

| Participant | Title | Abstract |
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| Tony Albers | Analyzing anomalous diffusion processes: the distribution of generalized diffusivities | We propose a new tool for analyzing data from normal or anomalous diffusion processes: The distribution of generalized diffusivities $\rho_{\alpha}(D, \tau)$ is defined as the probability density of finding a squared displacement of duration τ , rescaled by its asymptotic time dependence τ^{α} . It describes the fluctuations appearing during the diffusion process around the mean anomalous diffusion coefficient which can be obtained from the asymptotic behavior of the mean squared displacement (MSD) and is also equal to the first moment of the distribution $\rho_{\alpha}(D, \tau)$ for large τ . In comparison with the MSD we therefore obtain more information from the numerically or experimentally determined data. In this contribution we investigate with this new tool subdiffusive continuous time random walks which show weak ergodicity breaking. Further we apply our new tool to the diffusive transport in a simple low dimensional Hamiltonian system, the standard map, and show with the help of a modified Lévy walk model how the structures in phase space are reflected in the distribution of generalized diffusivities. |
| Gábor Borbély | Slow decay of correlations and CLT for the system of two falling balls | The system of two falling balls was first introduced by Wojtkowski [W]. He proved that it is hyperbolic and ergodic if the lower ball is heavier. However in discrete time hyperbolicity is not uniform as the lower ball may experience arbitrary many bounces on the floor before it collides with the upper one. For this reason subexponential decay rates of mixing are expected. Following the work of Young [Y], Chernov and Zhang [CZ] gave simplified conditions for polynomial mixing. We checked these conditions for our system (for an interval of mass ratios). This way we could obtain a polynomial bound on the decay of correlations and, as the bound is summable, we could also prove the central limit theorem. We found a lot of interesting phenomena while exploring the singularity structure and the main properties of the dynamics. If our application is accepted then probably I will give the oral presentation while my colleague will contribute the poster. [CZ] N. I. Chernov, H.-K. Zhang, On statistical properties of hyperbolic systems with singularities, Journal of Statistical Physics, Vol. 136., pp. 615-642, 2009. [W] M. P. Wojtkowski, A system of one dimensional balls with gravity, Comm. Math. Phys., Vol. 126, pp. 507-533, 1990. [Y] L.-S. Young, Statistical properties of dynamical systems with some hyperbolicity, The Annals of Mathematics, Second Series, Vol. 147., No. 3, pp. 585-650, 1998. |
| Andreas Dechant | Finite time dynamics and ergodicity breaking in logarithmic potentials | We derive the position autocorrelation function for overdamped Brownian motion in a logarithmic potential and use it to obtain an expression for the finite time fluctuations of the time average. Depending on the depth of the potential, we find either a stationary correlation function and ergodic behaviour or a super-ageing correlation function and broken ergodicity. |
| Weihua Deng | Ergodic Properties of Fractional Brownian-Langevin Motion | We investigate the time average mean square displacement $\overline{\langle \Delta^2(x(t)) \rangle} = \int_0^t \langle x(t')^2 \rangle dt'$ for fractional Brownian and Langevin motion. Unlike the previously investigated continuous time random walk model $\overline{\langle \Delta^2 \rangle}$ converges to the ensemble average $\langle x^2 \rangle$ in the long measurement time limit. The convergence to ergodic behavior is however slow, and surprisingly the Hurst exponent $H=3/4$ marks the critical point of the speed of convergence. When $H < 3/4$, the ergodicity breaking parameter $\mathfrak{M} = \langle \overline{\langle \Delta^2 \rangle} \rangle / \langle \Delta^2 \rangle$ converges to $9/16$ as $H \rightarrow 3/4$, when $H = 3/4$, $\mathfrak{M} \sim 1/\ln t$, and when $3/4 < H < 1$, $\mathfrak{M} \sim k(H) \Delta^H$. In the ballistic limit $H \rightarrow 1$ ergodicity is broken and $\mathfrak{M} \sim 2$. The critical point $H=3/4$ is marked by the divergence of the coefficient $k(H)$. Fractional Brownian motion as a model for recent experiments of sub-diffusion of mRNA in the cell is briefly discussed and comparison with the continuous time random walk model is made. |
| Alexander Dubkov | Transient dynamics of Verhulst model with Levy-stable fluctuations in resources | The nonlinear stochastic Malthus-Verhulst equation for the population density with fluctuating volume of resources is considered. Using the exact solution of the equation and previously obtained result for the characteristic functional of arbitrary non-Gaussian white noise (Fluct. Noise Lett. 2005. V.5, L267), the conditional probability density function for the white noise excitation having one-sided stable Levy distribution is calculated. The exact evolution of the population density distribution with the transient bimodality for the case of white noise with Levy-Smirnov stable distribution is analyzed. The analytical expressions for the probability density function in a steady state and the transient time from bimodality to unimodality are also derived. At last, it is shown that for the nonlinear system considered the correlation function of the population density in a steady state has a simple exponential form with the correlation time depending only on the Malthus factor. |
| Stephan Eule | Models of power-law adaptation based on anomalous kinetics | A characteristic feature of many biological sensing molecules is the ability to adapt to a persistent input stimulus, thereby increasing the range of sensitivity. A quantitative hallmark of adaptation is the return of the response to a stimulus to its prestimulus value even in the continued presence of the signal. In most studies response adaptation is analyzed and modeled assuming that the process is characterized by only a single or at most a few time constants. However, many biological systems exhibit a complex temporal behavior which often is better described by power-law, scale-free rather than exponential dependencies on time. In this contribution, we consider a simple model of gated ion-channels leading to neural adaptation. Assuming the kinetics of the ion channels to be anomalous, we show that the adaptation process is scale-free. We present an exact solution of our model for a sudden stimulus change and relate this solution to a model of fractional feedback control. |
| Daniela Froemberg | Asymptotic front velocity in an autocatalytic reaction-subdiffusion system | We discuss the front propagation in the A+B->2A reaction under subdiffusion which is modelled by continuous time random walks with a heavy-tailed power law waiting time probability density function. Two scaling regimes of the front propagation are discussed. The intermediate asymptotic regime is given by the front solution of the corresponding continuous reaction-subdiffusion equation, whereas the final asymptotics is fluctuation-dominated and the continuous description fails. The continuous reaction subdiffusion equation is shown to possess a front solution that decelerates and becomes narrow in the course of time. Consequently, the continuous picture breaks down for larger times when the front gets atomically sharp, and the fluctuation dominated regime sets in. The resultant velocity of the fronts then decays faster in time than in the continuous regime. |
| Paulo Paneque Galuzio | Onset of Turbulence in Damped and Forced RLW Equation | The Regularized Long Wave (RLW) equation is an important model from plasma physics and hydrodynamics. In the plasma physics context it represents the fluctuations of a scalar electrical potential in a toroidal plasma. In this work a damping and a forcing term are added a posteriori in order to enrich the dynamics and also take into consideration the effects of other possibly relevant modes. The equation is then integrated in Fourier space via the Pseudo-spectral method, and a transition from periodic behavior to temporal chaos and then to spatio-temporal chaos (turbulence) is analyzed. The analysis are made via the computation of the wave energy, the use of this scalar quantity is not only simpler in a highly dimensional space but it is also free of projection errors due to false neighbours. The transition to temporal chaos happens due to a doubling period route. And the transition to turbulence happens intermittently due to a dynamical system property called unstable dimensional variability (UDV), as suggested by the finite time Lyapunov exponent distribution of the system. The statistical properties of the intermittency are compatible with similar phenomena occurring in time only systems. In this case the intermittent behavior leads the system to a strong turbulent state, as characterized by the Fourier spectrum of energy which fits very well the Kolmogorov spectrum. |

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| Alexander Iomin | Fractional transport on a comb | Two examples of fractional diffusion and quantum motion are considered on a comb. The first example is subdiffusion on a fractal comb. A mechanism of subdiffusion with a transport exponent different from 1/2 is suggested. It is shown that the transport exponent is determined by the fractal geometry of the comb. The second example is the Schrodinger equation on the comb, which is a paradigm of quantum time fractional dynamics. |
| Georgie Knight | Chaotic diffusion in dynamical systems | We calculate the parameter dependent diffusion coefficient for a one-dimensional chaotic map and explain the mixture of fractal and linear structure that we find. |
| David Kocheim | Probabilistic limit theorems for infinite measure preserving systems | Many infinite measure preserving transformations exhibit stochastic properties parallel to those of null-recurrent Markov chains. While various interesting probabilistic results confirming this have been established in recent years, several basic questions seem to remain open, and call for novel approaches. Here we describe how ideas from "transient dynamics", that is conditionally invariant measures and conditional limit theorems can be used to obtain new results for null-recurrent dynamical systems. |
| Mirko Lukovic | Convex Hulls of Levy Walks | Recently there has been much debate in the scientific community as to whether the observed walk patterns of foraging animals are Levy-like or not. Even in cases where they are, it is not clear whether they are actually produced by a Levy walk process or a combination of more classical movements such as Brownian motion. This is mainly due to the questionable accuracy of the statistical methods employed to identify Levy behavior from collected data samples of animal trajectories. So far, strong evidence for Levy walks was found in nature only in cases where extremely large data sets were available (Humphries et al. Nature 2010). We propose the use of convex hulls (minimum convex polygon enclosing the recorded points - Boyle et al 2009., Majumdar et al. 2009) of the home range of animals as a robust and accurate method to discriminate between different types of foraging animal motions. This method is simple and robust and it can be used in cases where the data set available is sparse. It also provides information on how Levy-like a recorded trajectory is. |
| Günter Radons | see poster Tony Albers | see poster Tony Albers |
| Christian Rodrigues | Transport in randomly perturbed dynamics | We study the influence of random bounded perturbation to the transport properties of dynamics. First we shall discuss non-hyperbolic chaotic scattering, showing that the random perturbations may allow the escape of trajectories from regions of the phase space where otherwise they are expected to be trapped forever. The dynamics then gains hyperbolic-like characteristics, what is reflected in the distribution of escaping time as well as in the value of the estimated fractal dimension of singular sets. Then, we shall discuss random perturbation of dissipative dynamics with coexisting attractors. We show that, in contrast to unbounded perturbation, under bounded noise the dynamics can have multiples "invariant" measures. We show that there is a critical amplitude of the perturbations such that beyond this value there exist the possibility of escape for the random orbits. This gives rise to a hopping process among different basins. We statistically describe such transport in terms of a Markov tree, where the non-stable "attractors" play a role similar to that which KAM islands do in transport in Hamiltonian dynamics. |
| Trifce Sandev | Modeling anomalous diffusion by fractional diffusion equation and generalized Langevin equation with frictional memory kernels of the Mittag-Leffler type | The fractional diffusion equation (FDE) and the generalized Langevin equation (GLE) are used to model anomalous diffusion processes. The solution of a FDE with a Hilfer-generalized Riemann-Liouville time fractional derivative in both confined and unconfined space is obtained in terms of the Mittag-Leffler type functions and Fox's H-function. The asymptotic behavior of the solution is derived and the moments of fundamental solution obtained. The obtained results for the considered FDE may be helpful for the evaluation of data from complex systems, in particular, in the context of relaxation dynamics in glassy systems or aquifer problems. Also, exact results for velocity and displacement correlation functions of a GLE with frictional memory kernels of the Mittag-Leffler type are obtained by analyzing the relaxation functions. It is shown that for various values of the parameters of the frictional memory kernels, anomalous diffusive behavior occurs. The asymptotic behavior of the particle in the short and long time limit is obtained by using the Tauberian theorems. The proposed model may be used to model anomalous diffusive processes in complex media. |
| Yuzuru Sato | Noise-induced Phenomena in one-dimensional maps | Problems of complex behavior of random dynamical systems is investigated based on numerically observed noise-induced phenomena in Belousov-Zhabotinsky map (BZ map) and modified Lasota-Mackey map with presence of noise. We found that (i) both noise-induced chaos and noise-induced order robustly coexist, and that (ii) asymptotical periodicity of density is varied according to noise amplitude. Applications to time series analysis are also discussed. |
| Johannes Schulz | Continuous time random walk processes with correlations | We consider correlations in the waiting times and jump lengths of continuous time random walk models, but help of composition rules for stable random variables. |
| Soya Shinkai | Some examples of 2-dimensional area-preserving infinite ergodic systems | We will introduce some maps of 2-dimensional infinite ergodic systems. These maps preserve the Lebesgue measure. In other words, they have the area-preserving property, which is one of most important properties in Hamiltonian systems. Through the following two steps, the maps are constructed from 1-dimensional intermittent maps defined on a bounded interval: the first is a conjugate relation and the second is the natural extension. In the sense of physical measurements, though the maps are ergodic, conservative, area-preserving and invertible, an empirical measure generated by a trajectory is intrinsically random in contrast to the baker map, which is the simplest example of 2-dimensional fully chaotic area-preserving maps. We will report such an anomalous ergodic property, and discuss an infinite ergodic aspect of Hamiltonian systems. |
| Renat Sibatov | Charge carrier dynamics in colloidal quantum dot array and Levy stable statistics | A new statistical model of charge transport in colloidal quantum dot array is proposed. It takes into account Coulomb blockade forbidding multiple occupancy of nanocrystals and influence of energetic disorder of interdot space. The model explains power law current transients and the presence of memory effect. The fractional differential analogue of the Ohm law is found phenomenologically for nanocrystal arrays. The model combines ideas that were considered as conflicting by other authors: the Scher-Montroll idea about power law distribution of waiting times in localized states for disordered semiconductors is applied with taking into account Coulomb blockade, Novikov's condition about asymptotical power law distribution of time intervals between successful current pulses in conduction channels is fulfilled, carrier injection blocking predicted by Ginger and Greenham takes place. Different percolation schemes are discussed and corresponding Monte Carlo algorithms are proposed. |
| Oleksii Sliusarenko | Kramers Problem in Anomalous Dynamics | We study the stochastic problem of the escape from a potential well (Kramers problem) for two models of anomalous diffusion mechanisms: Levy flights and fractional Brownian motion. Using method of numerical integration of Langevin equations we reveal a power-law behaviour of mean escape time of a particle driven by white Levy noise as a function of noise intensity, being independent on the Levy index at low intensities. Considering three different types of potentials it was shown that the first escape time is mainly determined by the position of the absorbing boundary and not by the barrier's height or shape. The main result for fractional Brownian motion is that the escape is faster in case of subdiffusive motion, rather than of superdiffusive one. We also underline main differences and similarities between the laws of classical Kramers problem for ordinary Brownian motion and those of the Kramers problem for fractional Brownian motion and Levy flights. |
| Vladimir Uchaikin | Fractional approach to cosmic ray dynamics in the Galaxy | Possible formulations of cosmic ray propagation and acceleration in the interstellar galactic medium in terms of fractional differential equations have been discussed. Some of the models based on a fractional partial differential equation are criticized. A new model of propagation called the model of limited anomalous diffusion is proposed. Its main difference from the criticized models is the use of the material fractional derivative instead of partial space- and time-derivatives. This takes into account the finite free motion velocity and eliminates the paradox of instantaneous crossing the Galaxy by a cosmic ray particle. The CR acceleration process is considered as a jump process in the velocity space. The collisional concept of particle-field interactions admits instantaneous jumps in the velocity-space, but do not allow to replace the Stoss-integral by Laplacian in case the particles are accelerated by shock waves in the supernovae remnants. The fractional time-derivative in the acceleration equation appears as a result of fractal spatial distribution of the remnants in the Galaxy. character of interstellar magnetic fields. Applied technique fractal has been physically justified. A Fermi result for the energy spectrum of accelerated particles has been generalized. |
| Karina Weron | The role of compound counting processes in the continuous-time-random-walk approaches to fractional dynamics of complex systems. Experimental evidence | We show the importance of compound counting processes in modeling of the complex-system fractional dynamics within the anomalous-diffusion framework. We propose diffusion scenario based on random clustering of the spatio-temporal steps in a continuous time random walk (CTRW). Since the waiting time and the subsequent jump in the clustered CTRW are both random sums, with the same number of summands, the clustered CTRW is coupled, even if the original CTRW before clustering had no dependence between waiting times and jumps. The clustering of the random number of jumps into a single random-sized jump introduces hence a dependence between the waiting times and jumps that significantly affects the diffusion fronts. Moreover, such a construction substitutes the renewal counting process, used in the classical CTRW methodology, by a compound counting one. As a result, one obtains a subordinated under- and overshooting diffusive mechanisms governing the universal, fractional-two-power-law relaxation patterns. |

Our theoretical studies are motivated by the empirical evidence. We present such dielectric permittivity data which can be neither fitted nor explained by none of the well-known empirical relaxation laws. The analysis concerns semiconductor mixed crystals possessing deep metastable defects. The dielectric permittivity of such a material exhibits a non-standard relaxation pattern with the high-frequency power-law exponent equal to 1. This behavior is explained by the compound undershooting subordination of the Brownian motion to the deterministic operational time.