Ultracold neutral plasmas and Rydberg gases

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Abstract

We present two different theoretical approaches for the simulation of ultracold plasmas and Rydberg gases, namely a simple kinetic model including an approximate treatment of ionic correlations on the one hand and a hybrid molecular dynamics (MD) approach treating the electrons as a fluid but ions and atoms on a full MD level on the other hand. The simple kinetic model can be used in situations where the number of particles is so large that it prohibits an MD simulation, and, maybe even more importantly, it gives additional insight into the dynamics beyond that possible on the basis of MD simulations by providing simple evolution equations for the macroscopic parameters describing the state of the system. On the other hand, the MD approach provides a powerful method for the study of spatially resolved quantities, and it permits the study of scenarios where the ions are so strongly coupled that Coulomb crystallization occurs, which cannot be described by the kinetic model.

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1. Introduction

During the last few years, experimental and theoretical investigations of ultracold ($T \ll 1$ K) neutral plasmas and ultracold Rydberg gases have revealed interesting and partly unexpected behavior. Depending on initial conditions such as particle number, density, electronic excitation (i.e. principal quantum number in the case of Rydberg gases, kinetic energy in the case of plasmas) etc., a very broad range of phenomena can be observed. The interpretation of these effects requires methods from different subfields of physics, and in turn the present studies provide new stimulus for the respective fields. Ultracold Rydberg gases, for example, show certain analogies with amorphous solids since the extremely low temperature of the atoms means that they are practically immobile on typical atomic-physics timescales. Consequently, these systems have been called “frozen gases” [1,2], and many-body effects such as the diffusion of electronic excitation through such a gas have been investigated. It was pointed out long ago [3] that a Mott-type metal-insulator transition familiar from solid-state physics might be realizable in such a cold Rydberg gas by varying the level of electronic excitation. Generally, the huge amount of control possible with state-of-the-art trapping and cooling techniques promises exciting experiments addressing many-body physics problems traditionally rooted in condensed-matter physics.

On the other hand, ultracold neutral plasmas may provide a convenient laboratory realization of so-called strongly coupled plasmas, where the Coulomb coupling parameter $\Gamma = e^2/(a k_B T)$, i.e. the ratio of electrostatic potential energy $e^2/a$ ($a$ is the Wigner-Seitz radius) to thermal kinetic energy $k_B T$, is larger than unity. In this case, interesting ordering effects up to Coulomb crystallization, which is predicted to occur for $\Gamma > 174$ [4], can be observed. Coulomb coupling parameters of this order are realized in some dense astrophysical objects such as neutron stars, or in one-component plasmas such as ions in ion traps or ion beams in storage rings. The creation of a cold neutral plasma by photoionizing atoms in a magneto-optical trap [5] seems to offer a unique opportunity for a laboratory study of neutral plasmas where, depending on the electronic kinetic energy which may be varied by tuning the frequency of the ionizing laser, either one component (namely the ions) or both components (ions and electrons) may be strongly coupled. Moreover, the plasma is created in a completely uncorrelated state, i.e. far away from thermodynamical equilibrium. The relaxation of a strongly correlated system towards equilibrium is an interesting topic in non-equilibrium thermodynamics, the more so as the “equilibrium” is constantly changing.
due to the expansion of the plasma cloud into the surrounding vacuum. Thus, the physics of ultracold neutral plasmas and Rydberg gases lies at the interface between condensed-matter physics, plasma physics, statistical physics and atomic physics, and a broad range of different aspects can be probed with different experimental setups. In the following, we will focus on the plasma physics perspective explored in the type of experiments described in [5].

The first experimental realization of an ultracold neutral plasma\(^1\) (UNP) has been reported in [5]. The plasma was created by photoionizing a cloud of Xe atoms laser-cooled to sub-Millikelvin temperature and collected in a magneto-optical trap. In a follow-up experiment [6], the measurement of the rate of expansion of the plasma cloud into the surrounding vacuum revealed a first surprise. At low electron kinetic energies, the expansion was found to be faster than predicted on the basis of energy conservation, assuming a complete transformation of thermal kinetic energy into a radially directed outward motion. At the time, the source of this additional energy was unknown. Soon afterwards, the close connection between UNPs and Rydberg gases was demonstrated experimentally [7-9]. (This connection is also obvious from the experiments [10] at CERN, where antihydrogen atoms are formed by recombination from a plasma of positrons and antiprotons.) With the same experimental setup as in [5,6], a significant amount of recombination was shown to appear in the plasma, leading to the formation of Rydberg atoms [7]. On the other hand, complementary experiments starting with a Rydberg gas showed a spontaneous evolution of the gas into a UNP, with up to ninety percent of the initial Rydberg atoms being ionized [8,9]. Again, the experimental observations seemed to be in contradiction with basic assumptions about the relevant physical mechanisms.

Naturally, these early experiments prompted a number of theoretical efforts to model them and to explain the experimental observations [11-18]. Meanwhile, most of the questions raised by the experiments [6-9] have been answered. The enhancement of the expansion velocity of the plasma cloud at low electronic excess energies, e.g., can be explained by “recombination heating” of the plasma electrons, i.e. it is the binding energy of the recombined Rydberg atoms which is the source of the additional kinetic energy of the plasma particles [11]. As it turned out, no “new physics” had to be invoked in order to model the

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\(^1\) Strictly speaking, the plasma is not completely neutral since some electrons quickly evaporate until a sufficiently large space charge builds up to trap the remaining electrons. Hence, the plasma should rather be called quasi-neutral. However, typically more than ninety percent of the electrons created are trapped.
outcome of the experiments. In the meantime, theoretical models have reached the stage where they go beyond an a-posteriori modelling of experimental findings and reach predictive power, simulating situations that can not yet be realized experimentally. Nevertheless, our understanding of the physics of UNPs and Rydberg gases is far from complete, and new simulations and experiments raise as many new questions as they answer. In the last two years, we have developed two different methods for the description of UNPs, namely a simple kinetic approach on the one hand and a hybrid molecular dynamics code on the other [19]. Since the two methods complement each other, this two-sided approach provides a powerful tool to explore the physics of ultracold neutral plasmas.

2. Two theoretical approaches for the description of ultracold neutral plasmas

2.1.Kinetic approach

Given the long-range nature of the dominant Coulomb interactions on the one hand, and the rather low experimental densities with interparticle distances that are several orders of magnitude larger than the deBroglie wavelengths of the ions on the other, one may conclude that a classical treatment is appropriate for the description of UNPs. Nevertheless, due to the large number of particles ($N > 10000$) and the long timescales ($t > 100\mu s$) involved, a straightforward full molecular dynamics simulation is infeasible with current-day computer resources. Hence, the first theoretical description of a UNP as created in [5] over experimental timescales was achieved in [11] using a hydrodynamical model. Our kinetic approach is very similar to the ansatz made in [11], with the additional inclusion of ion-ion correlations. It is described in detail in [19]. Starting from the first equation of the Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy [20], one obtains the evolution equations for the one-particle distribution functions $f$ of the electrons and ions. It was shown in [11] that, due to three-body recombination heating of the electrons, the electrons are always weakly coupled during the plasma expansion, with $\Gamma_e \lesssim 0.2$ at all times. Thus, electron-electron as well as (free) electron-ion correlations are small, and we neglect them in our model, taking into account only ion-ion correlations beyond the mean-field level. Since the relaxation of the electrons towards thermodynamic equilibrium is much faster than the plasma expansion due to the small mass ratio $m_e/m_i \ll 1$, an adiabatic approximation may be employed for the electrons, assuming instant equilibration and a local Maxwell-Boltzmann distribution with an electronic temperature $T_e$. Together with a quasineutral approximation [21], this permits expressing the mean-field electrostatic potential in terms of the ionic
density, leading to a closed equation for the ion distribution function which contains the electron temperature as a parameter. A Gaussian ansatz for the ion distribution function,

\[ f_i \propto \exp \left( -\frac{r^2}{2\sigma^2} \right) \exp \left( -\frac{m_i (v - \gamma r)^2}{2k_BT_i} \right), \]

which corresponds to the initial state of the plasma cloud, is then inserted into the evolution equations for the second moments \( \langle r^2 \rangle \), \( \langle rv \rangle \) and \( \langle v^2 \rangle \) of the ion distribution function. In this way, evolution equations for the width \( \sigma \) of the cloud, the parameter \( \gamma \) of the hydrodynamical expansion velocity \( u = \gamma r \) and the ionic temperature \( T_i \) are obtained.

In the framework of a local density approximation together with a gradient expansion, the ion-ion correlation force can be expressed by the correlation energy \( u_{ii} \) of a homogeneous plasma. Analytical expressions for the value of \( u_{ii} \) for a plasma in thermodynamical equilibrium are available in the literature, however, in the experimental setup of [5] the plasma is created in a non-equilibrium state, with zero initial correlations \( u_{ii} = 0 \). No simple equation for the time evolution of \( u_{ii} \) exists, and a precise description of its relaxation behavior towards equilibrium in the framework of a kinetic theory is rather difficult. We therefore employ the so-called correlation time approximation [22], which assumes an exponential relaxation with a correlation time constant \( \tau \) given by the inverse of the ionic plasma frequency, \( \tau_{corr} = \omega_{\rho,i}^{-1} = \sqrt{m_i/(4\pi e^2 \rho_i)} \).

Finally, inelastic processes such as three-body recombination and re-ionization of the formed Rydberg atoms are taken into account on the basis of rate equations. In our calculations, we account for three-body recombination, electron-impact ionization, excitation and deexcitation, with the rates taken from [23]. If needed, other processes such as ionization by black-body radiation, which is essential for situations starting from a Rydberg gas rather than a plasma, are easily incorporated in the description provided the corresponding rates are available. For higher densities, e.g., one might anticipate that dipole-dipole interactions between Rydberg atoms become important, for the current experimental parameters we found the corresponding rates to be negligible. In order to take into account the influence of the Rydberg atoms on the expansion dynamics, we assume equal hydrodynamical velocities of ions and atoms. The final set of equations of our kinetic model then reads \( (N_i, N_a \text{ and } N_{a}(n)) \) are the number of ions, Rydberg atoms, and Rydberg atoms with principal quantum
number \( n \), respectively)

\[
\begin{align*}
\partial_t \sigma^2 &= 2\gamma \sigma^2 \\
\partial_t \gamma &= \frac{N_i \left( k_B T_e + k_B T_i + \frac{1}{3} U_{ii} \right)}{(N_e + N_i) m_e \sigma^2} - \gamma^2 \\
\partial_t k_B T_i &= -2\gamma k_B T_i - \frac{2}{3} \gamma U_{ii} - \frac{2}{3} \partial_t U_{ii} \\
\partial_t U_{ii} &= -\frac{U_{ii} - U_{ii}^{eq}}{\tau_{corr}} \\
\partial_t \rho_a(n) &= \rho_e \sum_p \left[ K(p, n) \rho_a(p) - K(n, p) \rho_a(n) \right] + \rho_e \left[ R(n) \rho_e \rho_i - I(n) \rho_a(n) \right]
\end{align*}
\] (2a)-(2e)

with \( U_{ii} = N_i^{-1} \int u_{ii} \rho_i \, dr \) and \( U_{ii}^{eq} \) taken from ref. [24]. The electronic temperature is determined by energy conservation,

\[
E_{tot} = N_i \left[ \frac{3}{2} \left( k_B T_e + k_B T_i \right) + \frac{3}{2} m_e \gamma^2 \sigma^2 + U_{ii} \right] - 1/2 \sum_n N_a(n) n^{-2} = \text{const.} .
\] (3)

The above equations (2,3) constitute a set of ordinary differential equations for the macroscopic parameters determining the state of the plasma, and they are quickly and easily solved numerically. The simulation of the experiment [6], e.g., takes about sixty minutes of CPU time on a modern single-processor Unix workstation. Since the numerical effort is independent of the number of particles involved, the kinetic approach provides a convenient method for simulations of large plasma clouds over long, i.e. millisecond, timescales. It can be used to quickly gain insight into the plasma dynamics by efficiently scanning a broad range of initial-state parameters, and it is able to simulate plasmas that are too large to be treated effectively with molecular dynamics methods. Moreover, and maybe even more importantly, the kinetic model provides physical insight by reducing the plasma dynamics to a few macroscopic parameters. Its basic equations (2,3) are sufficiently simple to be interpreted in terms of simple physical mechanisms, which are usually hidden behind the many parameters of a full molecular dynamics simulation. As an example, figure 1 shows the correlation energy \( U_{ii} \) (dot-dashed line) and the ionic temperature \( 3k_B T_i / 2 \) (dotted line) for a plasma of 40000 Sr ions with an initial average density of \( 10^9 \, \text{cm}^{-3} \) and an initial electron kinetic energy of 20 K. In this figure, the effect of “disorder-induced heating” or “correlation-induced heating” [13,25] of the ions becomes apparent. Initially, the plasma is created in a non-equilibrium, completely uncorrelated state. The following relaxation to thermodynamical equilibrium leads to the build-up of ionic correlations, converting electrostatic potential energy into
thermal kinetic energy, thus heating the ions and strongly decreasing the degree of coupling $\Gamma_i$. Figure 1 demonstrates that this effect heats up the ions by several orders of magnitude to a temperature of the order of 1 K, and that it is in fact the only mechanism responsible for the heating. The latter can be inferred from the observation that the negative of the correlation energy exactly equals the ionic temperature as long as the plasma has not yet started to expand, only at later times the plasma expansion leads to an adiabatic cooling of the ions and the two curves start to deviate from each other. Numerically, this equality of build-up of correlations on the one hand and heating of the ions on the other hand is also reproduced by more sophisticated molecular dynamics simulations, to be described below. However, the direct physical connection between the two effects remains hidden behind the numerics. In the kinetic model, on the other hand, it becomes immediately obvious from equation (2c). As long as the plasma expansion is still negligible ($\gamma = 0$), the change in ionic temperature is directly given by the last term in eq. (2c), i.e. by the change in correlation energy.

As a second and last example of the usefulness of the simple evolution equations provided by the kinetic approach, we consider the expansion behavior of the plasma cloud. Since the ionic temperature is given by the correlation energy $U_{ii}$ as demonstrated above, and since analytic approximations for the equilibrium value of the correlation energy are available, it is possible to show that $T_i \ll T_e$ over the whole time of the plasma expansion for experimentally relevant initial-state parameters. Neglecting $T_i$ and $U_{ii}$ against $T_e$ in equation (2b), one finds immediately that the expansion of the plasma is driven by the electron thermal motion. In situations where three-body recombination is also small, which is the case for sufficiently small $\Gamma_e$ ($\Gamma_e \lesssim 0.05$), equation (3) in turn relates the change of electron temperature to the expansion of the plasma. Under these circumstances, an analytical solution for the time dependence of the width of the plasma cloud can be found, namely $\sigma(t) = \sqrt{\sigma_0^2 + k_B T_{e0} t^2 / m_i}$ (with the lower index 0 indicating initial-state values). Consequently, defining a typical expansion timescale as the time $t_{\text{exp}}$ in which the width $\sigma$ of the plasma cloud changes by a factor $\sqrt{2}$, we find $t_{\text{exp}} = \sigma_0 \sqrt{m_i / (k_B T_{e0})}$. The latter expression can be rewritten, dropping constant numerical prefactors, as $t_{\text{exp}} = \sqrt{\Gamma_0 N_i^{1/3} \omega_{p,i}^{-1}}$. On the other hand, relaxation of the plasma towards thermodynamical equilibrium takes place on a timescale of $\tau_{\text{corr}} = \omega_{p,i}^{-1}$ as argued before. Hence, when studying the influence of plasma expansion on the relaxation behavior, similar behavior can be expected from plasmas with different initial conditions,
as long as the product \( \sqrt{\tau_{e0}} N_i^{1/3} \) remains constant. This fact was used, e.g., in [26] for the study of the evolution of UNPs created in an overcorrelated state, where the possibility of “correlation-induced cooling” has been demonstrated.

2.2. Hybrid molecular dynamics approach

With all the advantages of the kinetic approach highlighted above, some drawbacks of the model are immediately apparent. First, the inclusion of ionic correlation effects beyond the mean-field level is done in an approximate way, which might not be applicable for strongly coupled systems where correlations are predominant. (In particular, effects such as Coulomb crystallization obviously cannot be described with the one-particle distribution function underlying the kinetic approach.) Moreover, further approximations enter in the process of reducing the original evolution equations for the one-particle distribution functions to the much simpler hydrodynamical equations for \( \sigma \), \( \gamma \) and \( T_i \). For example, the quasineutral approximation may not be applicable in situations where the initial plasma density is so low or the electron temperature so high that a significant fraction of the free electrons created in the photoionization process immediately escapes. Alternatively, the selfsimilarity of the plasma expansion which enters the Gaussian ansatz for the ion distribution is disturbed by recombination, since the latter process is nonlinear in the electronic density and hence more efficient in the inner, denser part of the plasma. In order to test the kinetic model, and to be able to describe situations which are beyond the capabilities of the kinetic approach, e.g. very strongly coupled systems or systems where the quasineutral approximation fails, we have developed an alternative approach, namely a hybrid molecular dynamics (H-MD) method.

In this approach, electrons and ions are treated on different levels of sophistication, due to the large differences in timescales for relaxation and in the degree of coupling. As discussed above, electronic correlations are always small in the present type of experiments, hence we also neglect them in the H-MD ansatz. Moreover, due to the large ion-to-electron mass ratio, the adiabatic approximation for the electron distribution introduced above is well justified, and it is also kept in the H-MD treatment. Consequently, the electrons are still treated on a hydrodynamical level as in the kinetic model, which allows for the use of much larger timesteps than in a full MD simulation since the electronic motion does not need to be resolved. It is this adiabatic approximation for the electrons which makes a molecular dynamics treatment of the ionic motion in UNPs computationally feasible. In contrast to
the kinetic approach, however, we lift the quasineutral approximation of equal densities of ions and electrons. The electronic density is now determined self-consistently from the Poisson equation. The fact that the potential well created by the ions which is trapping the electrons has a finite depth is taken into account by using a King-type distribution [27] known from simulations of globular galaxy clusters rather than a Maxwell-Boltzmann distribution for the velocity distribution of the electrons. Finally, the ions and atoms are propagated individually in a molecular dynamics simulation, taking into account the electronic mean-field potential and the full interaction potential of the remaining ions. Technically, the most time-consuming part of the H-MD simulation, namely the calculation of the interionic forces, is done using a treecode procedure originally designed for astrophysical problems [28], which scales like \( N_i \ln N_i \) rather than \( N_i^2 \) with the number \( N_i \) of ions.

Simulations using the H-MD approach are much more time-consuming than the kinetic model. Typically, the simulation of an experiment as in [5] with 50000 ions takes about 2 weeks of CPU time on a Unix workstation. Nevertheless, the H-MD treatment provides a powerful method to go beyond the much simpler kinetic model and to test its predictions. Generally, for initial conditions corresponding to the experimental setup of [5,6], it is found that the kinetic model provides a surprisingly accurate description of the plasma dynamics, as long as macroscopic parameters such as plasma size or temperature are concerned. This can be seen in figure 1, where ionic temperature as well as correlation energy obtained from the kinetic approach and from the H-MD simulation, respectively, are compared. Other quantities, such as electron temperature or distribution of recombined Rydberg atoms, show equally good or even better agreement [19]. The comparison in fig. 1, however, serves a second purpose, namely a test of the quality of the description of ionic correlations in the kinetic approach, which was done in the framework of a correlation time approximation as described above. As can be seen in fig. 1(b), the early phase of relaxation is clearly non-exponential in the H-MD simulation. Rather, it shows a characteristic “overshoot” of the ion temperature connected with transient oscillatory behavior, also observed in previous theoretical and experimental studies [26,29,30,31]. However, once the system is sufficiently close to equilibrium, the correlation time approximation provides an accurate description of

\[ \text{In order to compare with a kinetic approach neglecting ionic correlations, such as in [11], it is also possible to neglect correlations in the H-MD approach by propagating the ions in the mean-field potential created by all charges rather than the full ionic interaction.} \]
the formation of ionic correlations.

On the other hand, as should be expected, spatially resolved quantities such as ionic density, ion velocities or local temperature show deviations from the behavior predicted by the simple kinetic model, as shown in figures 2 and 3 for the same initial conditions as in fig. 1. In figure 2(a), the formation of a density peak at the plasma edge becomes apparent, which is washed out again at later times (fig. 2(b)). The build-up of this spike is connected with deviations of the hydrodynamical expansion velocity $u$ from the simple linear behavior assumed in the kinetic model (fig. 2(c)). Moreover, the kinetic approach assumes a well-defined ionic temperature, i.e. thermodynamic equilibrium for the ions. Figure 3 shows the local temperature of the ions, defined for the inner, intermediate and outer third of the plasma cloud. As can be seen, the plasma is not in complete equilibrium. Due to the higher density in the central plasma region, the correlation energy is largest in this part, leading to a stronger heating of the ions than in the outer region of the plasma. However, the distribution of thermal velocities over the whole plasma cloud (fig. 3(d)) is well fitted to a Maxwell-Boltzmann distribution with a temperature intermediate between those of the inner and outer plasma part already at relatively early times, supporting the approximation of a global temperature in the kinetic model.

The H-MD approach provides a powerful method to test the applicability of simpler treatments, such as the kinetic model described above, and to study the evolution of UNPs in greater detail. For example, it allows for detailed studies of the relaxation behavior towards thermodynamic equilibrium, as apparent in the spatial dependence of the ionic temperature (fig. 3) or its oscillatory time-dependence (fig. 1(b)). Moreover, it permits the description of situations where the plasma is so strongly coupled that correlation effects dominate the plasma dynamics and the kinetic model is not applicable anymore, as has been demonstrated in [32]. There, we have simulated the dynamics of a UNP under the additional influence of a cooling laser cooling the ions during the plasma expansion, and we have demonstrated that a strongly coupled plasma can indeed be created in this way. Inclusion of binary electron-ion collisions changes the results of [32] somewhat quantitatively but not qualitatively [33]. It was predicted that Coulomb crystallization should be achievable, leading to the formation of shell structures and long-range radial as well as intra-shell ordering of the ions. This is demonstrated in fig. 4, where we show the radial ion density of a plasma with $\Gamma_e(t = 0) = 0.08$, consisting of $N_i = 80000$ ions which are cooled with a rate of $\beta = 0.2\omega_{p,i}$. With an initial
ion density of $5 \times 10^7$ cm$^{-3}$ and an initial electron temperature of 12.4 K, these parameters can be achieved, e.g., with Beryllium ions which are cooled on the $2s \ 2S_{1/2} - 2p \ 2P_{3/2}$ transition ($\lambda = 313$nm) [34] down to the minimum Doppler temperature of 0.5mK by using cooling lasers with an intensity of 50 mW/cm$^2$.

Interestingly, the dynamics of this shell-structure formation differs from the one in ion traps, where corresponding structures are also observed. There, concentric shells start forming at the outer rim of the plasma, and their spherical symmetry can be attributed to the symmetry of the trapping potential confining the ions. In the present case, no such external force influencing the crystallization process exists. The formation of shells proceeds from the center of the plasma cloud towards the edge, i.e. it starts in the region of highest density (i.e. strongest forces), and the spherical symmetry of the shells must be related to the symmetry of the density profile of the plasma cloud. Moreover, while the external confinement of the ions in a trap keeps the plasma density constant and allows for an arbitrarily long timescale for the cooling and crystallization process, the UNPs considered here are freely expanding, which limits the time available for the build-up of spatial correlations.

3. Outlook

In the present manuscript, we have emphasized the high potential of our two-sided approach for the description of ultracold plasmas. We believe that the two methods outlined above complement each other, providing physical insight on the one hand and a numerically accurate description of the dynamics on the other hand. At the present stage, it allows one to go beyond a simulation of the early experiments [5-7], and to provide guidance for new experiments by making quantitative predictions, such as, e.g., the Coulomb crystallization of laser-cooled UNPs [32]. However, our understanding of UNPs is far from complete, and several further applications and extensions of the models suggest themselves. Naturally, the relaxation behavior of the plasmas towards equilibrium needs to be studied in more detail, which should be an interesting problem at the interface between plasma physics and non-equilibrium thermodynamics. More generally, an extension towards the description of non-neutral plasmas is straightforward. Such systems could be realized by photoionization of ions in an ion trap rather than neutral atoms. In this case, one might anticipate a separation of the plasma into a purely ionic outer and a quasineutral inner region. Another obvious extension of the model is the inclusion of magnetic fields, which would aim not only at new experiments planned in several laboratories, but directly at the antihydrogen produc-
tion experiments at CERN [10]. Finally, an extension of the H-MD model to experimental setups starting from a Rydberg gas rather than a plasma, where the current approach leads to problems with the electron density in the initial phase of the system evolution due to the very low number of free electrons present, is called for.

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References

FIG. 1:
FIG. 2:
FIG. 3:
FIG. 4:
Figure 1: (a) Correlation energy (solid line: H-MD simulation, dot-dashed: kinetic result) and thermal ion kinetic energy (dashed: H-MD, dotted: kinetic) of a plasma of 40000 Sr ions with initial average density $\rho = 10^9 \text{ cm}^{-3}$ and initial electron kinetic energy $3T_e/2 = 20 \text{ K}$. (b) Ion thermal energy in the early stage of the relaxation process with its characteristic transiently oscillatory behavior.

Figure 2: Spatial densities $\rho_i$ (solid) and $\rho_a$ (dashed) of the ions and atoms, respectively, at $t = 3 \mu s$ (a) and $t = 31.3 \mu s$ (b), compared to the Gaussian profile assumed for the kinetic model (dotted). (c) and (d) show the hydrodynamic velocity $u(r)$ of ions (full circles) and atoms (open circles) at the same times, compared to the straight-line assumption of the kinetic model.

Figure 3: Distribution of thermal ionic velocities at $t = 1.5 \mu s$ sampled from three different regions of the plasma: $r \leq 1.3\sigma$ (a), $1.3\sigma < r \leq 2\sigma$ (b), and $r > 2\sigma$ (c), and from the total plasma volume (d). The solid lines show a fit to a Maxwell-Boltzmann distribution corresponding to the temperatures specified in the figure.

Figure 4: Radial density after a time of $\omega_{p,i} t = 216$ for a laser-cooled plasma of 80000 ions with $\Gamma_e(t = 0) = 0.08$ and a cooling rate of $\beta = 0.2\omega_{p,i}$. The inset shows the 5th shell of the plasma, demonstrating significant intra-shell ordering.