Weak Chaos, Infinite Ergodic Theory, and Anomalous Dynamics: Description of aims and structure of the event

1 What is it about?

Over the past few decades it was realized that deterministic dynamical systems involving only a few variables can exhibit *complexity reminiscent of many-particle systems* if the dynamics is chaotic, as is quantified by the existence of a positive Lyapunov exponent. Such systems provided important paradigms in order to construct a theory of nonequilibrium statistical physics starting from first principles [1]: Based on the chaotic hypothesis, which generalizes Boltzmann's ergodic hypothesis, SRB measures were studied as nonequilibrium equivalents of the Gibbs ensembles of equilibrium statistical mechanics. This novel approach led to the discovery of fundamental relations characterizing nonequilibrium transport in terms of microscopic chaos, such as formulas expressing transport coefficients in terms of Lyapunov exponents and dynamical entropies, equations relating the nonequilibrium entropy production to the fractality of SRB measures and fluctuation theorems, which are now widely studied as a fundamental property of nonequilibrium processes [2].

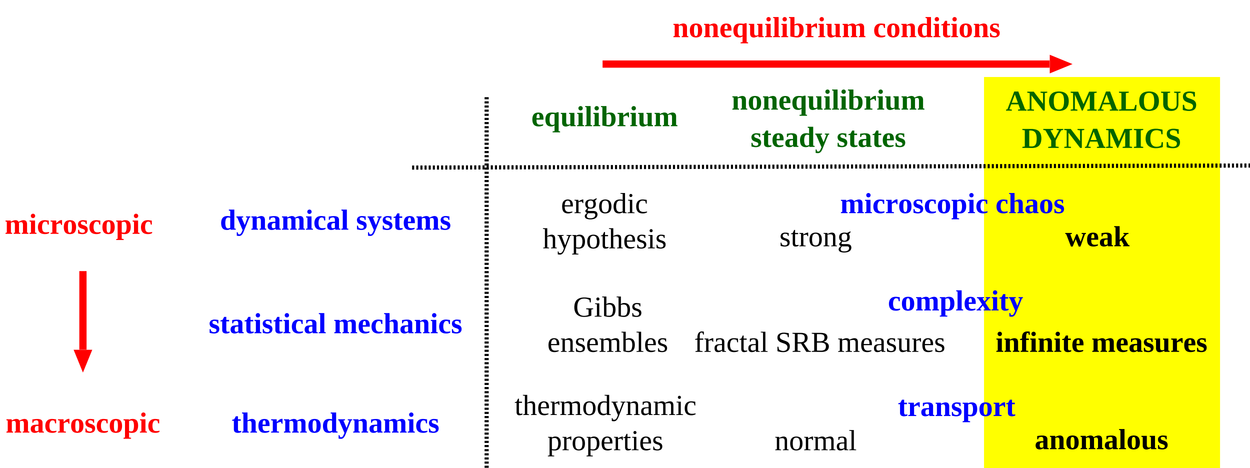

Figure 1: Conceptual foundations of a theory of nonequilibrium statistical physics based on dynamical systems theory, highlighting the emergence of a new field.

These findings are illustrated by the second column in the above figure, in analogy to the theory of equilibrium statistical mechanics. As is represented by the third column, however, more recently scientists learned that random-looking evolution in time and space also occurs under conditions that are weaker than requiring a positive Lyapunov exponent. It is now known that there is a wealth of systems exhibiting zero Lyapunov exponents, meaning that the separation of nearby trajectories is weaker than exponential. This class of dynamical systems is called *weakly chaotic*. Examples include transformations with indifferent fixed points, polygonal particle billiards, and Hamiltonian systems with sticky islands in phase space [2,3].

Theoretical physicists have discovered that weakly chaotic systems (and random walks driven by a mildly hyperbolic dynamics) exhibit *anomalous dynamics* characterized by novel properties such as ageing, which reflects an extremely weak (power-law) relaxation towards equilibrium involving more than one time scale in the decay of correlations. Other surprising properties are the non-equivalence of time and ensemble averages, called weak ergodicity breaking, and the existence of Lèvy-type probability distributions [3,4]. These physical phenomena were observed experimentally in a wide variety of systems, such as in the anomalous statistics of blinking quantum dots, in the anomalous diffusion of atoms in optical lattices, in plasma physics and even in cell and animal migration [3-5]. Long-range correlations have furthermore been reported from almost all complex real world signals, as is reminiscent in the development of the detrended fluctuation analysis of experimental time series [6]. These observations do not yet have any convincing theoretical explanation. Weak ergodicity breaking could provide a statistical framework for them, however, by itself it may not reveal the underlying mechanism.

On the other hand, recent work in mathematical ergodic theory has independently led to first rigorous results about some of the physically relevant phenomena mentioned above in a dynamics setup which goes beyond a purely probabilistic [7] study. In particular, the ergodic theory of skew products (generalized random walks) driven by (weakly) hyperbolic dynamical systems, e.g. [8], and of other infinite (non-normalizable) invariant measures [9] has the potential of providing a sound mathematical basis for some of them in form of what is called *infinite ergodic theory*.

In summary, the foundations of a novel theory of "strange" nonequilibrium processes are presently emerging. In contrast to standard nonequilibrium statistical mechanics, this dynamics is inherently non-stationary, due to the weak chaos by which it is generated. This mechanism leads to important physical consequences like anomalous transport, which can be tested in experiments. On the side of theoretical physics this approach asks for further generalizations of recently developed fundamental concepts, perhaps leading to a "weakly chaotic hypothesis", the identification of the physically relevant measures characterizing such systems, and to deriving experimentally measurable consequences such as generalizations of ordinary large deviation properties of Gallavotti-Cohen, Jarzynski, and Crooks-type. These questions also motivate further mathematical work in upcoming directions of infinite ergodic theory to provide a formal framework and rigorous results for parts of the physical theory.

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[8] D.Szasz, *Hard ball systems and the Lorentz gas* (Springer, Berlin, 2000)

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Scientific key topics are:

1. **Dynamical systems theory of weak chaos:** pseudochaos, nonhyperbolic dynamics, entropy concepts for weak chaos, zero Lyapunov exponent, polygonal billiards, weakly chaotic diffusion

2. **Infinite ergodic theory:** skew products (dynamically driven random walks), infinite invariant measures, distributional and functional limit theorems, mathematical billiards and Lorentz gas models, entropy, generalized ergodic theorems,

3. **Stochastic theory of anomalous dynamics:** continuous time random walks, anomalous diffusion, weak ergodicity breaking, fractional calculus, statistics of occupation times, anoma-

lous fluctuation relations

4. **Experimental applications:** anomalous statistics of blinking quantum dots, anomalous diffusion of atoms in optical lattices, anomalous biological dynamics, extreme events, detrended fluctuation analysis

2 What are we aiming at, and why is it timely?

In recent years it was realized by the physics community that weak chaos and anomalous dynamics cannot be described by using standard methods of statistical physics. Solving this problem by fusing different disciplines thus moved into the center of interest in current nonequilibrium statistical physics. However, maintaining the momentum of the development of such a highly interdisciplinary subject crucially depends on providing ample opportunities for scientific exchange across the fields involved in this endeavour.

More specifically, we are convinced that there is substantiated need for bringing together in particular two scientific communities, that of physicists working on both the deterministic and the stochastic aspects of weakly chaotic systems and anomalous dynamics, and that of mathematicians active in the relevant branches of dynamical systems and ergodic theory: Despite energetic research activities in either group, the significant overlap of their respective research interests has only recently become apparent (specific topics include weak ergodicity breaking, finer stochastic properties, and the proper definition and analysis of generalized observables, like entropies or Lyapunov exponents, quantifying weak chaos). This highlights the existence of a profound communication barrier between physicists and mathematicians.

The particular goal of this conference is therefore to bridge that gap, thereby providing an important and efficient stimulus for progress in either direction, through the exchange of alternative viewpoints, approaches, and techniques as well as motivations. While theoretical physicists already communicate with experimentalists working on anomalous dynamics, physicists eager to understand what mathematicians have already learned about the ergodic properties of such systems will value the opportunity to directly interrogate mathematical colleagues. We believe that such efforts will play an important role in shaping these emerging physical theories: For example, mathematicians have developed the profound concept of infinite invariant measures, where the measure underlying a dynamical system is non-normalizable. In the physics community such measures are often looked upon with suspicion, since a non-normalizable measure seems at first sight unphysical. Recently, however, a few examples of applications of infinite invariant measures have emerged in the physics community in the context of weak chaos. The mathematics of infinite invariant measures, on the other hand, reveals unexpected (and sometimes counterintuitive) inherent difficulties and pitfalls which should already be observed when aiming for consistent formulations of physically relevant concepts within mathematical models. Alas, so far, information on these matters is hardly accessible to non-specialists. Many mathematicians, in turn, regard other scientist's interest and questions, as well as additional information on the physics background, as being highly motivating for their own work. Our conference will thus explore the applicability of such mathematical concepts in physics and help to diffuse the ideas on infinite invariant measures and related infinite ergodic theory concepts among a larger community of scientists. We emphasize that the spreading of this knowledge has important implications beyond the theory of nonlinear dynamical systems: These topics are in fact at the heart of statistical physics, as is illustrated in our previous figure.

To foster communication between these different communities, we intend to bring together open-minded key players in both fields who have declared their interest in this kind of exchange, as well as a number of young scientists. We are confident that this mixture can initiate sustainable exchange and transfer of knowledge. This central goal is exemplified by the group of organizers: Zweimüller is a mathematician working on infinite ergodic theory, the other three organizers have a physics background. Klages is working at the interface between mathematical and physical aspects of dynamical systems theory, Barkai is an expert on stochastic anomalous dynamics, and Kantz has an international reputation for applying all this knowledge to the analysis of real world data.

3 The structure of the event

We propose to organize one week of workshop surrounded by two weeks of seminars. Each seminar day should feature no more than three talks per day to leave ample time for informal discussions. Topically, the guideline is to proceed from *ergodic theory* during week one and *dynamical systems theory of weak chaos* during week one and three to the *stochastic theory of anomalous dynamics* during week three. However, we emphasize that we are not planning to impose a strict separation between these three main topics, since the aim is to bring the different communities together rather than separating them! This will be accomplished by organizing a *series of advanced lectures*, to be given by selected experts in these three main fields: Saussol and Zweimüller will give introductions to infinite ergodic theory and applications of this theory in paradigmatic model systems; Artuso and Klages will summarize important concepts and results of dynamical systems theory related to weak chaos; and Sokolov and Barkai/Klafter will review fundamental methods of the stochastic theory of anomalous dynamics. These six lecture series should provide both reviews for researchers coming from these different fields and a training for young scientists. There should be about one to two lectures per seminar day supplemented by one to two short talks, the latter primarily to be given by young scientists.

While the seminars will serve for in-depth studies of fundamental concepts, the workshop week in-between should highlight the main theme of this event by bringing together world-leading experts from all three fields. Each workshop day should feature no more than about six hours of talks. Topically, the workshop should proceed along the same lines as the seminars. The main aim of the workshop will be to make scientists aware of important crosslinks between weak chaos, infinite ergodic theory, and anomalous dynamics, and to present the state of the art of research in the new field emerging from the fusion of these disciplines. Young researchers will be encouraged to present their research in form of short talks and in two poster sessions. There will furthermore be a special session in which each participant can annouce his/her poster presentation in a two-minute talk. We are planning to conclude the workshop with a roundtable discussion, for which we will invite a few experienced researchers to highlight what they consider as the most important results of this workshop, and to formulate important open problems for future research. We will also investigate the possibility to publish lecture notes for these talks, ideally such that they are accessible to a general public, thus serving as a general introduction to this new field of research for both physicists and mathematicians.

All of us have profound experience in successfully organizing conferences fulfilling the highest international standards. Previous conferences organized by us highlighted anomalous (Klages, Bad Honnef, 2006) and chaotic transport (Klages, MPIPKS, 2002), stochastic anomalous dynamics (Barkai, Jerusalem, 2008), ergodic theory (Zweimüller, Vienna, 2007) and extreme events (Kantz, MPIPKS, 2006, and Palma, 2008). We emphasize that the topic of the proposed conference will be a completely new endeavor for all of us and will bring together, for the very first time, very different communities of mathematicians and physicists. We finally remark that a somewhat related conference on “pseudochaos” took place in Autumn 2009 in Trieste. None of us was involved in the organization, however, one of us (Klages) participated as a keynote speaker. While highlighting the problem of understanding pseudochaotic dynamics in general, apart from Klages’ two talks this event did not address any crosslinks to infinite ergodic theory or the stochastic modeling of anomalous dynamics. To do so will be the purpose of our planned meeting, which therefore will be very different from the Trieste conference.