Studies of the threshold electron impact ionization of helium

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Abstract. Studies have been made of the two outgoing electrons in ionization events for which the energy difference $E$ between the total energy and the ionization energy is in the range from 0.2 to 3.0 eV. A coincidence time-of-flight technique has been used to measure their energy distribution and angular correlation functions, and a non-coincidence technique has been used to measure the partial cross section for the production of very slow electrons. The energy distribution function has been found to be uniform in the range of $E$ from 0.2 to 0.8 eV and the width of the angular correlation function has been found to increase with the energy $E$ in a way which is consistent with a $E^{4/3}$ dependence. The yield of very low energy electrons is consistent, in the range of $E$ from 0.2 to 1.7 eV, with a total ionization cross section having the energy dependence $E^n$, where $n = 1.131 \pm 0.019$. These results are all consistent with the Wannier–Peterkop–Rau theory.

1. Introduction

The energy dependence of the cross section for single ionization of neutral atoms by electrons, at energies $E_i$ near the ionization energy $U$, has usually been expressed in the form

$$\sigma_{\text{ion}} \propto E^n$$

where $E = E_i - U$. Various theoretical analyses have given values of $n$ which are either constant (in the range from 1.0 to 1.5) or which vary with $E$. Some of these analyses also make predictions about the probability distribution $P_\theta(\theta_{12})$ of angles $\theta_{12}$ between the two outgoing electrons, and the distribution $P_E(E_1)$ of their energies $E_1$ and $E_2$ (where $E_1 + E_2 = E$).

All the experimental tests which have been carried out so far have aimed at finding the value of the exponent $n$ by measuring the energy dependence of the ionization cross section. In the present paper we shall present the results of the first investigation into the distributions $P_\theta$ and $P_E$ very near the ionization threshold. We shall also present information about the exponent $n$ obtained from measurements of a partial ionization cross section.

The threshold ionization process has been treated theoretically in several different ways. In the well known work of Wannier (1953) (later extended by Vinkalns and Gailitis 1967) it is assumed that the electrons move classically when outside some inner reaction zone, and that the trajectories from the inner zone fill the available phase space with a smooth and nonsingular probability. The predictions of this classical theory are that $n = 1.127$, that $P_\theta(\theta_{12})$ has a maximum at $\theta_{12} = 180^\circ$ and a width which is $\sigma_{\text{ion}} \propto E^n$.
proportional to $E^{1/4}$, and that $P(E)$ is constant and independent of $E$ over the range of possible values from 0 to $E$. We shall see below that the present experimental results are all consistent with these predictions.

The earlier quantum mechanical treatments (Rudge and Seaton 1964, 1965) of the threshold ionization process showed that $n = 1$ and that $P_E$ is uniform. Later treatments (Temkin 1966, Temkin et al. 1968, Kang and Kerch 1970) have given values of $n$ from 1.0 to 1.5, but have not been specific about $P_0$ and $P_E$. In the most recent quantum mechanical treatment Rau (1971) has obtained the classical value 1.127 for the exponent $n$, and has also suggested that $P_E$ is uniform and that $P_0$ has a width which is proportional to $E^{1/4}$.

Another method has been to treat the electron motion semiclassically by using the WKB approximation (Peterkop and Liepins 1969, Peterkop 1971); this has given the Wannier exponent $n = 1.127$, but does not give information about $P_0$ and $P_E$. Methods in which classical trajectories are integrated numerically have also been used (Banks et al. 1969, Peterkop and Tsukerman 1969, 1970, Grujic 1972); these have shown that $n$ is approximately 1.127 and that $P_E$ is uniform, but have not yielded the form of $P_0$.

These theoretical methods can be summarized by saying (i) that when the energy distribution function $P_E$ has been studied it has been found to be uniform, (ii) that when the angular correlation function $P_0$ has been studied it has been found to have a maximum at $\theta_{12} = 180^\circ$ and to have a width which is proportional to $E^{1/4}$, (iii) that the predicted values of $n$ range from 1.0 to 1.5 and (iv) that the range of values of $E$ over which these predictions are valid is not known.

Experimental results tend to give values of $n$ which are near to 1.127 (e.g. McGowan and Clarke 1968, Brion and Thomas 1967, 1968, Krige et al. 1968, Marchand et al. 1969). In the experiments of Marchand et al. (1969), for example, $n$ has been found to be $1.16 \pm 0.03$ within 1 eV of threshold, and to decrease to $1.02 \pm 0.02$ at 12 eV above threshold. There have been no experimental measurements of the energy and angular distributions $P_E$ and $P_0$ near threshold, although Ehrhardt et al. (1972) have measured $P_0$ for particular values of $E$ and $E_1$ at higher energies (6 eV and more above the threshold).

As was mentioned above, three features of the threshold ionization process have been studied in the present work, namely the distributions $P_E$ and $P_0$ and the partial cross section for the production of very slow electrons. The first two features have been studied by a coincidence time-of-flight technique, and the third by a non-coincidence threshold technique. Each of these three measurements has its own energy range of validity. We shall start by describing the coincidence technique and the measurement of $P_E$, and will then proceed to discuss the measurements of $P_0$ and the partial ionization cross section.

2. The coincidence time-of-flight technique and the measurement of $P_E$

The apparatus which has been used is basically the same as that used previously for studying the threshold excitation spectra of bound states of He, Ar and N_2 (Cvejanović and Read 1974), although for the present set of measurements its geometry and mode of operation have been changed.

Figure 1 shows a schematic diagram of the target region and electron detectors. The target shield is cylindrical, with a radius of 1 cm and a length of 2 cm, and is constructed of gold plated copper mesh (with 80 lines per cm and 95% transparency) mounted on gold plated wire supports. The figure shows two holes (of diameter 1.4 mm)
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Figure 1. A section through the target chamber and detection systems. The shaded area represents the collision region. A is the shielding mesh, B are outer focusing electrodes, C are cylindrical plate deflector analysers, D are channel multiplier detectors, and E are their shields. A detailed description is given in the text.

in the mesh, through which scattered electrons can pass, and these holes are traversed by a few strands of very fine wire, in order to preserve the electrostatic shielding (this is the main difference from the apparatus as used previously, Cvejanović and Read 1974). The holes subtend half angles of 10° at the scattering centre. Outside each of the holes is an electrode (which is usually 2 or 3 volts higher in potential than the shielding mesh) in which is a hole of diameter 2.2 mm covered by a very transparent mesh of thin wires. These outer electrodes focus the emerging electrons onto electrostatic energy analysers which are designed to give a transmission efficiency which is large and insensitive to energy for low energy electrons, but which is small for photons and elastically scattered electrons. They have the form of single cylindrical plates of radius 1 cm and length 1.5 cm and are constructed from thin wires. Their semi-empirical mode of operation has been described previously (Cvejanović and Read 1974). The electrons are finally detected by channel electron multipliers. The time differences between the detection of electrons in the two multipliers are measured by a time to pulse height converter, and the spectrum of these time differences is recorded and stored by a multichannel pulse height analyser.

The incident electron beam enters and leaves the target region through holes in the flat end plates of the cylindrical mesh (that is, normal to the plane of figure 1). The energy width of this beam is controlled by an electrostatic hemispherical energy selector of the same general design and size as that described by Comer and Read (1971), but with a redesigned lens system incorporating triple cylinder asymmetric voltage lenses (Adams and Read 1972) and ‘zero-gap’ triple aperture lenses (Read 1971). This selector produces a beam of $5 \times 10^{-9}$ amp with an energy spread of 40 meV (fwhm) and a diameter which is less than 1 mm.

The target gas is in the form of an atomic helium beam which issues from a gold plated platinum–iridium hypodermic needle of internal diameter 0.3 mm projecting through a hole in one side of the cylindrical target mesh. The needle can be heated to improve the energy resolution of the system (as was done in a similar apparatus described
by Joyez et al 1973), and the end of the needle projects to within 2 mm of the electron beam.

Let us suppose that in the reaction
\[ e + \text{He} \rightarrow \text{He}^+ + e + e \]  
the two outgoing electrons have energies \( E_1 \) and \( E_2 \) respectively, and velocities \( v_1 \) and \( v_2 \) respectively. Suppose also that these two electrons travel in opposite directions towards the exit holes in the shielding mesh. If \( l \) is the radius of the shield (1 cm in the present work) the flight times of the electrons inside the shielded region are \( l/v_1 \) and \( l/v_2 \) respectively, and the difference in the times of arrival at the exit holes is
\[ t = \frac{l}{v_1} - \frac{l}{v_2}. \] (3)

Having passed through the shielding grid the electrons are quickly accelerated, analysed and detected, and the time difference contributed at this stage is small compared with that given by equation (3), provided that \( E_1 \) and \( E_2 \) are sufficiently small. The time difference between the detection of the two electrons is therefore given by equation (3), and if the combined energy \( E \) of the two electrons is known (as it is in the present experiments) the individual energies \( E_1 \) and \( E_2 \) can be immediately deduced from the measured value of \( t \). This is the principle of the present coincidence time-of-flight technique for measuring the distribution of energies \( E_1 \).

There are many experimental difficulties which must be considered, but before discussing these it is convenient to develop the method of analysing the data. The primary data is in the form of spectra of time differences \( t \), for various known total energies \( E \), and these time distributions \( P_T(t) \) must be converted into energy probability distributions \( P_E(E_1) \). To do this it is convenient to use the dimensionless variables
\[ \epsilon = \frac{E_1}{E} \] (4)
and
\[ \tau = \frac{t}{t_0} \] (5)
where
\[ t_0 = l \left( \frac{2E}{m} \right)^{1/2}. \] (6)

Then
\[ \tau = \epsilon^{-1/2} - (1 - \epsilon)^{-1/2} \] (7)
and
\[ P_E(\epsilon) = \frac{d\tau}{d\epsilon} P_T(\tau) = \frac{1}{2} (\epsilon^{-3/2} + (1 - \epsilon)^{-3/2}) P_T(\tau). \] (8)

We shall use this expression to obtain \( P_E \) from the measured distributions \( P_T \).

This type of experiment and analysis are valid only if certain conditions are satisfied, and we shall see that these conditions impose restrictions on the ranges of energies which can be studied. Firstly the electrons must travel in a field-free region of length \( l \). This is obviously not exactly true because there may be patch fields caused by variations of the surface potential of the shielding grid, because the hypodermic needle from which the atomic beam issues may have a surface potential different from that of the shielding
grid, and because outside fields penetrate slightly through the shielding wires at the exit holes in the grid. For these reasons we consider that our results would be invalid if the energy $E_1$ were too low (less than about 0.05 eV). The second condition which must be satisfied is that the efficiency of detection of the electrons must be independent of their energy, which means that the analysers shown in figure 1 must have a poor energy resolution. The characteristics of these analysers have been described previously (Cvejanović and Read 1974). For the present purpose they have been adjusted to give a transmission and detection efficiency which is approximately constant over the widest possible energy range. The actual range depends on the minimum value of $E_1$ which one wishes to observe. For example if this minimum value is 0.1 eV the range extends to 0.7 eV, and this setting is suitable for use when the combined energy $E$ is about 0.8 eV. Higher values of $E$ are not accessible because the width of the range of constant efficiency cannot be extended, but lower values of $E$ can be studied by using a range having lower end points. However, since the lower limit to the measured value of $E_1$ is 0.05 eV, for the reasons given previously, the lower limit to the combined energy is about 0.2 eV. The third and last condition which must be satisfied is that the differences in flight times outside the shielded region must be small compared with the differences $t$ between the free flight times inside this region. In fact it can be calculated that the ratio of the added time difference to the difference $t$ is approximately independent of $E_1/E$, but decreases with $E$ approximately as $E^{-1.3}$. When $E$ is 0.4 eV the value of the ratio is approximately 0.04. It has been possible (see below) to allow for this added time difference in the analysis of the results.

We may summarize these considerations by saying that the experimental data and the proposed method of analysis are valid if the combined energy $E$ lies between about 0.2 and 0.8 eV, and if the individual energies $E_1$ and $E_2$ lie between about 0.05 and 0.7 eV.

Figure 2 shows two time distribution spectra, for energies $E$ lying well within the range of validity of the experiment. Typical counting rates for such spectra are about one true coincidence per second. The background of random coincidences from unrelated events is typically about 10 per second. Typical flight times range from 20 ns (for 0.7 eV electrons) to 75 ns (for 0.05 eV electrons), and typical delayed coincidence times $t$ range from 0 to about 50 ns, as can be seen in the figure.

The curves going through the spectra of figure 2 have been computed from equation (8), with the assumption that $P_E$ is uniform. These curves have been fitted to the data by a least squares analysis, the only variables being the position of the centre, the height of the peak and the height of the constant background of random coincidences. Before fitting the curve given by equation (8) it has been properly averaged and convoluted to allow for (i) the finite energy spread (40 meV) of the incident electron beam, (ii) the random error (3 ns) in measuring the time differences, (iii) the finite size (0.5 mm radius) of the collision region and (iv) the small time differences introduced by the analysers. None of these effects change $P_T$ appreciably.

It can be seen that the calculated curves fit the data well, showing that the assumption of a uniform energy distribution $P_E$ is justified. To obtain further information about $P_E$ we have divided the observed coincidence yield (after subtracting the background of random coincidences) by the calculated function $P_T$, over time intervals corresponding to given energy intervals in $E_1$. The results of doing this are shown in the right hand part of figure 2. We have not plotted points corresponding to small values of $E_1$ or $E_2$, because these would be outside the range of validity. It can be seen that there is no obvious deviation from a uniform $P_E$. This same conclusion can be drawn from other
measurements which we have made at other values of $E$ in the range from 0.2 to 0.8 eV, where we have found that $P_E$ is constant to within about 15%. We have looked carefully to see if $P_E(E_i)$ might have maxima close to $E_i/E = 0$ and 1, and a minimum at $E_i/E = 0.5$, since this is the known form of $P_E$ at higher energies, but have found no systematic evidence for this for values of $E$ less than 0.8 eV. There are indications that $P_E$ starts to become nonuniform when $E$ is greater than about 2 eV, but for the reasons stated above the present technique is not suitable for measuring the form of the energy probability function at these higher energies.

Before leaving the subject of the energy distribution function, it should be noted that the time spectra of figure 2 contain only coincidences from ionization electrons, and do not contain any measurable contributions from other types of coincidences. This has been established by showing that the yield of coincidence counts is zero when the incident energy is less than the ionization energy. In preliminary experiments coincidences were observed between inelastic electrons and photoelectrons ejected by decay photons, but these have been eliminated in the present arrangement by positioning of the channel electron multipliers as shown in figure 1 and by constructing the cylindrical plate deflectors from transparent grids of thin wires.

3. Measurements of the angular correlation function $P_\theta$

In the measurements described in the previous section the angle $\theta_{12}$ between the two outgoing electrons was in the range from 160 to 200°, and the mean directions of the outgoing electrons, the incident beam and the atomic beam were mutually at right angles. At energies near the ionization threshold one expects the partial differential
ionization cross sections to depend only on the angle $\theta_{12}$, and not on the individual angles $(\theta_1, \phi_1)$ and $(\theta_2, \phi_2)$. In this case the angular correlation function $P_\theta(\theta_{12})$ can be measured by changing the direction of observation of either of the two detectors, in any plane. We have assumed this to be true for the energies used in the present experiments. To be exact one should take into account the dynamical effects caused by the motion of the centre of mass of the system, but although the electrons which we observe have a low energy their speeds are nevertheless much faster than that of the centre of mass and so these effects are negligible.

In fact information about $P_\theta$ has been obtained by constructing and using a different shielding mesh having an extra exit hole for one of the analysers and detectors, and by making this analyser and detector system rotatable. In this way the range of angles $\theta_{12}$ was made $180^\circ \pm 20^\circ$ for one position of the detector and $150^\circ \pm 20^\circ$ for the other position, the direction of this second position being at right angles to the incident electron beam, and in the plane containing the atomic beam and the direction of the first detector position. It was not possible to use more than two values of $\theta_{12}$, nor to make $\theta_{12}$ continuously variable, because of the difficulty of maintaining a field-free region inside the shielding mesh, and because the rotatable detector system had a limited freedom of movement. We have therefore only been able to measure the ratio $R$ of the coincidence yield at these two values of $\theta_{12}$.

To measure the ratio $R$ at various fixed total energies $E$ of the two electrons we have measured the areas under time spectra such as those of figure 2 (with the background of random coincidences subtracted) at the two values of $\theta_{12}$. This has been done for 5 values of $E$ from 0.2 to 3.0 eV. These measurements are valid at higher energies than those described in the previous section because although the time spectra are attenuated and distorted at the higher energies the effects should be independent of the angle $\theta_{12}$ and the ratio of areas should be unaffected, provided that the combined energy is not greater than about 3 eV.

An obvious problem which might exist in these measurements is that the detection efficiency might depend on the detector position. To take account of this two different methods were used to normalize the coincidence counting rate. In the first of these the product of the strength of the incident current, the collection time and the pressure in the atomic beam were used to normalize the measured yields at the two different angles. The second method of normalization, which could be used for the point of highest energy, was less direct but more accurate. It is known (eg Ehrhardt et al 1968, Sanche and Schulz 1972) that resonances exist at 22.5 eV and higher in elastic and inelastic electron helium scattering, and they have been seen with the present apparatus in the yield of low energy electrons. The observed strength of these resonance structures therefore depends on the product of incident beam current, collection time, gas beam density and detection efficiency, and can be used to normalize the coincidence data at the two detector positions. In fact the two methods of normalization agreed to within 5%.

The upper part of figure 3 shows the ratio $R$ of the yield at $150^\circ$ to the yield at $180^\circ$. This ratio was measured more than once at each of the five values of $E$, and the errors shown have been estimated empirically from the observed spread of values at each point. These spreads of values are caused both by possible errors in retuning the analysers at the different values of $E$ and by the low counting rates and high random counting errors, although it would seem from figure 3 that the estimates of the errors may be too small. The statistical accuracy of this series of observations has been insufficient to allow the form of $P_\theta$ to be determined at the second value ($150^\circ$) of $\theta_{12}$, but we have assumed in interpreting the data that $P_\theta$ is the same in both orientations.
We have converted the measured values of $R$ into a more comprehensible form by considering the full width at half maximum, $\theta_{1/2}$, of the function $P_\theta$. This has been done by assuming that the function is Gaussian in shape and by calculating the ratios $R$ which would be observed (by averaging over the ranges of $\theta_{1/2}$ at the two detector positions) as a function of $\theta_{1/2}$. The resulting values of $\theta_{1/2}$ corresponding to the observed values of $R$ are shown in the lower part of figure 3. The curve going through these points is of the form

$$\theta_{1/2} = aE^{1/4} \quad (9)$$

and, although the experimental errors are rather large, it can be seen that the observations are not inconsistent with this form of energy dependence, although other forms are clearly possible. A least squares analysis shows that for a power law dependence $E^b$, the exponent $b$ has the value $0.19 \pm 0.07$. When the exponent is 0.25 the constant $a$ of equation (9) has the value $39 \pm 3$, if $\theta_{1/2}$ is measured in degrees and $E$ in eV. The gaussian form of $P_\theta$ was chosen because it is implied by the work of Rau (1971), and may be more general. The assumption of a triangular shape still leads however to the same conclusion that the observations are consistent with a one quarter power law.

4. Measurement of a partial ionization cross section and the exponent $n$

Finally we have carried out a non-coincidence experiment to obtain information about the exponent $n$. The counting rates in this series of measurements are much higher than in the two series described above, and consequently the result is more precise.
For these measurements the apparatus was modified and used in the way described previously (Cvejanović and Read 1974). The necessary modifications are (i) only one analyser and detector is used, (ii) the shielding wires across the exit holes in the shielding mesh are removed to allow a small electrostatic field to penetrate into the scattering region and (iii) the analysers are tuned to the 'negative energy' mode in which they have the highest transmission efficiency for very low energy electrons. With these modifications the detection system has its highest sensitivity for electrons of energy less than about 15 meV, and a much reduced sensitivity for energies above about 50 meV. This is because the combined effects of the penetrating electrostatic field and the 'negative energy' mode of the analyser cause an enhancement of the collection and detection efficiencies of slow electrons and a rejection of faster electrons. The slowest electrons are collected with a high efficiency over the $2\pi$ solid angle on that side of the target chamber (see figure 1) which is covered by the active detector.

In this mode of operation of the apparatus the measured yield is proportional to a partial ionization cross section defined as

$$\sigma_{\text{partial}}(E) = 2 \int \frac{d\sigma_{\text{ion}}(E)}{dE_1} f(E_1) \, dE_1$$

(10)

where $f(E_1)$ is the detection efficiency for electrons of energy $E_1$. The factor 2 arises because there are two outgoing electrons, either of which may be detected. Figure 4 illustrates this partial ionization cross section. The total ionization cross section is proportional to the total area shown, but the only part of this which is detected is that represented by the shaded area. The efficiency $f$ is independent of the energy $E$, and it has a constant mean width $\Delta E$ (actually about 15 meV). Further details of this method of detecting slow electrons can be found in the previous publication (Cvejanović and Read 1974).

If the energy distribution function $P_E$ is constant, as suggested by the results described in §2 above and as predicted by all the theoretical work in which this point has been considered, then $d\sigma/dE_1$ is uniform, as drawn in figure 4. In this case the measured
partial ionization cross section is given by
\[ \sigma_{\text{partial}}(E) \propto \frac{\Delta E}{E} \sigma_{\text{total}} \propto E^{-1} \]
provided that \( E_m \) is less than \( E \).

Figure 5 shows the yield of low energy electrons obtained in this way. Below the ionization threshold low energy inelastically scattered electrons are detected and peaks appear at the thresholds of excited states of helium; these features have been discussed by Cvejanović and Read (1974). Above the ionization threshold the yield is proportional to the partial ionization cross section defined above; the curve which has been drawn through these points is proportional to \( E^{-0.127} \).

![Figure 5. A spectrum showing the measured yield of very low energy (\( \leq 50 \text{ meV} \)) electrons as a function of the incident energy. The curve drawn through the points above the ionization threshold is proportional to \( E^{-0.127} \), where \( E \) is the energy excess above the ionization energy.](image)

Clearly a 0.127 power law fits this data very well, but to find the range of possible exponents we have used a least squares method to find the exponent \( m \), and have found that it is given by \( m = 0.121 \pm 0.017 \). The mean of this and other measurements gives
\[ m = 0.131 \pm 0.019. \]

The quoted error includes contributions from (i) the random statistical errors, (ii) the uncertainty in the level of the background counting rate, (iii) the uncertainty in the energy scale and (iv) the uncertainties which arise from fitting over different energy ranges. The latter three systematic errors provide the greater part of the uncertainty in \( m \).

The lower and upper limits of the energy range over which we have fitted a power law to the data are 0.2 and 1.7 eV respectively. It was not possible to use the data
nearer than 0.2 eV to threshold because (i) the base width $E_m$ of the detection efficiency curve is not known sufficiently well, (ii) the yield in this energy region would be affected by the energy width of the incident beam and (iii) the excitation functions of the highly excited states of helium might spill over into this region of energy. The high energy limit of about 1.7 eV exists because the incident current and the background counting rate cannot be kept constant over wide ranges of the incident energy. We have not found any systematic dependence of the exponent $m$ on the range of energies used, within the limits 0.2 to 1.7 eV.

5. Discussion of results

The three series of measurements which have been described have shown (i) that the energy distribution $P_E$ is uniform to within about 15% for values of $E$ from 0.2 to 0.8 eV, (ii) that the variation of the angular correlation $P_\theta$ with $E$ is consistent with a one quarter power law for the angular width, in the range of $E$ from 0.2 to 3.0 eV, and finally (iii) that the partial cross section for the production of very slow electrons can be fitted by a power law having the exponent $0.131 \pm 0.019$, in the range of $E$ from 0.2 to 1.7 eV.

The observation of a uniform function $P_E$ is in agreement with all extant theories, and if it can be assumed that $P_E$ is in fact exactly uniform at these low energies then the measured energy dependence of the partial cross section implies (equation (11)) a total cross section which increases as the power $1.131 \pm 0.019$. This is therefore consistent with the power law 1.127 obtained by Wannier (1953), Peterkop (1971) and Rau (1971). It should be noted that we have managed to measure this exponent $n$ accurately by obtaining the value of $(n-1)$, rather than $n$ itself as in earlier experiments (eg McGowan and Clarke 1968, Brion and Thomas 1967, 1968, Krige et al 1968, Marchand et al 1969). The previous measurements of $(n-1)$ by the SF$_6$ scavenger technique (eg Brion and Olsen 1970), were invalidated by the dominance of chemi-ionizing collisions (see Cvejano-vić and Read 1974 for a discussion of this point).

If $P_E$ is not uniform equation (11) would be incorrect and the exponent $n$ could not be derived from the present measurements. The results which we have obtained do not therefore definitely confirm the predictions of Wannier (1953), Peterkop (1971) and Rau (1971), although the present measurements of $P_E$ and $m$ are certainly consistent with these theories. The value of $n$ which we have derived is also consistent with that obtained in the previous experimental work quoted above.

The measurements of the energy dependence of the width of the angular correlation function $P_\theta$ are also consistent with the theory of Wannier (as extended by Vinkalns and Gailitis 1967) and that of Rau (1971). In the former case the value of the proportionality constant $a$ in equation (9) has not been given, but in the latter case it has the value $2^{3/2} 15^{-1/4} \ln(2)^{1/2}$ in atomic units, or $84.9 \text{deg eV}^{-1/4}$, which is greater by a factor $2.2 \pm 0.2$ than the value which has been obtained from the present measurements. Perhaps our measured magnitude for this constant may provide a crucial test of the various theories.

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