) = (1 - g) \delta(t) + g \theta(t) \frac{1 + \omega_0^2 \tau_R^2}{\omega_R^2 \tau_R^2} e^{-i \omega_R t} \sin(\omega_R t),$$

$$\rho = W(I) (\rho_{in} - \rho) + \alpha \rho_l / (1 - \rho / \tau_{rec}),$$

where $\mathcal{E}$ is the propagation variable and $t$ is the retarded time of the pulse in a frame moving at group velocity. The operator $D = \sum_{n \geq 2} (k^{(n)} / n!) (i \partial_t)^n$ accounts for dispersion formally involving the derivatives $k^{(n)} = \partial^2 k / \partial o^n |_{o_0}$. It includes the coefficient $k^{(2)} = k''$ for group-velocity dispersion (GVD). The operator $T = 1 + (i / o_0) \mathcal{D}$ corrects the slowly varying envelope approximation and is responsible for space-time focusing and self-steepening [23]. $n_2$ denotes the nonlinear Kerr index and $\rho_c$ the critical plasma density. The nonlinear Kerr response contains only an instantaneous contribution $\mathcal{R}(t) = \delta(t)$ when the pulse propagates in argon. It also includes a Raman-delayed contribution with ratio $g = 0.15$ in silica [24] and $g = 0.5$ in air [7]. Dispersion curves follow Dalgarno and Kingston’s [25] refractive indices $n(o)$ for argon and that of Peck and Reeder [26] for air. Exemplary values for the propagation of the glass windows are taken from Ref. [22]. Plasma generation is modeled using the Perelomov, Popov, and Terent’ev (PPT) ionization rate $W(I)$ [27] in argon and air. Alternatively, the Keldysh rate for crystals is employed to model ionization in glass [28]. Besides the GVD coefficient and the constituents of the overall Kerr response Eq. (2), other physical parameters such as the initial density of neutral species $\rho_{in}$, the avalanche cross sections $\sigma$, ionization potentials $U_i$ and electron recombination times $\tau_{rec}$ have been recalled in Table I. Argon is assumed to have a uniform pressure of 0.5 bar, leading to $n_2 = 30 \times 10^{-20}$ cm$^2$/W at 800 nm. The Kerr index for glass was chosen as $n_2 = 3.2 \times 10^{-16}$ cm$^2$/W, and $n_2 = 2.5 \times 10^{-19}$ cm$^2$/W in air at ambient pressure. In this regard we tested smaller values of $n_2$ in glass ($\sim 2.2 \times 10^{-16}$ cm$^2$/W) that may be more appropriate for pure silica at 800 nm, but found only minor quantitative differences in the results. Concerning the Kerr coefficient of air, we note that the selected value is widely consistent with the one used in Refs. [29,30]. Given that the ionization potential of N$_2$ is higher than that of O$_2$ molecules, we only consider the latter species responsible for generating an electron plasma. Several ionization models (PPT with ion charge equal to 1 or 0.53 [31]), optionally supplemented by higher-order (quintic) optical nonlinearities, were checked to yield behaviors comparable with those described below.

The propagation model constituted by Eqs. (1)–(3) will be integrated numerically by using Gaussian pulses, initially focused at the entrance $z = z_{in}$ of the gas cell.

$$\mathcal{E}(r,z = z_{in},t) = \sqrt{\frac{2 P_{in}}{\pi w_0^2}} r^{-1} w_0 f^{-1} \left| e^{-i \omega r^2 / 2 \sigma} F e^{-i \omega r^2 / \sigma^2} \right|,$$

with input power $P_{in}$, beam waist $w_0$, $1/e^2$ pulse half-width $\sigma$, and focal length $f$ in vacuum. Here $F$ denotes Fourier transform in time. This modeling of a lens should be applied for a field envelope subject to space-time focusing ($T = \nabla^2 \mathcal{E}$). It makes the pulse frequency $\omega$ affect the initial parabolic phase curvature introduced by the lens. Equation (4) ensures that all frequency components of the pulse focus at $z = z_{in} + f$, provided that the lens is situated in vacuum. Describing a focusing lens effect by simply multiplying the incident envelope by $\exp(-i \omega r^2 / 2 \sigma^2)$ and including space-time focusing in the propagation equation introduces an artificial lens error that stretches the pulse in the focal region.

Numerical simulations have been performed in full three-dimensional (3D) geometries ($\nabla^2 = \nabla_{\perp}^2 + \nabla_{\parallel}^2$) and radially symmetric ($\nabla_{\perp}^2 = r^{-1} \partial_r r \partial_r$) geometries. We shall mainly present results from radial simulations, except in the section discussing inclusion of multifilamentation in the window. The starting pulse is focused by a lens with $f = 50$ cm into a cell of argon with maximum length of 1.5 m. The input power $P_{in}$ is equal to one critical power $P_{cr}$ in argon, where $P_{cr} = \lambda_0^2 / 2 \pi n_2 \rho_c$. The input beam waist is $w_0 = 500$ μm, and the

<table>
<thead>
<tr>
<th>Parameters vs medium</th>
<th>Ar</th>
<th>Silica</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k''$ (fs$^2$/cm)</td>
<td>0.1</td>
<td>370</td>
<td>0.21</td>
</tr>
<tr>
<td>$n_2$ (10$^{-19}$ cm$^2$/W)</td>
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<td>3200</td>
<td>2.5</td>
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<tr>
<td>$P_{cr}$ (GW)</td>
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<td>0.002</td>
<td>4.1</td>
</tr>
<tr>
<td>$g$</td>
<td>0</td>
<td>0.15</td>
<td>0.5</td>
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<tr>
<td>$\tau_R$ (fs)</td>
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<td>76.9</td>
<td></td>
</tr>
<tr>
<td>$\omega_R^2$ (fs)</td>
<td>11.9</td>
<td>62.5</td>
<td></td>
</tr>
<tr>
<td>$\rho_{in}$ (10$^{19}$ cm$^{-3}$)</td>
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<td>2100</td>
<td>0.54</td>
</tr>
<tr>
<td>$U_j$ (eV)</td>
<td>16</td>
<td>7.8</td>
<td>12.1</td>
</tr>
<tr>
<td>$\sigma$ (10$^{-20}$ cm$^2$)</td>
<td>5.026</td>
<td>57.13</td>
<td>5.457</td>
</tr>
<tr>
<td>$\tau_{rec}$ (fs)</td>
<td>$\infty$</td>
<td>150</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

TABLE I. Physical parameters at 800 nm for argon at 0.5 bar pressure, glass, and air at 1 bar pressure. The critical plasma density amounts to $\rho_{cr} = 1.11 \times 10^{21} / \lambda_0^2$ μm$^{-3}$. The reduced (electron-hole) mass of silica is equal to 0.635.
initial pulse extent is $t_p=25$ fs. Radially symmetric simulations used $6144 \times 6144$ points in $(r,z)$, yielding respective resolutions of $2.44 \, \mu m$ and $0.25$ fs for an adaptive step size along $z$.

III. PULSE REFOCUSING OUTSIDE THE NONLINEAR SAMPLE

Recently, a couple of papers [15,32] identified that maximum shortening in time occurs when the filament starts to diffract, after the shortest wavelength has been attained in the supercontinuum. Here we consider the same self-compression scheme. We initiate filamentation in argon, in such a way that a minimum full width at half maximum (FWHM) duration is attained by self-compression of the filament after the focus point. Special attention will next be given to the filament dynamics in the silica window and in air.

For the sake of clarity, numerical results referring to the argon cell initiating the filamentation will be plotted with thin red curves, while those corresponding to the transient propagation in glass will be plotted in black. Results describing propagation in air will be plotted with thick blue curves. Because we mainly examine the pulse dynamics inside the cell and beyond, the coordinate $z=0$ of the pulse will be defined at the inner face of the output window. Coordinates locating the preceding propagation stage inside the cell will thus be counted negatively from this origin.

A. Pulse dynamics through the Ar-glass-air interfaces

In argon, the input pulse only conveys one critical power with an initial intensity about one decade below the clamping value. Its subsequent dynamics is thus to increase in amplitude until being saturated by plasma generation. Initially close to the clamping value fixed by the balance between Kerr response and photoionization, the pulse slowly diffracts beyond the focus. Here, the filament core keeps a narrow shape in space (FWHM radius <200 $\mu m$) and attains its shortest duration in time. Typically, this scenario follows a “light bullet” dynamics [32], characterized by a primary focusing sequence followed by a limited refocusing and eventually a long, slow diffraction stage. Examples are given by the thin red curves in Figs. 1(a) and 1(b). The pulse is analyzed after propagating over two different distances in the cell, namely, 1 m [Fig. 1(a)], where the peak intensity attains $I_{\text{max}}=20$ TW/cm$^2$, and 1.5 m, where $I_{\text{max}}=3$ TW/cm$^2$ [Fig. 1(b)]. These two distances define the effective length of the argon cell, i.e., the position of the exit window located at the origin $z=0$. In the following, the pulse propagating over 1 m in argon will be referred to as pulse A and the pulse at larger distance as pulse B.

Let us next investigate the crossing of the silica window in some more detail. In Fig. 1, we chose a thickness of 0.5 mm for the glass sample, a typical value used in experiments [12]. The domain of the silica window is represented by black ellipses. The on-axis fluence in the argon cell ($F = \int |E|^2 \, dt$) reaches 0.2 J/cm$^2$ for pulse A and 0.05 J/cm$^2$ for pulse B. Following Ref. [33], only part of the forward pulse can be transmitted through a silica sample, depending on the initial pulse duration and maximum fluence. At high enough fluences >1.5 J/cm$^2$, the generated plasma is also expected to cause an important change in the refractive index and to filter the leading edge of the pulse, modifying thereby the pulse duration [34]. For the much weaker fluences reached here, we evaluate that 90% of pulse A and 100% of pulse B can go beyond the glass surface. Note that in this case intensities stay below the damage threshold of silica, i.e., $I_d \leq 2 \times 10^{13}$ W/cm$^2$ [35], and the temporal pulse profiles should be preserved when they impact the window.

Because diagnostics are usually located tens of centimeters away from the exit window, the beam finally propagates in the atmosphere. Blue curves in Figs. 1(a) and 1(b) show maximum intensities and peak electron densities as the pulses A and B exit the 0.5 mm thick window and are fully transmitted in air. Unexpectedly, the beam refocuses in air. Naively, this behavior could be attributed to self-focusing. Indeed, pulses A and B contain time slices with maximum powers of $\sim 3.5 \, P_{cr}$ in air, such that they are potentially able to self-focus, even if pulse B involves weak intensities. In this respect, we checked that collapse is not arrested by plasma generation, but instead by Raman scattering, which efficiently inhibits the wave blowup. However, we will show that refocusing systematically takes place even if the third medium is replaced by vacuum. Hence, this phenomenon has a different origin than pure Kerr self-focusing.

For deeper insight, Fig. 2 shows the filament intensity and associated plasma density inside the silica window for both pulses A and B. Over the first few millimeters, the intensity rapidly drops. This follows from the prevailing action of GVD in glass, which is the key player acting against self-

FIG. 1. (Color online) Peak intensities (solid curves, left-hand side axis) and electron densities (dashed curves, right-hand side axis) for a 500 $\mu m$-waisted Gaussian pulse with $t_p=25$ fs focused in (a) a 1-m-long argon cell (pulse A) and (b) a 1.5-m-long argon cell (pulse B) at 0.5 bar pressure. The thin red curves refer to the propagation stage in argon. The black ellipses indicate the position of a 0.5-mm-long zone of the silica window detailed in Fig. 2. The thick blue curves represent refocusing in air. Distances in the argon cell are counted negatively with respect to the origin $z=0$ locating the inner face of the exit window.
focusing (see Sec. IV). In pulse A configuration, the beam enters the window with a mean waist of 240 μm and exhibits time slices with maximum power of ~7100 \( P_{cr} \); here \( P_{cr} = 2.19 \) MW corresponds to the critical power in glass. Pulse B starts with a lower intensity and disperses less over the same distance in the window. The maximum input power, however, is still about 8000 \( P_{cr} \); the beam waist entering the sample is ~0.6 mm. Note that Fig. 2 shows propagation in silica over 1.5 mm, although in Fig. 1 we assume a window of 0.5 mm only.

Figures 3(a) and 3(b) describe the evolutions of the on-axis temporal profiles of pulses A and B, respectively. After 1 m long propagation in the argon cell, the on-axis pulse profile has contracted to a FWHM duration of ~7.8 fs. After 0.5 mm propagation in silica, the same pulse evolves into a wider structure of ~33 fs FWHM duration, forming a pedestal that extends far into the trailing region. Once exited from the window, the pulse refocuses in air, where it attains a 10.3 fs FWHM duration at \( z = 0.25 \) m. This duration is relatively stable, sustaining values of ~10.6 fs at \( z = 0.5 \) m and ~10.8 fs at \( z = 1 \) m (not shown). For comparison, pulse B initially exhibits a temporal extent of 12.7 fs at the argon-glass interface. Subsequently, it broadens to 27.8 fs with 0.5 mm passage through glass. In the atmosphere, the pulse eventually refocuses and recompresses. Typical durations are 16.6 fs at \( z = 0.5 \) m and 18.6 fs at \( z = 1 \) m.

Ending this subsection, Fig. 4 summarizes the history of the on-axis spectra and spectral phases around the pump frequency (\( \omega_0 = 2.35 \) THz) along the different propagation steps. Figures 4(a) and 4(b) describe pulse A, passing through the glass window and being transmitted to air. The first focusing event in argon creates redshifted and blue-shifted spectral wings caused by self-phase modulation (thin red curve). GVD subsequently dominates the propagation in the dielectric, which lowers the saturation intensities and weakens self-phase modulation, so that the blueshifted wing drops out (thick blue curve). Once in air, the pulse refocuses, which enhances the blue wing again (black curve). This blueshift is less pronounced than in Ar, as peak intensities are smaller. In the spectral phase, oscillations appear on the red side during the air stage. These local phase fluctuations cannot be exploited to recompress significantly filaments by means of linear compression techniques. Figures 4(c) and 4(d) provide the same pieces of information for pulse B. With weaker intensities, the spectral history basically follows that of pulse A, although the properties expected from linear dispersion become more important and plasma has no influence. Consequently, on-axis spectra remain closer to those exiting argon, and the phase keeps a positive curvature corresponding to normal dispersion in all three propagation media.

B. Generality of the “self-restoration” process

To test the generality of the recompression process, we enlarged the thickness of the glass window up to 1.5 mm for, e.g., pulse A. Results are shown in Fig. 5. Generally, the larger the propagation distance in glass, the stronger one expects GVD to induce temporal broadening of the pulses. This trend is, however, only partly true. For instance, pulse A’s FWHM duration attains 51 fs over a 1 mm long path in glass, but reduces to 33.2 fs over 1.5 mm. Such variations depend on the effective interplay between the Kerr response of glass and GVD at a given propagation distance [see, e.g., the slight intensity increase in pulse A around \( z = 1 \) mm in Fig. 2(a)]. Moreover, we have to consider the particular definition of the FWHM duration. For complicated pulse shapes exhibiting...
modulations, as those shown in Fig. 5, FWHM durations become of limited significance.

Despite these differences, pulses always tend to recompress in air. FWHM durations indeed reduce from 51 to 42.5 fs after traversing a 1 mm thick window [Fig. 5(a)], and from 33.2 fs down to 18.6 fs for a 1.5 mm thick window [Fig. 5(b)]. Note that, although the pulse refocuses and recompresses in the former case, the temporal distribution develops several peaks making the pulse duration exceed the initial one ($t_p=25$ fs, $t_{\text{FWHM}} \sim 30$ fs). Thus, the efficiency of the self-restoration mechanism may vary from one experimental setup to another one.

More surprising results are revealed by Fig. 6. Here, instead of air, we simulated the final gaseous medium by an argon cell with varying pressure. Corresponding curves are plotted in black. Increasing the pressure up to 3 bar produces double-peaked temporal profiles (not shown), as high power ratios ($P_{\text{in}} = 3.8 \ P_c$) and no Raman response cause significant plasma generation that splits up the pulse. High pressures may thus weaken the efficiency of the self-restoration process. We also decreased the pressure to $10^{-5}$ bar, practically a perfect vacuum (dashed curve). While pressures $p \gtrsim 1$ bar guarantee a local peak power above the critical power for self-focusing in argon, beam propagation becomes definitively subcritical at pressures $p < 0.5$ bar. Nevertheless, we still observe refocusing in these extreme situations. Even though Kerr self-focusing does no longer contribute to recompress the pulses, the evolution patterns in the $(t,z)$ plane are persistent at subcritical powers [Figs. 6(b) and 6(c)], and refocusing takes place at distances comparable with Fig. 1. The beam indeed forms a focus at $z \approx 2.5$ cm for pulse A and contracts its temporal distribution down to 12.7 fs at $z=0.25$ m, even in vacuum. Hence, the observed self-restoration process does not depend on the Kerr response of the second gas. We will further see that the phase accumulated by the pulse through the glass window actually causes this refocusing event.

IV. INTRAPULSE DYNAMICS IN THE GLASS WINDOW

We first address the point of multiple filamentation in glass and therefore focus on 3D simulation results of pulse A. These were shown to provide propagation patterns similar to radially symmetric computations in an earlier publication [17]. Next, we shall apply a variational principle to a reduced model capturing the key players of the intrapulse dynamics inside the window. This model will reveal phase anomalies
accumulated along the window path and triggering the self-
restoration mechanism.

A. Inhibiting multiple filamentation

An interesting feature of the propagation dynamics is the total absence of multifilamentation (MF) in the silica win-
dow. Despite the very high peak powers ($P_{\text{max}}/P_{\text{cr}}>7000$) encountered in the dielectric, three-dimensional numerical simulations displayed no amplification of perturbations that could have led to a beam breakup. Estimates from modula-
tional instability theory [19], however, predict an onset of MF and subsequent collapse upon very short distances in the order of

$$\Delta z_{\text{MF}} \approx \frac{n_0 P_{\text{cr}}}{\lambda_{\text{max}}} \sim 0.1 \ \text{mm},$$

for peak intensities $I_{\text{max}} \sim 3 \ \text{TW/cm}^2$ impinging on the glass surface. Nevertheless, neither MF nor beam collapse occur over distances $\approx 4.5 \ \text{mm}$, beyond which pulse A was seen to refocus and blow up. We performed several simulations, both in 3D and in radial geometries, in order to sort out the key player arresting the collapse inside the glass window. First, plasma generation here plays only a negligible role as the nonlinear refractive index

$$\Delta n_{\text{NL}} = n_{\text{max}} - \frac{2 I_{\text{max}}}{n_0 P_{\text{cr}}} \sim 1 - 5 \times 10^{-3}$$

remains small and positive at maximum intensity. This finding indicates that Kerr nonlinearities always prevail along the beam propagation. We also numerically checked that dispersion orders higher than 2, pulse steepening, and Raman scattering were of minor influence. More precisely, while suppression of high-order dispersion and of the Raman re-
sponse for pulse A does not visibly change the propagation pattern, setting $T=1$ (i.e., no steepening terms) slightly ac-
celerates the beam collapse over several millimeters in glass. In contrast, removing all dispersion orders, including GVD, causes an early refocusing of the beam around $z \approx 0.7 \ \text{mm}$. Hence, the only competitor for the Kerr self-focusing ap-
pears to be GVD, which rapidly damps the peak intensity and prevents the amplification of periodic spatial modula-
tions.

Figures 7 and 8 actually demonstrate that temporal disper-
sion is capable of arresting the growth of the modulational instability, which would break the homogeneity of the beam distribution otherwise. Numerical noise is sufficient for trig-
-gering multiple filaments at peak powers above 7000 $P_{\text{cr}}$.

Figure 7 displays evidence of the emergence of local modula-
tions on top of the maximum field distribution. These modulations develop in the $x,t$ plane from $z=0.5 \ \text{mm}$ on-

FIG. 6. (Color online) Pulse A propagating in
a second argon cell after exiting a 0.5-mm-thick
window. (a) Peak intensity at 1 bar pressure ($P_{\text{max}}/P_{\text{cr}}=1.25$, solid curve) and vacuum (dashed curve). On-axis temporal evolutions when the last medium is (b) a 1 bar argon cell and (c) vacuum. FWHM durations evaluated at $z =0.25 \ \text{m}$ are 10.3 fs and 12.7 fs, respectively.

FIG. 7. (Color online) Field distribution of pulse A in the ($x,t$) plane of the glass window, computed from a three-dimensional code discarding chromatic dispersion and steepening operators ($T \to 1$). Results are shown at the distances (a) $z=0.5 \ \text{mm}$ and (b) $z =0.75 \ \text{mm}$.
ward whenever dispersion and pulse steepening are artificially switched off. After further 0.25 mm propagation in glass they are amplified in the x direction and form regularly spaced, small-scaled filaments causing the beam collapse just afterward. Their amplification length remains on the order of 0.1 mm, in agreement with the theoretical expectation Eq. (5). In contrast, Fig. 8 shows the field distribution when all derivatives in time are kept. No beam breakup takes place upon even longer ranges (z=0.8 mm). GVD, which is the key player in chromatic dispersion, broadens the pulse along time and maintains the intensity to levels for which modulations cannot be amplified, which, in turn, prevents MF and beam collapse.

B. Variational approach

This subsection aims at determining the changes in the pulse amplitude and phase, which are responsible for the self-restoration mechanism. For convenience, we limit this analysis to second-order dispersion. Ignoring plasma nonlinearities and dispersion orders n>2 and letting T→1, we introduce the rescaled variables z→4z0,c0, r→rμ0, t→τp, and the field E→√Pcr/4πμ0w0^2ψ, from which Eq. (1) simply reduces to the nonlinear Schrödinger equation

\[ i\partial_t\psi + r^2\partial_r^2\psi - \partial_r\psi + |\psi|^2\psi = 0, \]

where \( \delta = 2z_0k^*/t_p^2 \) depends on both the dispersion length \( \tau_p^2/k^* \) and the Rayleigh distance \( z_0 = \mu_0\omega_0^2/\lambda_0 \) of the beam impacting the window entrance. Functional relations describing the mean-square spread of the pulse in space and time can then be derived in the form [36]

\[ d_z^2 \int r^2|\psi|^2 d\tilde{r} dt = 8 \int |\nabla_r|\psi|^2 d\tilde{r} dt - 4 \int |\psi|^4 d\tilde{r} dt, \]

\[ d_z^2 \int r^2|\psi|^2 d\tilde{r} dt = 4\delta \left[ 2\delta \int |\partial_r\psi|^2 d\tilde{r} dt + \frac{1}{2} \int |\psi|^4 d\tilde{r} dt \right], \]

the solutions of which are approximated by the self-similar substitution

\[ \psi = R_z x^2/4R_z + T_z r^2/4\delta T_z. \]

Plugging Eqs. (10) and (11) into Eqs. (8) and (9) yields the dynamical system for the transverse and temporal pulse scales \( R_z = R(z) \) and \( T_z = T(z) \):

\[ \frac{1}{4} R_z^2 \partial_z R_z = 1 - \frac{p}{2T_z}, \quad \frac{1}{4} T_z^2 \partial_z T_z = \delta + \frac{T_p}{2R_z^2}. \]

FWHM durations are given by \( \tau(z) = 2\sqrt{2}\tau(z) \) and \( p = P_{\text{cr}}/P_{\text{cr}} \). By comparing the initial condition (4) and Eq. (10), \( R(z) \) and \( T(z) \) are fixed at the entrance of the glass window as \( R(0) = T(0) = 1/\sqrt{2} \). Their derivatives express as \( R, T = -\sqrt{2}\omega_0\omega_0^2/cf \), where \( f \) is the focal length of the spatial phase curvature in vacuum, and \( T_r(0) = 2\sqrt{2}\delta C \) may serve to introduce a chirp with coefficient \( C \).

Reciprocally, the pulse ansatz (10) serves to evaluate a focal length or a chirp occurring at the window exit. In the present context, we employ the averaged waist of either pulse A or B when they enter the piece of glass. This is accomplished through a Gaussian fit of the radial field distribution associated with the most powerful time slice of the pulse. We evaluated \( \omega_0 = 240 \mu m \) for pulse A with \( P_{\text{cr}} = 7100 \mu m \) c, while \( \omega_0 = 600 \mu m \) for pulse B having \( P_{\text{cr}} = 7985 \mu m \) c. The normalized GVD coefficient \( \delta \) then takes very large values, i.e., \( \delta_A = 554 \) and \( \delta_B = 1306 \), when using durations and waists of pulses A and B, respectively.

From the previous data, it turns out that both GVD and the Kerr response are responsible for the strong broadening of the pulse profile. Equations (12) illustrate this property. Since the growth of \( T(z) \) is driven by \( \delta \) (dispersion) and \( \delta \times p \) (Kerr response), beam collapse characterized by the vanishing \( R(z) \) at finite \( z \) can efficiently be delayed in the limits \( \delta, p \gg 1 \). In this case, the more \( T(z) \) increases, the less \( R(z) \) decreases.

Figure 9 shows the evolution of “variational” parameters for pulses A and B when they transmit through the silica window. In Fig. 9(a), the maximum intensity reproduces rather well the numerical curves shown in Fig. 2. Thick and thin curves refer to pulse A and B, respectively. In Figs. 9(b) and 9(c), the solid curves indicate the evolution of the physical waists and FWHM pulse durations computed from the variational model. The dashed curves show the evolutions of the dimensionless derivatives, \( R_z \) and \( T_z \).

Over 1.5 mm of propagation in glass, the waist of pulse A is kept almost constant. In contrast, its FWHM duration monotonously increases from ~8 fs to 25 fs at 0.5 mm and to ~50 fs at 1 mm, which agrees with the direct numerical simulations. Linear dispersion alone, yielding \( \tau_{\text{lin}}(z) = \tau_0/\sqrt{1+16(\ln 2)/k^*\Delta z/\tau_0^2} \) [22], would only allow for a temporal broadening limited to ~10 fs. For this pulse, temporal broadening is effectively more than double by the Kerr response. Hence, normal GVD and Kerr effect combine to cause significant pulse stretching in time. Simultaneously, \( R_z \) decreases to \( R_c = -14 \) and \( 18 \) at the same distances, \( z = 0.5 \) and 1 mm, respectively. Expressed in physical quantities, we find \( R_z/R_c = -2\omega_0\omega_0^2/cf_c \), where \( f_c \) denotes the equivalent lens associated with the newly formed spatially-quadratic phase \( R_z r^2/4R_z \). For pulse A, the focal length accumulated in a 0.5-mm-thick window yields \( f_c = 4.6 \) cm. With
a spatial phase varying parabolically in \(r\), the phase curvature is thus \(\frac{\partial^2 \varphi}{\partial r^2} = R_z \approx -172\, \text{mm}^{-2}\). Similar observations can be done for pulse B (thin curves in Fig. 9). Over 0.5 mm in glass, the pulse duration attains 16 fs. \(R_z = -3.6\) yields \(f_s \approx 111\, \text{cm}\), and the phase curvature \(\frac{\partial^2 \varphi}{\partial r^2} = -7.1\, \text{mm}^{-2}\). Despite the large values reached by \(T_z\), we observed that pulse chirping in time has no significant consequence on the location of the beam focus in the second gas if \(C \ll 10\). This condition holds for the present configurations.

Although discrepancies exist and even though they are more pronounced for pulse B, the values of \(f_s\) lie in the order of magnitude of the refocusing distances predicted by direct simulations, namely, \(z_{\text{ref}} = 2.5\, \text{cm}\) for pulse A and \(z_{\text{ref}} \approx 25–30\, \text{cm}\) for pulse B. Discrepancies originate from the fact that the Gaussian ansatz (10) cannot reproduce the complex space-time structure of the pulse. In particular, the beam chosen in the variational calculation may not correspond to that associated with the pulse time slice having either maximum power or maximum intensity.

To illustrate these causes of discrepancies, let us return to the direct numerical simulations. Figure 10 shows surface plots of phases in the \((r, t)\) plane corresponding to the numerical pulses A and B sampled at \(z = 0.5\, \text{mm}\) in the glass window. In pulse A configuration, the phase develops a parabolic shape with negative slope (\(\frac{\partial^2 \varphi}{\partial r^2} < 0\)) around \(r = 0\) in the trailing pulse \((t > 0)\). This negative slope is responsible for refocusing in the third propagation medium. Maximum intensity is located in the trailing pulse as well [\(I_{\text{int}} \approx 13\, \text{fs}\), see Fig. 3(a)]. Moreover, as we will see in Sec. V, maximum intensity and strongest phase curvature do not necessarily coincide at the same time slice. For comparison, the Gaussian ansatz (10) forces the phase curvature of the solution to Eq. (7) to be the same for all time slices, which is source of discrepancies in the predicted focal length. Also, the width of the numerical spatial profiles varies along time. In particular, profiles in the trailing pulse appear narrower than the Gaussian test function. In pulse B configuration, the phase develops a less pronounced curvature as expected, located again in the trailing pulse. We find similar differences between the variational and numerical profiles.

### V. RECOMPRESSION IN THE SECOND GAS

In Sec. IV we analyzed the phase accumulated when the pulse propagates through a glass window. This phase carries a spatial curvature that provides the pulse exiting the window with an effective finite focal length \(f_s\). In our variational model, the phase curvature results from the decrease in \(R_z\), which, however, barely affects the value of the beam waist over millimeter-long distances [see Fig. 9(b)]. The evolution of \(R_z\) is dictated by both the high GVD value and the large power ratio \(P_{\text{in}}/P_{\text{cr}} \gg 1\). However, reality is more complicated than our simple variational description. For instance, Fig. 10 suggests that the phase shape becomes more involved and may not be characterized by a unique focal length \(f_s\). Nevertheless, it still acts like a “focusing lens.” To illustrate the importance of the accumulated phase, Fig. 11 shows the peak intensities of pulses A and B when their phase has been artificially canceled before the beam starts to propagate into air or vacuum. In air, pulses A and B both exhibit supercritical powers \((P_{\text{in}}^A \approx 3.2\, P_{\text{cr}}, P_{\text{in}}^B \approx 3.7\, P_{\text{cr}})\), and they slightly refocus at long distances (20 and 60 cm, respectively), far beyond the refocusing points reported in Figs. 1(a) and 1(b). Their intensity growth is also limited by the Raman-delayed response, as recalled above. In vacuum, refocusing is absent,
and the pulse simply diffracts. For comparison, we also display the same quantity for the case that pulse A directly passes from an argon cell with 0.5 bar pressure into vacuum, without traversing the exit window. Hence, this pulse only accumulated phase in the argon cell, and there is no refocusing in this case either. These results thus imply that the nonlinear phase accumulated in the dielectric is the main mechanism forcing the beam to refocus in air.

Because the self-healing process also develops in vacuum, the field distribution emerging just after the exit window contains all necessary ingredients to produce a temporally short on-axis structure when further propagated. Accordingly, we find it worth analyzing the spatial spectral amplitude versus time (Fig. 12). These amplitudes are indeed related to the far field and do not change upon propagation, if the influence of the operator $T^{-1}$ in front of the diffraction term in Eq. (1) is negligible. We numerically checked that this assumption holds. Therefore, the far field provides direct mapping of the pulse time slices that will support on-axis shortening or not. In Fig. 12(a) we show this information for pulse A just after the exit window. A dominant short peak in the back of the pulse is clearly visible, compatible with Fig. 3(a) and Figs. 6(b) and 6(c). Following linear diffraction theory, beams with small extent in $k_\perp$ diffract slower than those with a large one. Therefore, we find a different radial power flow in each time slice. Together with the total power contained in each slice, this flow determines the evolution of the on-axis intensity profile. In the configuration of Fig. 12(b) with the canceled phase, extents in $k_\perp$ are not varying much along the pulse, and all time slices diffract at a similar rate. We find a temporally more extended structure around $k_\perp=0$, thus a long on-axis pulse duration in the far field. As a result, we can conclude that the accumulated phase plays a crucial role in the observed recompression mechanism. Finally, Fig. 12(c) shows the spatial spectral amplitude of pulse A before the exit window. In a setup where the pulse passes directly from argon into vacuum, Fig. 12(c) corresponds to the far field. Qualitatively, Figs. 12(a) and 12(c) look similar, which indicates that the strong perturbations introduced by the glass window become less pronounced upon long propagation distances. Hence, as far as the far field is concerned, the effect of the exit window is limited.

As indicated above, the phase curvature $\frac{\partial^2}{\partial r^2} \arg(E)$ in the numerical simulations is dependent on the time $t$. Figure 13 shows on-axis phase curvature and intensity profiles of pulse A vs time, at the distances $z=5$ and 25 cm, where the shortest FWHM duration is measured, in air as well as in vacuum. At the output of the silica window, we find a strong negative curvature decreasing with increasing $t$. This asymmetry with respect to the corresponding intensity profile is mainly due to

FIG. 13. (Color online) On-axis temporal profiles and phase curvatures [$\frac{\partial^2}{\partial r^2} \arg(E)$] along time for pulse A in air (thick blue curves) and in vacuum (black curves) at [(a) and (b)] $z=0$ (dashed curves), $z=5$ cm (solid curves) and [(c) and (d)] $z=25$ cm. Note the change of scales between (a) and (b), and (c) and (d).
the action of self-steepening in the window. However, the “overall” curvature leads to an effective focal length of 2.5 cm, which was already reported in Fig. 6. At $z=5$ cm, i.e., at about twice the refocusing distance, we find a positive phase curvature as expected from linear diffraction theory. With increasing $z$, this positive curvature decreases toward zero [see $z=25$ cm, Fig. 13(d)]. The observed on-axis recompression is again a spatiotemporal effect. Time slices that strongly refocus due to a more pronounced negative phase curvature diffract more rapidly after the focus.

For comparison with variational results and to confirm generality of our observations, we also simulated Gaussian pulses propagating over 0.5 mm silica and then into vacuum. Emphasis is here given to pulse A, which has been modeled by a Gaussian condition [Eq. (4) with $w_0=240$ $\mu$m, $t_p=6.8$ fs and no initial lens]. As seen from Fig. 14, the dynamics remains comparable with those shown in Fig. 13, apart from a shift of the on-axis profiles and phases by $\sim 15$ fs in the time domain. Despite the discrepancies introduced by the underlying assumptions of the variational methods, the range of phase curvatures issued from these computations are in rather good agreement with the variational estimates at the window exit, with $\partial^2 \varphi = -250$ $\text{mm}^{-2}$ at the instant $t=0$. In vacuum, durations of 20 fs are reached at $z=20$ cm. However, we observe that the pulse starts to split and that the front component diffracts faster than the rear one. At $z=80$ cm, a FWHM duration of 12.5 fs is measured [see Fig. 14(c)].

In experiments, the actual length of the gas cell determines the beam waist and intensity at the output window (compare, e.g., pulses A and B). Moreover, varying input pulse parameters or the setup geometry can reduce the efficiency of the self-compression mechanism, such that longer pulses pass through the exit window. In order to get an idea of what may happen under those circumstances, we choose an input Gaussian pulse at the exit window with either the beam waist doubled compared to its reference value of 240 $\mu$m or with a doubled duration, while keeping the input pulse energy constant. For the large beam waist, refocusing takes place at about 20 cm, beyond which the pulse can be recompressed down to 14 fs at $z=40$ cm. The longer effective focal length $f_e$ follows from both the larger beam waist and the lower intensity, in agreement with the discussion in Sec. IV B. When the pulse duration is doubled, self-restoration again occurs at larger distances along the optical path, which can be attributed to the lower intensity. Temporal broadening in the silica window is less pronounced due to a smaller value of $\delta$. Finally, we performed additional simulations starting with the same input intensity as pulse A while varying the beam waist and pulse duration. For wide waists, the same behaviors as above occur over longitudinal scales shifted by the squared ratio between the new and old initial waist values. The dynamics are identical along the axis $z/f_e$.

Examining longer pulses with same input intensity, we observed the emergence of small-scale periodic modulations, indicating that multiple filamentation may further start in the window. Subsequent propagation in vacuum also leads to recurrent formation of split pulses, which prevents an efficient pulse recompression.

**VI. CONCLUSION**

In conclusion, numerical simulations cleared up the dynamics of self-compressed filaments when they are produced in a noble gas, propagate over a few millimeters inside a silica window, and when they are released into air before they are being characterized after tens of centimeters distance. In the window, dispersion competes with Kerr self-focusing within a high-power regime along which multiple filamentation is inhibited and pulse durations significantly increase. Despite this large stretching in time, temporal profiles with durations close to 10 fs are finally restored when the pulse recompresses in the second gas. By changing the nature and the pressure of this second gas (air, argon with decreasing pressure), we highlighted that self-compression is mainly restored from a second refocusing event caused by...
the nonlinear spatial phase accumulated throughout the silica window. This important finding has been explained by a two-scaled variational approach that qualitatively supports the comparison with results from direct numerical computations. A parabolic spatial phase forms in the trailing pulse, which forces the latter to refocus over short distances in the consecutive medium. Through this mechanism the trailing pulse recompresses and can restore durations down to a few optical cycles. Although nonlinear self-focusing may enhance pulse recompression to some extent, the phase accumulation cycles. Although nonlinear self-focusing may enhance pulse recompression to some extent, the phase accumulation mechanism, however, is pinpointed as the key process being responsible for the self-restoration phenomenon. This phase accumulation is apparently unavoidable in nearly all gas cell setups employed for investigating filamentation in noble gases and pulse self-compression. More importantly, the action of the glass window and subsequent propagation allow for the self-healing of the pulse spatiotemporal distortions. Our investigations emphasize as being the mechanism behind the remarkably short pulse durations that were still seen meters behind the filament itself. Clearing up this often ignored final step in filamentary propagation therefore provides much better understanding of self-compression experiments.

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